P.R. GOVERNMENT COLLEGE

KAKINADA

An Autonomous Institution & NAAC Accredited with "A" Grade with CGPA 3.17

BOARD OF STUDIES

2021-2022

MATHEMATICS

DEPARTMENT OF MATHEMATICS

P.R. GOVERNMENT COLLEGE (A), KAKINADA

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P. R. GOVERNMENT COLLEGE (A), KAKINADA, E. G. Dt.

Department of Mathematics

The Board of Studies meeting for **Mathematics** subject during the academic year 2021-2022 is conducted at the Dept. of Mathematics on 12.11.2021 at 10:00 AM with Sri.G.Moses, Lecturer In-charge in Mathematics & Statistics the chair along with the following members.

…………………………………………………………………………………………………

Name with Designation and Address **Signature(s)**

…………………………………………………..……………………………………

1. Chair Person

Sri. G.Moses
I/C of Mathematics and Statistics
P. R. Govt. College (A), Kakinada. Chair Person

2. University Nominee

Dr. V.Ananatha Lakshmi
Principal
A.S.D.Govt degree college for (w),(A) University Nominee
Kakinada

3. Members Nominated by Executive Committee of the College

i. Dr. P. Subhashini,
Principal
Government Degree College, Subject expert
Pithapuram

ii. Sri. K. Chittibabu,
Lecturer in Mathematics,
Government Degree College, Subject expert
Ramachandrapuram.

4. From Alumni

Sri. P. S. R. Subrahmanyam,
Rtd. HOD of Mathematics, Alumni Member
Ideal College of Arts & Science (A),
Kakinada

5 Members from the College

Faculty Members:

1) G .Shyam Prasad Reddy Contract Faculty
2) G. Prasada Rao Contract Faculty
3) K.S.I.Priyadarshini Contract Faculty
4) L.S.B.R.Bhanu Contract Faculty
5) K. Samrajyam Contract Faculty
6) V. Haripriya Guest Faculty
7) A. Geetha Sowjanya Guest Faculty

Student Members:

a. N. Bala Gangadhara varma (III MPCS)
b. S.Bhuvaneswari (III MECS)
c. G.Suryapriya (III-MCAC)
d. V.Vijaymouli (IMCAC)

Dr.B.V.Tirupanyam

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

BOARD OF STUDIES MEETING IN MATHEMATICS ON 12.11.2021 at 10:00AM

Agenda:

1. **Revamping Of Syllabus For I,III &V Semesters.**
2. **Model Question Papers And Blue Print**
3. **Panel Of Question Paper Setters And Examinars**
4. **Pass Minimum In Internal Assessment**
5. **Choice Based Credit System For I,II &III Year Students**
6. **Introducing Of New Courses Of Study And The Possibilities .**
7. **Admission Criteria For Programmes Offered By The Departments .**
8. **Proposals For Community Service /Extension Activities/ Projects For The Benefit Of The Society.**
9. **Any Other Proposal /Item With The Permission Of The Chair**

Resolutions taken :

The following resolutions are approved by university nominee and all the members of BOS

1. **It is resolved to revamping of syllabus for I,III and V semesters**
2. **It is resolved to follow the existing model question papers and blue print of papers.**
3. **It is resolved to follow the modified panel of question paper, setters and examiners.**
4. **It is resolved to follow pass minimum in internal assessment as per norms**
5. **It is resolved to follow Choice Based Credit System for I,II & III year students .**
6. **It is resolved to introduce new courses of study whenever necessary.**
7. **It is resolved to follow the admission criteria for the programmes offered by the department**
8. **It is resolved to conduct extension lectures by the eminent persons.**
9. **It is resolved to**

   Student Project / Assignment - 10 marks (**Assignment) /** Final year **V SEM (Project** all students)

Blue Print of C.B.C.S. Model Curriculum in B.Sc. Mathematics

Yr.	Course & Theory / Lab	Paper	Title	Workload Hrs / Week	Credits	Max. Marks Intrnl	Extrnl	Total
I	Sem I	I	Differential Equations	6 Hrs	5	50	50	100
	Sem II	II	Three Dimensional Analytical Solid Geometry	6 Hrs	5	50	50	100
II	Sem III	III	Abstract Algebra	6 Hrs	5	40	60	100
	Sem. Students	Life Skill Course:	Analytical Skills	2 Hrs	2	-	50	50
	Sem IV	IV	Real Analysis	6 Hrs	5	40	60	100
		V	Linear Algebra	6 Hrs	5	40	60	100
III	Sem V	V	Ring Theory & Vector Calculus	5 Hrs	5	40	60	100
		VI	Linear Algebra	5 Hrs	5	40	60	100
	Sem VI	VII	**Elective (any one)*** A. Laplace-Transformations B.Numerical Analysis C. Number Theory D. Graph Theory	5 Hrs	5	40	60	100
		VIII A	1.Integral Transformations 2. Special Functions 3.Project	5 Hrs (for each paper)	5	40	60	100
		VIII B	1.Advanced Numerical Analysis 2. Special Functions 3.Project	5 Hrs (for each paper)	5	40	60	100
		VIII C	1.Principles of Mechanics 2. Fluid Mechanics 3. Project	5 Hrs (for each paper)	5	40	60	100
		VIII D	1.Applied Graph Theory 2.Discrete Mathematics 3.Project	5 Hrs (for each paper)	5	40	60	100
	Project		Project work	5 Hrs	5			100

Total number of hours for each paper in the academic year 2021-2022:

Paper I & II : 180 Hrs

Paper III & IV : 270 Hrs

Paper V & VI : 150 Hrs

Paper VII : 75 Hrs

Paper VIII (for each paper) : 75 Hrs

Project : 75 Hrs

Analytical Skills (F.C) : 30 Hrs

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

Objectives of the Department

- To impart knowledge on various Mathematical concepts like Differential Equations, Solid Geometry, Group Theory, Real Analysis, Ring Theory and Vector Calculus, Linear Algebra, Numerical Analysis and Special Functions.
- To equip our students with good quality to appear for competitive examinations.
- To make the students to understand the needs of Mathematics in Science and Technology.
- To inculcate research atmosphere among students by assigning projects.

The Department of Mathematics is offering B.Sc. courses involving mathematics (10 courses), B.Sc. Professional (B. Voc.) for undergraduate courses.

PROGRAMME OUTCOMES

For every degree program expectations are listed out by the institution under the Program Outcomes. For all Degree Streams the following are set as Programme Outcomes.

PO 1. Knowledge and Understanding of:

1. All concepts in under graduate level.
2. Real life applications of these concepts and relationship between them.

PO 2. Intellectual skills – be able to:

1. Think logically and arrange real life situations to mathematical problems.
2. Assimilate knowledge and ideas based on wide reading and through the internet.
3. Transfer of appropriate knowledge and methods from one topic to another within the subject.
4. Understand the evolving state of knowledge in a rapidly developing field.

PO 3. Transferable skills:

1. Use of IT (word-processing, use of internet for doing project).
2. Ability to work as part of a team.
3. Ability to use library resources/Equipment.
4. Time management.

PO 4. Problem analysis:

1. Conversion of real life problem to Mathematical problem to get solution.
2. Conduct investigations of complex problems: Use research-based knowledge.

PO 5. Ethics:

1. Apply ethical principles, commit environment and responsibilities among students.

PO 6. Individual and team work:

1. Function effectively as an individual and as a member in diverse teams and in multidisciplinary settings.

PO 7. Communication:

1. Communicate effectively on complex group activities and with society at large. Speak, read, write and listen clearly in person and through electronic media in English.

PO 8. Critical Thinking:

1. Take informed actions after identifying the assumptions that frame our thinking and actions, checking out the degree to which these assumptions are accurate and valid, and looking at our ideas and decisions (intellectual, organizational, and personal) from different perspectives.

PO 9. Effective Citizenship:

1. Demonstrate empathetic social concern and equity centred national development, and the ability to act with an informed awareness of issues and participate in civic life through volunteering.

PO 10. Life-long learning:

1. Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.
2. Able to acquire domain knowledge for self-employment and vertical mobility.
3. Able to acquire skills to serve the community
4. Able to gain the concept of a sustainable environment.

Programme Specific Outcomes of Mathematics Stream Courses

PROGRAMME	Program Specific Outcomes
MPC	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Physics and Chemistry
	PSO 2: To identify and understand the theoretical concepts of physical and chemical properties of materials and the role of mathematics in dealing with them in a quantitative way.
	PSO 3: To learn problem solving techniques related to Mathematics, Physics and Chemistry
	PSO 4: To gain insights procedures of safe handling of Chemicals and Equipments.
	PSO 5: To carry out hands on experiments and to analyze results.
MPE	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Physics and Electronics.
	PSO 2: To identify and understand the mechanism behind various electronic and physical systems and quantify them with mathematical tools.
	PSO 3: To learn problem solving techniques related to Mathematics, Physics and Electronics.

	PSO 4: To gain skills needed to handle the instruments and design circuits with analysis of results.
MPCs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Physics and Computer Science.
	PSO 2: To identify and understand the concepts of Mathematics, Physics and Computer Science and then relate them in numerical programming of physical system models.
	PSO 3: To learn problem solving techniques related to Mathematics, Physics and Computer Science.
	PSO 4: To gain skills required to develop programming techniques and implementation of numerical algorithms by using various programming languages.
MCPC	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Chemistry and Petro Chemicals.
	PSO 2: To identify and understand the theoretical concepts of Mathematics and Chemistry and utilize them in Petro Chemicals.
	PSO 3: To examine the Mathematical Modelling and Chemical procedures in the field of Petro chemicals.
	PSO 4: To get the employability skills in chemical industries as well as petro chemical industries.
MECs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Electronics and Computer Science.
	PSO 2: To identify and understand the concepts of Mathematics and Computer Science and utilize them in numerical programming of electronical system models.
	PSO 3: To gain insights to design circuits and provide mathematical modelling.
	PSO 4: To design circuits and understand the variations by simulation.
MCCs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Chemistry and Computer Science.
	PSO 2: To analyse the concepts of Mathematics, Chemistry and Computer Science and identify the relation among them like deriving the equations in chemistry, mathematical modelling of chemistry.
	PSO 3: To carry out problem solving and to demonstrate the real life applications of Mathematics and Chemistry in Computer Science.
	PSO 4: To gain insights procedures of safe handling of Chemicals and Equipments.
MSCs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Statistics and Computer Science.
	PSO 2: To identify and analyse the concepts of mathematics, statistics and computers science and then to find their applications in different areas like physical sciences, life sciences, various industries, etc.
	PSO 3: To solve various real life problems by developing mathematical model and applying various statistical tools with the help of computer programming knowledge.
	PSO 4: To develop thinking about research to solve critical problems.
MSAs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Statistics and Actuarial Science.
	PSO 2: To identify and analyse the concepts of mathematics, statistics and Actuarial science and then to find their applications in different areas like physical sciences, life sciences, various industries,

	Insurance, etc.
	PSO 3: To solve various real life problems by developing mathematical model and applying various statistical tools with the help of suitable economic, finance and risk policies.
	PSO 4: To acquire the skill of collection of data, analyzing it and to give conclusions
	PSO 5: To develop thinking about research to solve critical problems.
MCAc	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Chemistry and Analytical Chemistry.
	PSO 2: To identify and understand the concepts of Mathematics, Chemistry and Analytical chemistry and then to understand the relation among them like mathematical modelling of chemistry and derivation of chemical equations.
	PSO 3: To gain insights procedures of safe handling of Chemicals and Equipments
	PSO 4: To get the employability skills especially chemical industries.
ME.IOT	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Electronics and Internet of thinking.
	PSO 2: To identify and understand the concepts of Mathematics and Electronics and utilize them in Internet programming system models.
	PSO 3: To gain insights to design networking.

Courses (Papers) offered under B.Sc. Mathematics Stream

S. No.	Sem. No.	Domain Specific course/Clusters	Title
1	I	General Core	**Differential Equations**
2	II	General Core	**Three Dimensional Analytical Solid Geometry**
3	III	General Core	**Abstract Algebra**
4	IV	General Core	**Real Analysis**
5		General Core	**Linear Algebra**
6	V	General Core	**Ring Theory & Vector Calculus**
7		General Core	**Linear Algebra**
8	VI	Elective B	**Numerical Analysis**
9		Cluster Elective B1	**Advanced Numerical Analysis**
		Cluster Elective B2	**Special Functions**
		Project	**Related to mathematics / Interdisciplinary involved with mathematics**

MATHEMATICS COURSE OUTCOMES

Year	Semester	Title of the Paper	Course Outcomes
I	I	Differential Equations	CO 1. Solve linear differential equations CO 2. Convert non exact homogeneous equations to exact differential equations by using integrating factors. CO 3. Know the methods of finding solutions of differential equations of the first order but not of the First degree. CO 4. Solve higher-order linear differential equations, both homogeneous and non homogeneous, with constant coefficients. 5. Understand the concept and apply appropriate methods for solving differential equations.
	II	Three Dimensional Analytical Solid Geometry	CO 1. Get the knowledge of planes. CO 2. Basic idea of lines, sphere and cones. CO 3. Understand the properties of planes, lines, spheres and cones. CO 4**.** Express the problems geometrically and then to get the solution.
II	III	Abstract Algebra	CO 1. To analyse the abstract algebraic concept Group theory. CO 2. To understand the concepts in group theory like groups, subgroups, normal subgroups, permutation groups and cyclic groups with examples. CO 3. To understand the theorems on these concepts and also to solve problems on it. CO 4. To analyse and understand the applications of group theory in various fields.
	IV	Real Analysis-Paper-IV	CO 1. To get clear idea about the real numbers and real valued functions. CO 2. To obtain the skills of analyzing the concepts and applying appropriate methods for testing converges of a sequence or series. CO 3. To analyse the concepts of continuity, differentiability and Riemann integrability of a function and also to gain the skills about how to test these conditions of functions defined on the subsets of the real line. CO 4. To know the Geometrical interpretation of mean value theorems.
		Linear Algebra – Paper-v	CO 1. To understand the different concepts of linear algebra. CO 2. To analyse the concepts of vector space, subspace and homomorphism between them. CO 3. To understand how to solve the system of linear equations and this concept used in balancing of chemical equations.

			CO 4. To analyse the concepts of eigen values, inner product spaces and orthogonality and also gain the problem solving ability on them.
III	V	Ring Theory and Vector Calculus	CO 1. To understand the ring theoretic concepts with the help of knowledge in group theory and to prove the theorems on it. CO 2. To understand the applications of ring theory in various fields. CO 3. To get knowledge of vector differentiation, differential operators, line, surface and volume integrals. CO 4. To analyse the concepts of derivation and integration of vectors in rectangular, cylindrical and spherical coordinate systems.
		Linear Algebra	CO 1. To understand the different concepts of linear algebra. CO 2. To analyse the concepts of vector space, subspace and homomorphism between them. CO 3. To understand how to solve the system of linear equations and this concept used in balancing of chemical equations. CO 4. To analyse the concepts of eigen values, inner product spaces and orthogonality and also gain the problem solving ability on them.
	VI	Numerical Analysis and Advanced Numerical Analysis	CO 1. To analyse the error incumbent in any such numerical approximation and to understand different concepts in Numerical Analysis. CO 2. To gain the procedure of finding the solution of equations and derivation of interpolation formulae. CO 3. To understand the concepts curve fitting, solving system of linear equations and solving ordinary differential equations. CO 4. Compare the possibility of different approaches to the numerical solution of problems arising in roots of solution of non-linear equations, interpolation and approximation, numerical differentiation and integration, solution of linear systems.
		Special Functions	CO 1. To obtain Hermite polynomial, Laguerre polynomial, Legendre's polynomial and Bessel's functions by solving the concerned differential equations and to solve problems on these functions. CO 2. To know the concepts of Beta and Gamma functions, the relation between theorems and solving the problems. CO 3. To apply all these methods in different fields.
		Project	CO 1. To get basic idea of research. CO 2. To know how to search data in internet, understand, analysis and write about specific concepts. CO 3. To learn about team work.

INTERNAL ASSESSMENT

<u>Paper I, II, III, IV, V, VI, VII & VIII:</u>

Weightage for Internal Assessment is 50 marks.

For Mid Semester Examinations - 25 marks

For Continuous Assessment - 25 marks

Two Mid Semester Examinations will be conducted for 50 marks (1 hours) in the following.

Question Paper pattern:

Short answer Questions (5mark)	: 05 out of 3	: 3 x 5 = 15 marks
Eassy answer question (10 marks)	: 02 out of 1	: 1 x 10 =10 marks
		= 25 marks

The average of two mid examination marks are to be taken for 25 marks.

For continuous assessment – 25 marks distributed in the following way:

Student Project / Assignment - 10 marks (**Assignment) /** Final year **V SEM (Project** all students)

Seminar - 5 marks

Viva voce exam - 10 marks

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2021-22 Admitted Batch)

Total Hrs. of Teaching-Learning: 75 @ 6 hr/Week **Total credits: 05**

Course Outcomes: After successful completion of this course, the student will be able to

- Solve linear differential equations
- Convert non exact homogeneous equations to exact differential equations by using integrating factors.
- Know the methods of finding solutions of differential equations of the first order but not of the first degree.
- Solve higher-order linear differential equations, both homogeneous and non homogeneous, with constant coefficients.
- Understand the concept and apply appropriate methods for solving differential equations.

COURSE SYLLABUS:

UNIT – I: Differential Equations of first order and first degree **(12 Hours)**

Linear Differential Equations; Differential equations reducible to linear form; Exact differential equations; Integrating factors; Change of variables.

UNIT – II: Orthogonal Trajectory and Differential Equations of first order but not of the first degree (12 Hours)

Orthogonal Trajectories, Equations solvable for *p*; Equations solvable for *y*; Equations solvable for *x*; Equations that do not contain x (or y); Equations homogeneous in x and y; Equations of the first degree in *x* and *y* – Clairaut's Equation.

UNIT – III: Higher order linear differential equations-I **(12 Hours)**

Solution of homogeneous linear differential equations of order n with constant coefficients; Solution of the non-homogeneous linear differential equations with constant coefficients by means of polynomial operators. General Solution of f(D)y=0. General Solution of f(D)y=Q when Q is a function of x, $\frac{1}{f(D)}$ is expressed as partial fractions.

P.I. of f(D)y = Q when $Q = be^{ax}$

P.I. of f(D)y = Q when Q is $b\sin ax$ or $b\cos ax$.

UNIT – IV: Higher order linear differential equations-II **(12 Hours)**

Solution of the non-homogeneous linear differential equations with constant coefficients.

P.I. of f(D)y = Q when $Q = b\ x^k$

P.I. of f(D)y = Q when $Q = e^{ax}\ V$**,** where V is a function of x.

P.I. of f(D)y = Q when $Q = x\ V$**,** where V is a function of x.

P.I. of f(D)y = Q when $Q = x^m\ V$**,** where V is a function of x.

UNIT –V: Higher order linear differential equations-III **(12 Hours)**

Method of variation of parameters; Linear differential Equations with non-constant coefficients; The Cauchy-Euler Equation, Legendre's linear equations.

Co-Curricular Activities: **(15 Hours)**

Seminar/ Quiz/ Assignments/ Applications of Differential Equations to Real life Problem /Problem Solving.

Prescribed Text Book:

Differential Equations and Their Applications by Zafar Ahsan, published by Prentice-Hall of India Pvt. Ltd, New Delhi-Second edition.

Reference Books :

1. A text book of Mathematics for B.A/B.Sc, Vol 1, by N. Krishna Murthy & others, published by S. Chand & Company, New Delhi.
2. Ordinary and Partial Differential Equations by Dr. M.D,Raisinghania, published by S. Chand & Company, New Delhi.
3. Differential Equations with applications and programs – S. Balachandra Rao & HR Anuradha- Universities Press.
4. Differential Equations -Srinivas Vangala & Madhu Rajesh, published by Spectrum University Press.

BLUE PRINT FOR QUESTION PAPER PATTERN

COURSE-I, DIFFERENTIAL EQUATIONS

Unit	**TOPIC**	**S.A.Q**	**E.Q**	**Marks allotted to the Unit**
I	Differential Equations of first order and first degree	2	1	20
II	Differential Equations of first order but not of the first degree	2	2	30
III	Higher order linear differential equations-I	1	1	15
IV	Higher order linear differential equations-II	1	1	15
V	Higher order linear differential equations-III	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10marks)

Short answer questions : 4X5= 20M

Essay questions : 3X10=30M

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Total Marks = 50M

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P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2021-22 Admitted Batch)
Model Paper (w.e.f. 2021-2022)

Time: 2Hrs 30 min **Max. Marks: 50M**

PART - I

Answer any FOUR questions. Each question carries FIVE marks. **4X5=20M**

1. Solve $\left(y - e^{sin^{-1}x}\right)\frac{dx}{dy} + \sqrt{1-x^2} = 0$
2. Solve $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$
3. Solve $y + px = p^2x^4$
4. Find the Orthogonal trajectories of family of curves $r = a(1 + cos\theta)$.
5. Solve $(D^2 - 3D + 2) = \cos hx$
6. Solve $(D^3 + 2D^2 + D)y = e^{2x}$
7. Solve$(D^2 - 4D + 4)y = x^3$

PART - II

Answer any THREE questions. Each question carries TEN marks. **3X10=30M**

1. Solve $x\frac{dy}{dx} + y = y^2 \log x$
2. Solve $p^2 + 2py\cot x = y^2$
3. Find the orthogonal trajectories of the family of curves $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, where a is the parameter.
4. Solve $(D^3 + D^2 - D - 1)y = \cos 2x$
5. Solve $(D^2 - 2D + 4)y = 8(x^2 + e^{2x} + \sin 2x)$
6. Solve $(D^2 - 2D)y = e^x \sin x$ by the method of variation of parameters.

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2021-2022 Admitted Batch)

QUESTION BANK
Short Answer Questions

Unit-I

1. Solve $x\frac{dy}{dx}+2y-x^2\log x=0$

2. Obtain the equation of the curve satisfying the differential equation

   $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$.

3. Solve $\frac{dy}{dx}+2xy=e^{-x^2}$.

4. Solve $\frac{dy}{dx}+x\sin 2y=x^3cos^2y$.

5. Solve $(1+y^2)dx=(tan^{-1}y-x)dy$.

6. Solve $x\frac{dy}{dx}+y\log y=xye^x$

7. Solve $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

8. Solve $(\sin x\ \cos y+e^{2x})dx+\ (\cos x\sin y+\tan y)dy=0$

9. Solve $(x^2+y^2+2x)dx+2y\ dy=0$.

10. Solve $(1+xy)y\ dx+(1-xy)x\ dy=0$.

11. Solve $(x^2-2xy-y^2)dx-\ (x+y)^2dy=0$.

12. Solve $(4x+3y+1)dx+(3x+2y+1)dy=0$.

13. Define an Integrating factor and solve $xdy-ydx=xy^2dx$.

14. Solve $xdy=\left[y+xcos^2\left({}^{y}/_{x}\right)\right]dx$

15. Solve $ydx-xdy+(1-x^2)dx+x^2\sin y\ dy=0$

16. Solve $\frac{dy}{dx}+\frac{y}{x}=y^2x$

17. Solve $\frac{dy}{dx}-y\ tanx=\frac{sinxcos^2x}{y^2}$

Unit-II

18. Find the Orthogonal trajectories of the family of coaxial circles $x^2+y^2+2fy+1=0, f$ being the parameter.

19. Find the orthogonal trajectories of the family of $r=a(1-\cos\theta)$ where 'a' is the parameter.

20. Find the Orthogonal trajectories of family of circles $x^2+y^2=a^2$, where a is the parameter.

21. Find the Orthogonal trajectories of family of straight lines in a plane and passing through the origin.

22. FindtheOrthogonal trajectories of family of coaxial circles $x^2+y^2+2gx+c=0$,where g is parameter.

23. Find the Orthogonal trajectories of family of curves $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

24. Find the Orthogonal trajectories of family of curves $r = a(1 + cos\theta)$.

25. Solve $y = 2px + x^2p^4$.

26. Solve $= xp^2 + p$.

27. Solve $2xp^3 - 6yp^2 + x^4 = 0$.

28. Solve $x^2(y - px) = p^2y$.

29. Define Clairaut's equation and solve $(px - y)(py + x) = 2p$.

Unit-III

30. Solve$(D^2 - 3D + 2)y = Coshx$.

31. Solve $(D^2 + 9)y = Cos\ \ 3x$.

32. Solve$(D^2 - 5D + 6)y = e^{4x}$.

33. Solve$(D^2 - D - 2)y = Sin2x$.

34. Solve$(D^2 + 4)y = Sin2x$.

35. Solve $(D^2 + 4)y = \tan 2x$

36. Solve $(D^2 + 1)y = cosec\ x$.

37. Solve $(D^2 + 9)y = cos^3 x$.

Unit-IV

38. Solve$(D^2 - 2D + 1)y = x^2e^{3x}$.

39. Solve$(D^2 + 4)y = xSinx$.

40. Solve $(D^3 + 2D^2 + D)y = e^{2x} + x$.

41. Solve$(D^2 - 4D + 4)y = x^3$

42. Solve$(D^4 + D^2)y = 4Sinx - 2Cosx$

43. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}. \sin 2x$

44. Solve $(6D^2 - D - 2)y = xe^{-x}$

Unit-V

45. Solve$(D^2 + 1)y = Secx$ by method of variation of parameters.

46. Solve$(D^2 - 3D + 2)y = Cos\ e^{-x}$ by the method of variation of parameters.

47. Solve$(x^4D^3 + 2x^3D^2 - x^2D + x)y = 1$

48. Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$.

49. Solve $(x^2\mathrm{D}^2 - 3\mathrm{xD} + 5)\mathrm{y} = \mathrm{x}^2\sin(\log \mathrm{x})$.

50. Solve $(x^2\mathrm{D}^2 - \mathrm{xD} + 1)y = 2\log x$.

51. solve$(x^2\mathrm{D}^2 + \mathrm{xD} - 4)\mathrm{y} = \mathrm{x}^2$.

Essay Answer Questions

Unit - I

1. Solve $x\, dy/dx + (x\, sinx + cosx)y = 1$
2. Solve $Cos^2x\, \frac{dy}{dx} + y = \tan x$.
3. Define Bernoulli's equation of first order and solve $x\frac{dy}{dx} + y = y^2 \log x$.
4. Solve $\frac{dy}{dx}\left(x^2y^3 + xy\right) = 1$
5. Solve $ydx - xdy + (1 - x^2)dx + x^2 \sin y\, dy = 0$
6. Solve $x^2y\, dx - (x^3 + y^3)dy = 0$.
7. Solve $y^2dx + (x^2 - xy - y^2)dy = 0$.
8. Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.
9. Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.
10. Solve $\left(x^2y^2 + xy + 1\right)ydx + (x^2y^2 - xy + 1)xdy = 0$
11. Solve $(xy \sin xy + \cos xy)y\, dx + (xy \sin xy - \cos xy)x\, dy = 0$.
12. Solve $(x^3y^3 + x^2y^2 + xy + 1)y\, dx + (x^3y^3 - x^2y^2 - xy + 1)x\, dy = 0$.
13. Solve $(1 + xy)x\, dy + (1 - xy)ydx = 0$.
14. Solve $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{1}{4}(x + xy^2)dy = 0$.
15. Solve $(x^3 - 2y^2)dx + 2xy\, dy = 0$.
16. Solve $\left(xy^3 + y\right)dx + 2(x^2y^2 + x + y^4)dy = 0$
17. Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

Unit - II

18. Show that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal, where λ is the Parameter.
19. Show that the orthogonal trajectories of the parabola $y^2 = 4a(x + a)$ belongs to the same itself, a being parameter.
20. Find the Orthogonal trajectories of the family of curves in polar coordinates $r = \frac{2a}{(1+\cos\theta)}$ Where 'a' is the parameter?
21. Find the orthogonal trajectories of the family of curves in polar coordinates $r = a(1 + \cos\theta)$
22. Find the orthogonal trajectories of the family of curves in polar coordinates $r \sin 2\theta = \lambda$
23. Solve $p^2 + 2py \cot x = y^2$
24. Solve $y + px = p^2x^4$.
25. Solve $y^2 \log y = xpy + p^2$.

26. Solve $2px = 2\tan y + p^3 cos^2 y$.

27. Solve $xy(p^2+1) = (x^2+y^2)p^2$.

Unit-III

28. Solve $(D^2 - 4D + 3)y = sin3xCos2x$.

29. Solve $(D^2 - a^2)y = secax$

30. Solve $(D^2 - a^2)y = tanax$

31. Solve $(D^2 + 4)y = e^x + sin2x + Cos2x$.

32. Solve $(D^2 + 4)y = sin2x$

33. Solve $(D^2 - 4)y = e^x + \sin 2x + cos^2 x$

Unit-IV

34. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}sin2x$

35. Solve $(D^2 + 4)y = x^2e^{3x} + e^xCos2x$

36. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}sin2x$

37. Solve $(D^4 + 2D^2 + 1)y = x^2Cosx$

38. Solve $(D^2 - 2D + 1)y = xe^xsinx$

39. Solve $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$.

40. Solve $(D^2 - 1)y = x^2 \sin 3x$

Unit-V

41. Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10(x + \frac{1}{x})$.

42. Solve $(x^2D^3 + 3xD^2 + D)y = x^2 \log x$.

43. Solve $x^2D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$.

44. Solve $x^2y'' - 2x(1+x)y' + 2(1+x)y = x^3$.

45. Solve $(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\ \frac{dy}{dx} + y\cos x = 0$.

46. Solve $\frac{d^2y}{dx^2} - \cot x\frac{dy}{dx} - (1 - \cot x)\, y = e^x \sin x$.

47. Solve $(D^2 + a^2)y = \sec ax$ by method of variation of parameters

48. Solve $y'' + 4y = 4\tan 2x$ by method of variation of parameters

49. Solve $(D^2 + 1)y = cosecx$ by method of variation of parameters

50. Solve $(D^2 + a^2)y = \tan ax$ by method of variation of parameters

51. Solve $(D^2 - 2D)y = e^x \sin x$ by method of variation of parameters

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P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - II Semester
Mathematics Course-II: Three Dimensional Analytical Solid Geometry
(w.e.f. 2021-22 Admitted Batch)

Total Hrs. of Teaching-Learning: 75 @ 6 hr/Week **Total credits: 05**

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Course Outcomes: After successful completion of this course, the student will be able to

- Get the knowledge of planes.
- Basic idea of lines, sphere and cones.
- Understand the properties of planes, lines, spheres and cones.
- Express the problems geometrically and then to get the solution

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COURSE SYLLABUS:

UNIT – I: The Plane **(12 Hours)**

Equation of plane in terms of its intercepts on the axis, Equations of the plane through the given points, Length of the perpendicular from a given point to a given plane, Bisectors of angles between two planes, Combined equation of two planes, Orthogonal projection on a plane.

UNIT – II: The Line **(12 Hours)**

Equation of a line; Angle between a line and a plane; The condition that a given line may lie in a given plane; The condition that two given lines are coplanar; Number of arbitrary constants in the equations of straight line; Sets of conditions which determine a line; The shortest distance between two lines; The length and equations of the line of shortest distance between two straight lines; Length of the perpendicular from a given point to a given line.

UNIT – III: The Sphere **(12 Hours)**

Definition and equation of the sphere; Equation of the sphere through four given points; Plane sections of a sphere; Intersection of two spheres; Equation of a circle; Sphere through a given circle; Intersection of a sphere and a line; Power of a point; Tangent plane; Plane of contact; Polar plane; Pole of a Plane; Conjugate points; Conjugate planes.

UNIT – IV: The Sphere and Cones **(12 Hours)**

Angle of intersection of two spheres; Conditions for two spheres to be orthogonal; Radical plane; Coaxial system of spheres; Simplified from of the equation of two spheres.

Definitions of a cone; vertex; guiding curve; generators; Equation of the cone with a given vertex and guiding curve; equations of cones with vertex at origin are homogenous; Condition that the general equation of the second degree should represent a cone.

UNIT –V: Cones **(12 Hours)**

Enveloping cone of a sphere; right circular cone: equation of the right circular cone with a given vertex, axis and semi vertical angle: Condition that a cone may have three mutually perpendicular generators; intersection of a line and a quadric cone; Tangent lines and tangent plane at a point; Condition that a plane may touch a cone; Reciprocal cones; Intersection of two cones with a common vertex.

Co-Curricular Activities: **(15 Hours)**

Seminar/ Quiz/ Assignments/ Applications of Differential Equations to Real life Problem /Problem Solving.

Prescribed Text Book:

Analytical Solid Geometry by Shanti Narayan and P.K. Mittal, published by S. Chand & Company Ltd. 7th Edition.

Reference Books :

1. A text book of Mathematics for BA/B.Sc Vol 1, by V Krishna Murthy & Others, published by S. Chand & Company, New Delhi.
2. A text Book of Analytical Geometry of Three Dimensions, by P.K. Jain and Khaleel Ahmed, published by Wiley Eastern Ltd., 1999.
3. Co-ordinate Geometry of two and three dimensions by P. Balasubrahmanyam, K.Y. Subrahmanyam,
4. G.R. Venkataraman published by Tata-MC Gran-Hill Publishers Company Ltd., New Delhi.

Additional Inputs :

Definition of Cylinder and Right Circular Cylinder .

BLUE PRINT FOR QUESTION PAPER PATTERN

COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**The Plane**	2	1	20
II	**The Line**	2	1	20
III	**The Sphere**	1	1	15
IV	**The Sphere and Cones**	1	2	25
V	**Cones**	1	1	15
	Total	7	6	95

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10marks)

Short answer questions : 4X5= 20M

Essay questions : 3X10=30M

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Total Marks = 50M

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P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - II Semester
Mathematics Course-II: Three Dimensional Analytical Solid Geometry
(w.e.f. 2021-22 Admitted Batch)
Model Paper (w.e.f. 2021-2022)

Time: 2Hrs 30 min **Max. Marks: 50M**

PART - I

Answer any FOUR questions. Each question carries FIVE marks. **4X5=20M**

1. Find the equation of the plane through the point (-1,3,2) and perpendicular to the planes x+2y+2z=5 and 3x+3y+2z=8.
2. Find the equation to the plane through the points (1, 1, 1), (1, -1, 1) and (-7, -3, -5). Show that it is parallel to y- axis.
3. Find the image of the point (2,-1,3) in the plane 3x-2y+z =9.
4. Find the equations of the line through the point (1, 1, 1) and intersecting the lines $2x - y - z - 2 = 0 = x + y + z - 1$; $x - y - z - 3 = 0 = 2x + 4y - z - 4$.
5. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.
6. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$
7. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).

PART - II

Answer any THREE questions. Each question carries TEN marks. **3X10=30M**

1. A plane meets the coordinate axes in A, B, C. If the centroid of ΔABC is (a, b, c), show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$
2. Find the shortest distance between the lines $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.
3. Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z = 2$;
   $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere.
4. Find the limiting points of the coaxial system of spheres
   $x^2 + y^2 + z^2 - 8x + 2y - 2z + 32 = 0$, $x^2 + y^2 + z^2 - 7x + z + 23 = 0$.
5. Prove that if the angle between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\pi/2$, then $a + b + c = 0$ and is $\pi/3$, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$
6. Prove that the equation $\sqrt{fx} \mp \sqrt{gy} \mp \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone.

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - II Semester
Mathematics Course-II: Three Dimensional Analytical Solid Geometry
(w.e.f. 2020-21 Admitted Batch)

QUESTION BANK
Short Answer Questions
Unit-I

1. Find the equation of the plane passing through the points (2, 2, -1), (3, 4, 2), (7, 0, 6).
2. Find the equation of the plane through the point (-1, 3, 2) and perpendicular to the planes x+2y+2z =5 and 3x+3y+2z =8.
3. Find the equation to the plane through the points (1, 1, 1), (1, -1, 1) and (-7, -3, -5). Show that it is parallel to y- axis.
4. Find the equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y-z=-4$ and is parallel to X-axis.
5. Find the equation of the plane through the line of intersection of the planes $x+y+z-1=0,\ 2x+3y+4z-5=0$ and perpendicular to the plane $x-y+z=0$.
6. If a plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r) then show that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$.

UNIT-II

7. Find the equation of the line through the point $(1, 2, 3)$ and parallel to X- axis.
8. Find the point of intersection of the line through $(2,-3,1), (3,-4,5)$ with the plane $2x+y+z=7$.
9. Find the image of the point $A(1, 3, 4)$ in the plane $2x-y+z+3=0$.
10. Write the equation of the line $x=ay+b,\ z=cy+d$.
11. Find the symmetric form of the equation of the line $x+y+z+1=0=4x+y-2z+2$.
12. Find the equations of the line through the point (1, 1, 1) and intersecting the lines

$$2x-y-z-2=0=x+y+z-1;\ x-y-z-3=0=2x+4y-z-4.$$

13. Show that the lines $2x+y-4=0=y+2z$ and $x+3z-4=0=2x+5z-8=0$ are coplanar.
14. Find the length and equations to the line of the shortest distance between the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $x+y+2z-3=0=2x+3y+3z-4$.

UNIT-III

15. Find the equation of the sphere through (0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c) in A, B, C. Prove that the centroid of the ΔABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$
16. Find the equation of the sphere circumscribing the tetrahedron whose faces are $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

17. Find the equation of the sphere passing through the circle $x^2 + y^2 = 4, z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ in a circle of radius 3.
18. Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0, x - y + z = 2; x^2 + y^2 + z^2 + x - 3y + z - 5 = 0, 2x - y + 4z - 1 = 0$ lie on the same sphere.
19. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$, and find the point of contact.

UNIT-IV

20. Find the power of the point $(1, -1, 2)$ with respect to the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z - 5 = 0$
21. Find the polar plane of the point (0, -1, 1) with respect to the sphere $x^2 + y^2 + z^2 - 2x + 4y + 6z - 11 = 0.$
22. Find the limiting points of the coaxal system of spheres determined by $x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0,\ x^2 + y^2 + z^2 + 2x - 4y + 2z + 6 = 0$
23. If r_1 and r_2 are the radii of the orthogonal spheres, then show that the radius of the circle of their intersection is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.
24. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts orthogonally to the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.
25. Show that the general equation of the cone of the second degree which pass through Coordinate axes is $fyz + gzx + hxy = 0$.

UNIT-V

26. Find the enveloping cone at the origin and generators touching the sphere $x^2 + y^2 + z^2 - 2x + 4z - 1 = 0$
27. Find the vertex of the cone $7x^2 + 2y^2 + 2z^2 + 10zx + 10xy + 26x - 2y + +2z - 17 = 0$.
28. Find the equations of the cone touches the 3 co-ordinate planes and the plane $x + 2y + 3z = 0, 2x + 3y + 4z = 0$.
29. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).
30. Find the equation to the right circles cone whose vertex is P(2, -3, 5), axis PQ which makes equal angles with the axes and which passes through (1, -2, 3).
31. Find the reciprocal cone of $9x^2 + 4y^2 - 7z^2 = 0$.
32. Find the equation of the right circular cone whose vertex is (1, -2, -1), axis is the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z+1}{5}$ and the semi- vertical angle $60°$.
33. Show that the reciprocal cone of $ax^2 + by^2 + cz^2 = 0$ is the cone $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$

Essay Questions

UNIT-I

1. A variable plane is at a constant distance 3p from the origin and meets the axes in A, B, C. Show that the locus of the centroid of Δ ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
2. A variable plane makes intercepts on the axes, the sum of whose squares is k^2 (a constant). Show that the locus of the foot of the perpendicular from origin to the plane is $(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2)^2 = k^2$.
3. Find the planes bisecting the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$. Point out which of the planes bisects the acute angle and which bisects the obtuse angle in which the origin lies.
4. Show that the plane $8x - 14y - 13 = 0$ bisects the obtuse angle between the plane $4x + 3y - 5z + 1 = 0$ and $12x + 5y - 13z = 0$.
5. Show that the equation $x^2 + 4y^2 + 9z^2 - 12yz - 6zx + 4xy + 5x + 10y - 15z + 6 = 0$ represents a pair of parallel planes and find the distance between them.

UNIT-II

6. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.
7. Show that the lines $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ and $x + 2y + 3z - 8$=2x+3y+4z-11 are intersecting, find the point their intersection and the equation to the plane containing them.
8. Prove that the points (1, 2, 3), (4, 0, 4), (-2, 4, 2), (7, -2, 5) are collinear.
9. Find the image of the point (2, -1, 3) in the plane 3x - 2y + z = 9.
10. Find the image of the point $(2, -2, 3)$ in the plane $3x - 2y - z = 9$.
11. Find the length and equations of shortest distance between the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ *and* $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.
12. Find the length and equation of the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$.
13. Find the equation of the plane containing line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z - 12 = 0$.

UNIT-III

14. A sphere of radius k passes through the origin and meet the axes in A, B, C. Show that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$
15. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

16. Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z = 2$; $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere , and find its equation .

17. Find the radius and centre of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2y - 4z = 11$ and the plane $x + 2y + 2z = 5$.

18. Find the equation of the sphere passing through the circle $x^2 + y^2 = 4$, $z = 0$ and is intersected by the plane $x + 2y + 2z = 0$ in circle of radius 3 .

19. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact.

20. Find the pole of the plane $x - y - z + 9 = 0$ with respect to the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$.

21. Find the pole of the plane $x - y + 5z - 3 = 0$ with respect to the sphere $x^2 + y^2 + z^2 = 9$.

UNIT-IV

22. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts orthogonally to the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

23. Show that the radical line of the spheres $x^2 + y^2 + -4x - 2y + 3 = 0$, $x^2 + y^2 + z^2 - 6y + 3 = 0$, $x^2 + y^2 + z^2 + 4x + 2y - 4z + 3 = 0$, $x^2 + y^2 + z^2 + 4x + 2y - 4z + 3 = 0$ is $\frac{x}{3} = \frac{y}{2} = \frac{z}{7}$.

24. Find the radical centre of the spheres $x^2 + y^2 + z^2 + 4y = 0$, $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$, $x^2 + y^2 + z^2 + 3x - 2y + 8z + 6 = 0$, $x^2 + y^2 + z^2 - x + 4y - 6z - 2 = 0$

25. If r_1 , r_2 are the radii of two orthogonal spheres then the radius of the circle of their intersection is $\frac{r_1 r_2}{\sqrt{{r_1}^2 + {r_2}^2}}$.

26. If (-2 , 1 , -1) is a limiting point of coaxal system for which $x + y + 2z = 0$ is the radical plane then show that the other limiting point is (-1 , 2 , 1).

27. Find the limiting points of the coaxal system of spheres determined by
$x^2 + y^2 + z^2 + 3x - 3y + 6 = 0, x^2 + y^2 + z^2 - 6x - 6y - 6z + 6 = 0$.

28. Find the limiting points of the coaxal system of spheres determined by
$x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0, x^2 + y^2 + z^2 + 2x - 4y + 2z + 6 = 0$.

29. Prove that if the angle between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\pi/2$, then $a + b + c = 0$ and is $\pi/3$, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

30. Show that two lines of intersection of the plane $ax + by + cz = 0$ with the cone $yz + zx + xy = 0$ will be perpendicular if $1/a + 1/b + 1/c = 0$.

31. Find the angle between the lines of section of the plane $3x + y + 5z = 0$ and the cone $6yz - 2zx + 5xy = 0$.

UNIT-V

32. The equation of a right circular cone with vertex at (α, β, γ) semi vertical angle θ and axis having direction ratios (l, m, n) is $[l(x-\alpha)+m(y-\beta)+n(z-\gamma)]^2 = (l^2+m^2+n^2)((x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2)\cos^2\theta$.
33. Find the equation to the right circular cone whose vertex is P(2,-3,5) and axis PQ which makes equal angles with the axis and which passes through A(1,-2,3)
34. Prove that the equation $2y^2 + 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $\left(\frac{-7}{6}, \frac{1}{2}, \frac{5}{6}\right)$.
35. Find the vertex of the following cones .
    i) $4x^2-y^2+2z^2+2xy-3yz+12x-11y+6z+4=0$
    ii) $X^2-2y^2+3z^2-4xy+5yz-6zx+8x-19y-2z-20=0$.
36. If $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone then prove that $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$.
37. Show that the general equation to a cone which touches the three co ordinate planes is $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$.
38. The semi-vertical angle of a right circular cone having three mutually perpendicular
    (i) generators is $tan^{-1}(\sqrt{2})$.
    (ii) tangent planes is $tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II year B.Sc., Degree Examinations - III Semester
Mathematics Course-III: ABSTRACT ALGEBRA

(w.e.f. 2020-21 Admitted Batch)

Total Hrs. of Teaching-Learning: 75 @ 6 hr/Week **Total credits: 04**

Objective:

- To learn about the basic structure in Algebra
- To understand the concepts and able to write the proofs to theorems
- To know about the applications of group theory in real world problems

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Course Outcomes:

After successful completion of this course, the student will be able to;

- acquire the basic knowledge and structure of groups, subgroups and cyclic groups.
- get the significance of the notation of a normal subgroups.
- get the behavior of permutations and operations on them.
- study the homomorphisms and isomorphisms with applications.
- Understand the ring theory concepts with the help of knowledge in group theory and to prove thetheorems.
- Understand the applications of ring theory in various fields.

UNIT I : **(12 Hours)**

GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II: **(12 Hours)**

SUBGROUP:Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroups. Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Co-sets and Lagrange's Theorem: Cosets Definition

– properties of Cosets–Index of a subgroups of a finite groups–Lagrange's Theorem.

UNIT III: **(12 Hours)**

NORMAL SUBGROUPS: Definition of normal subgroup – proper and improper normal subgroup–Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group –quotient group – criteria for the existence of a quotient group

UNIT IV: **(12 Hours)**

HOMOMORPHISM : Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

PERMUTATIONS: Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

UNIT V:

(12 Hours)

RINGS

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings, Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field. Sub Rings.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

TEXT BOOK :

1. A text book of Mathematics for B.A. / B.Sc. by B.V.S.S. SARMA and others, published byS.Chand & Company, New Delhi.

REFERENCE BOOKS :

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

Additional Inputs ;

Cyclic groups definition and number of generators of a finite cyclic groups .

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	2	2	30
II	Subgroups	2	2	30
III	Normal subgroups	1	2	25
IV	Homomorphism, Permutations	2	2	30
V	Rings	1	2	25
Total		8	10	140

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 4 X 10 = 40

……………….

Total Marks = 60

………………..

P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course: Abstract Algebra
Paper III (Model Paper w.e.f. 2020-21)

Time: 2Hrs 30 min **Max. Marks: 60**

PART - I

Answer any FOUR questions. Each question carries FIVE marks. **4 X 5 M=20 M**

1. Prove that the set Z of all integers form an abelian group w.r.t. the operation defined by $a * b = a + b + 2, \forall\, a, b \in Z$
2. If G is a group,for $a, b \in G$ prove that $(ab)^{-1} = b^{-1}a^{-1}$
3. If a non empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
4. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab \in H$.
5. Define Normal subgroup. Prove that a subgroup H of a Group (G,.) is a normal subgroup of G if and only if $xHx^{-1} =$ H ∀ x ∈ G.
6. If f is a homomorphism of a group G into a group G',then prove that the kernel of f is a normal subgroup of G.
7. Write down the following permutation as product of disjoint cycles
$$f = (1\ \ 3\ \ 2\ \ 5)(1\ \ 4\ \ 3)(2\ \ 5\ \ 1).$$
8. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.

PART - II

Answer ALL questions. Each question carries Eight marks. **5 X 8 M = 40 M**

9 **(a)** Prove that the set Z of all integers form an abelian group w.r.t. the operation is defined by $a * b = a + b + 2\ \forall\, a, b \in Z$.

(OR)

(b) Show that the set Q_+ of all positive rational numbers forms an abelian group under the composition defined by 'o' such that $a\ ob = (ab)/3\ \ for\ a, b\ \in\ Q_+$.

10 (a)Prove that a non empty complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$.

(OR)

(b) State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true

11 **(a)** If H is a normal subgroup of a group (G,.) then prove that the product of two right (or) left cosets of H is also a right (or) left coset of H in G.

(OR)

(b) If H is a subgroup of G and N is a normal subgroup of G, then prove that
(i) $H \cap N$ is a normal subgroup of H (ii) N is a normal subgroup of HN

12 .(a) Prove that the necessary and sufficient condition for a homomorphism f of a group G onto a group G' with kernel K to be an isomorphism of G into G' is that $K = \{e\}$

(OR)

(b) $f = (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8)$, $g = (4\ \ 1\ \ 5\ \ 6\ \ 7\ \ 3\ \ 2\ \ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.

13 .(a) A finite integral domain is a field .

(OR)

(b) Prove that the ring of integers Z is a principal ideal ring.

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–III: ABSTRACT ALGEBRA

Short Answer Questions

UNIT-1

1. Prove that in a group the identity element is unique and the inverse of every element is unique.
2. If G is a group,for $a, b \in G$ prove that $(ab)^{-1} = b^{-1}a^{-1}$
3. Prove that the cancellation laws hold in a group.
4. Prove that a semi group satisfying cancellation laws is a group.
5. Prove that in a group $G(\neq \emptyset)$, for $a, b, x, y \in G$,the equations $ax = b, ya = b, \forall\, a, b \in G$ have unique solutions.
6. Show that a group $(G, .)$ is abelian if and only if $(ab)^2 = a^2b^2 \;\forall\, a, b \in G$.
7. If $(G, .)$ is a group such that $(ab)^n = a^nb^n, \forall\, a, b \in G$ for three consecutive positive integers n, then show that $(G, .)$ is an abelian group.
8. Show that the fourth roots of unity form an abelian group under multiplication.

UNIT-II

9. If H and K are two subgroups of a group G then show that $H \cap K$ is also a subgroup of G.
10. If a non empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
11. Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.
12. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab \in H$.
13. $(Z_6 = \{0,1,2,3,4,5\}, +_6)$ is a group. Prove that $S = \{0,2,4\}, T = \{0,3\}$ are subgroups of Z_6 and $S \cup T$ is not a subgroup of Z_6.
14. If G is a group and $a \in G$, then prove that the normalizer $N(a) = \{x \in G / ax = xa\}$ is a subgroup of G.
15. Prove that any two left cosets of a subgroup H in a group $(G, .)$ are either disjoint or identical.
16. State and Prove Lagrange's theorem.

UNIT-III

17. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N.
18. Define Normal subgroup. Prove that a subgroup H of a Group (G,.) is a normal subgroup of G if and only if $xHx^{-1} = H \;\forall\, x \in G$.
19. Prove that a subgroup H of a Group (G,.) is a normal subgroup of G if and only if every left coset of H in G is a right coset of H in G.
20. Prove that a subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.
21. Prove that every subgroup of an abelian group is normal.
22. If G is a group and H is a subgroup of index 2 in G, then prove that H is a normal subgroup of G.
23. If G is a group then the centre Z of G is a normal subgroup of G

UNIT-IV

24. Prove that the homomorphic image of an abelian group is abelian.
25. If f is a homomorphism of a group G into a group G',then prove that the kernel of f is a normal subgroup of G.
26. Prove that the necessary and sufficient condition for a homomorphism f of a group G onto a group G' with kernel K to be an isomorphism of G into G' is that $K = \{e\}$.
27. Let a be a fixed element of a group G then prove that the mapping $f_a: G \to G$ defined by $f_a(x) = a^{-1}xa \ \ for\ every\ \ x \in G$ is an automorphism.
28. If f is a homomorphism from a group G to a group G' then prove that $f(a^{-1}) = [f(a)]^{-1}$.
29. Let G be a group. If $f: G \to G$ defined by $f(x) = x^2 \ \forall\, x \in G$ is a homomorphism then show that G is abelian.
30. Show that if $f: G \to G^{'}$ defined by $f(a) = a^{-1}\ \forall a \in G$ is an automorphism if and only if G is abelian.

31. $f = (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8)$, $g = (4\ \ 1\ \ 5\ \ 6\ \ 7\ \ 3\ \ 2\ \ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.
32. Write down the following permutation as product of disjoint cycles $f = (1\ \ 3\ \ 2\ \ 5)(1\ \ 4\ \ 3)(2\ \ 5\ \ 1)$.
33. Find the regular permutation group isomorphic to the multiplicative group $G = \{1, -1, i, -i\}$.
34. Show that every cyclic group is an abelian group.
35. Show that the group $(G = \{1,2,3,4,5,6\}, X_7)$ is a cyclic group and write down all of its generators.
36. Find the number of generators of cyclic groups of orders 5, 6, 8, 12.

UNIT-V

1. If R is a Boolean ring, then prove that
   i. $a + a = 0, \forall\, a \in R$
   ii. $a + b = 0 \ \Rightarrow a = b$
   iii. R is commutative under multiplication
2. Prove that, a ring R has no zero divisors iff the cancellation laws hold in R.
3. Prove that every field is an integral domain.
4. Prove that, the intersection of two subrings of a ring R is a subring of R.
5. If R is a commutative ring and $a \in R$, then prove that $Ra = \{ra: r \in R\}$ is an ideal of R.
6. Prove that, the characteristic of a Boolean ring is 2.
7. Prove that the set of integers is a subring of the set of rational numbers w.r.to usual addition and multiplication.
8. Prove that, every subring is an ideal. Explain briefly.
9. If U is an ideal of a ring R with unity element and $1 \in U$, then $U = R$.
10. Prove that, the intersection of two ideals of a ring R is an ideal of R.
11. Prove that the characteristic of an integral domain is either a prime or zero.
12. Let $U = \{0, 3\}$ be an ideal of the ring Z_6, then write the quotient ring Z_6/U

Essay Questions

UNIT-1

1. Prove that a semi group $(G,.)$ is a group if and only if the equations $ax = b, ya = b \quad \forall\, a, b \in G$ have unique solutions in G.
2. Show that the set of integers is an abelian group w.r.to 'o' where 'o' is defined as $a\ o\ b = a + b + 1 \forall a, b\ \in Z.$
3. Show that the set Q_+ of all positive rational numbers forms an abelian group under the composition defined by 'o' such that $a\ ob = (ab)/3\ \ for\ a, b\ \in\ Q_+$.
4. Prove that the set Z of all integers form an abelian group w.r.t. the operation is defined by $a * b = a + b + 2\ \forall\, a, b \in Z$
5. Prove that the set $\mathrm{G} = \mathrm{Q} - \{1\}$ is an abelian group with respect to 'o' defined by $aob = a + b - ab\ \forall a, b \in G.$
6. Prove that the set of n^{th} roots of unity forms an abelian group w.r.t. '.'
7. Prove that the set $G = \{A_\alpha / \alpha \in R\}\ where\ A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ forms a group with respect to '·'.
8. Show that all groups of order 4 and less are commutative.

UNIT-II

9. Prove that a non empty complex H of a group G is a subgroup of G if and only if

   $(i)\ a, b \in H \Rightarrow ab \in H \quad (ii)\ a \in H \Rightarrow a^{-1} \in H.$
10. Prove that a non empty complex H of a group G is a subgroup of G if and only if

    $a, b \in H \Rightarrow ab^{-1} \in H.$
11. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if

    $a, b \in H \Rightarrow ab \in H.$
12. State and prove the Necessary and sufficient condition for a finite complex H of a group G.
13. Prove that the union of two subgroups of a group is a subgroup if and only if one is contained in the other.
14. If N is a normal subgroup of a group G. Show that G/N is abelian if and only if $x, y \in G, xyx^{-1}y^{-1} \in N.$
15. State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true.
16. Prove that $(\{0,3,6,9,12\}, +_{15})$ is a subgroup of $(Z_{15}, +_{15})$. Find the left cosets of the above subgroup in Z_{15}. Find the index of subgroup in Z_{15}.

UNIT-III

17. If H is a subgroup of G and N is a normal subgroup of G, then prove that

    (i) $H \cap N$ is a normal subgroup of H (ii) N is a normal subgroup of HN.
18. If H is a normal subgroup of a group (G,.) then prove that the product of two right (or) left cosets of H is also a right (or) left coset of H in G.
19. If H is a normal subgroup of G. The set $\frac{G}{H}$ of all cosets of H in G with respect to coset multiplication is a group.
20. If N, M are normal subgroups of G, then prove that NM is also a normal subgroup of G.

UNIT-IV

21. Prove that the homomorphic image of a group G is isomorphic to some quotient group of G.
22. (Z,+) is a group of integers. Show that $f: Z \to Z$ defined by $f(x) = 2x \ \forall x \in Z$ is a homomorphism. Find Ker f and show that it is not an onto homomorphism.
23. Let a be a fixed element of a group G then the mapping $f_a: G \to G$ defined by $f_a(x) = a^{-1}xa \ \ for\ every\ \ x \in G$ is an automorphism.
24. Let (R^+, .) be a group and $f: R^+ \to R^+$ defined by $f(x) = x^2,\ \forall x \in R^+.$ Prove that f is an automorphism.
25. Let G be a group. Prove that the set A(G) of all Automorphisms on G is a group with respect to the composition of mappings.
26. Show that the group $(G = \{0,1,2,3\}, +_4)$ and the group $(G' = \{1, -1, i, -i\}, .)$ are isomorphic.
27. $f = (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8)$, $g = (4\ \ 1\ \ 5\ \ 6\ \ 7\ \ 3\ \ 2\ \ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.
28. Find the regular permutation group of the multiplicative group $G = \{1, -1, i, -i\}$.
29. Prove that the set A_n of all even permutations form a normal subgroup of the group of permutations S_n.
30. State and prove Cayley's theorem.
31. Prove that every subgroup of a cyclic group is cyclic.
32. Prove that a cyclic group of order ' n' has $\varphi(n)$ generators.

UNIT-V

33. Let Z be the set of all integers, then prove that $(Z, *, o)$ is a commutative ring with unity, where the operations $*$ and o are defined by $a * b = a + b - 1$ and $aob = a + b - ab$.

Prove that every finite integral domain is a field.

35. Prove that the set of even integers is a ring commutative without unity under usual addition & multiplication.
36. Prove that the set $Z(i) = \{a + ib : a, b \in Z, i^2 = -1\}\ of$ Gaussion integers is an Integral domain with respect to addition & multiplication of numbers is a field.
37. Prove that $Q\sqrt{2} = \{a + b\sqrt{2} :\ a, b \in Q\}$ is a field w.r.to ordinary addition and multiplication of numbers.

Prove that the characteristic of a field is either a prime or zero.

If R is a division ring show that C(R) = $\{x \in R /\ xa = ax,\ \forall a \in R\}$ is a field.

Prove that every ideal of Z is a principal ideal.

If p is a prime, then prove that Z_p, the ring of residue classes modulo p is a field.

42. Let S be a non empty subset of a ring R. Then prove that S is a subring of R iff $a - b \in S$ and $ab \in S$, for all $a, b \in S$.

Prove that every field has no proper ideals.

Prove that, a commutative ring R with unity element is a field, if R has no proper ideals.

45. If U_1 and U_2 are two ideals of a ring R, then prove that $U_1 \cup U_2$ is an ideal of R iff $U_1 \subseteq U_2$ or $U_2 \subseteq U_1$.

Prove that the ring of integers Z is a principal ideal ring.

47. If U is an ideal of the ring R, then prove that the set $R/U = \{x + U : x \in R\}$ is a ring w.r.t. the induced operations of addition(+) and multiplication(.) of cosets defined follows
$(a + U) + (b + U) = (a + b) + U$ and $(a + U).(b + U) = (a.b) + U$.

I year B.Sc., Degree Examinations - III Semester
Mathematics Course-III: ANALYTICAL SKILLS
(w.e.f. 2020-21 Admitted Batch)

Course Objective: Intended to inculcate quantitative analytical skills and reasoning as an inherent ability in students.

Course Outcomes:

After successful completion of this course, the student will be able to

- Understand the basic concepts of arithmetic ability, quantitative ability, logical reasoning, business computations and data interpretation and obtain the associatedskills.
- Acquire competency in the use of verbal reasoning.
- Apply the skills and competencies acquired in the related areas
- Solve problems pertaining to quantitative ability, logical reasoning and verbal ability inside and outside the campus.

UNIT – 1 (10 Hrs)

Arithmetic ability:

Algebraic operations BODMAS, Fractions, Divisibility rules, LCM& GCD(HCF).

Verbal Reasoning:

Number Series, Coding & Decoding, Blood relationship, Clocks, Calendars.

UNIT – 2: (10 Hrs)

Quantitative aptitude:

Averages, Ratio and proportion, Problems on ages, Time-distance–speed.

Business computations:

Percentages, Profit & loss, Partnership, simple compound interest.

UNIT – 3: (07 Hrs)

Data Interpretation:

Tabulation, Bar Graphs, Pie Charts, line Graphs. Venn diagrams.

Recommended Co-Curricular Activities **(**03 Hrs)

Surprise tests / Viva-Voice / Problem solving/Group discussion.

Text Book:

Quantitative Aptitude for Competitive Examination by R.S. Agrawal, S.Chand Publications.

Reference Books:

1. Analytical skills by Showick Thorpe, published by S Chand And Company Limited, Ramnagar,New Delhi-110055.
2. Quantitative Aptitude and Reasoning by R V Praveen, PHI publishers.
3. Quantitative Aptitude for Competitive Examination by Abhijit Guha, Tata Mc Graw Hill Publication

BLUE PRINT FOR QUESTION PAPER PATTERN

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Arthimetic ability&Verbal reasoning	3	2	35
II	Quantitative aptitude&Business computations	3	2	35
III	Data interpretation	2	2	30

SEMESTER-III

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10marks)

Short answer questions : 4 X 5 = 20M

Essay questions : 3X 10 = 30M

……………….

Total Marks = 50

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P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Foundation course:Analytical Skills
(Model Paper w.e.f. 2020-21)

Time: 2 Hrs **Total Marks: 50M**

SECTION –A

Answer any 4 questions. Each question carries 5 marks **4×5 = 20M**

1. Simplify (i) $36 \times 15 - 56 \times 784 \div 112 = ?$
   (ii) $5163 - 4018 + 3209 = ?$
2. How many numbers between 11 and 90 are divisible by 7 ?
3. What was the day of the week on 16^{th} April 2000 ?
4. Divide Rs.1162 among A,B,C in the ratio 35:28:20.
5. The sum of the three consecutive odd numbers is 285. What is the ratio of the smallest andthe largest numbers respectively ?
6. One-fourth of two-fifth of 30% of a number is 15. What is 20% of that number ?
7. Prasad sold his work tools for Rs.1850 and earned a profit of 25%.At what price did prasad buy the work tools ?
8. Population in Millions

City	Total Population	Male Population
A	12	6.5
B	15	7.2
C	17	9.0
D	19	9.8
E	22	10.8

What is the average female population in million ?

Section-B

Answer all the questions.Each question carries 10 marks. **(3×10 = 30 M)**

9. Evaluate (i) $784 \times \sqrt{256} \times 343 = 4^4 \times 7^?$
   (ii) LCM of two numbers is 120 and their HCF is 10.What is the sum of those two numbers?

(OR)

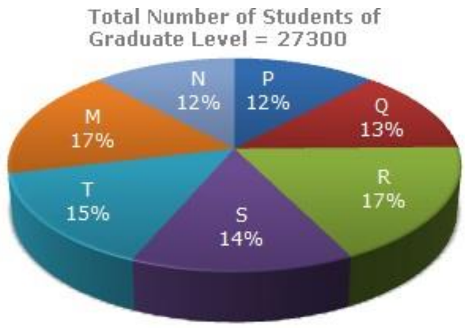
10. What was the day of the week on 15^{th} August 1947 ?

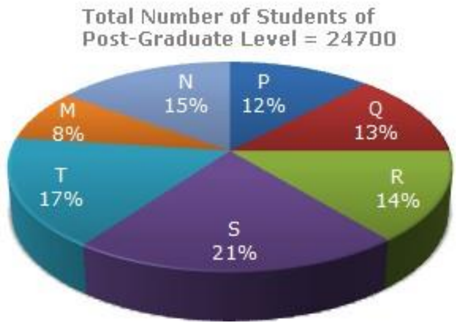
11. A car covers a distance of 540 km in 9 h. Speed of the train is double the speed of the car. Two – third of the speed of the train is equal to the speed of a bike. How much distancewill the bike cover in 5 h ?

(OR)

12. The Simple interest accrued on an amount of Rs.2500 at the end of 6 yr is Rs.1875. What would be the simple interest accrued on the amount of Rs.6875 at the same rate and sametimeperiod ?

13. The following pie-charts show the distribution of students of graduate and post-graduate levelsin seven different institutes in a town.





Distribution of students at graduate and post-graduate levels in seven institutes:

(i) What is the total number of graduate and post-graduate level students is institute R?

(ii) What is the ratio between the number of students studying at post-graduate and graduatelevels respectively from institute S?

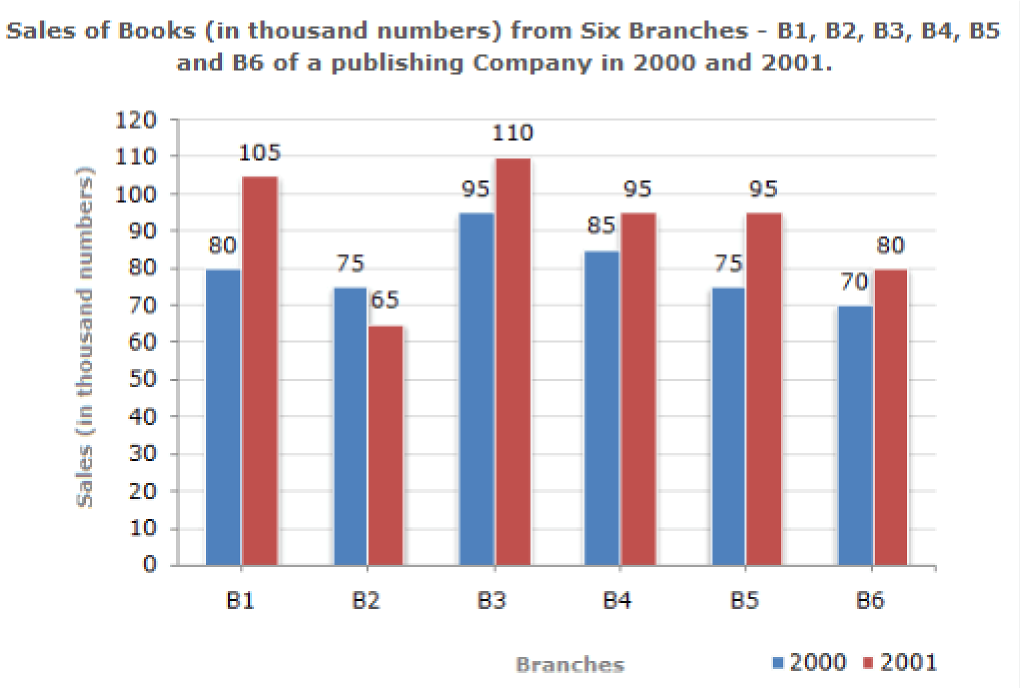
(iii) How many students of institutes of M and S are studying at graduate level?

(iv) What is the ratio between the number of students studying at post-graduate level frominstitutes S and the number of students studying at graduate level from institute Q?

(v) Total number of students studying at post-graduate level from institutes N and P is?

(OR)

14. The bar graph given below shows the sales of books (in thousand number) from sixbranches of a publishing company during two consecutive years 2000 and 2001.



(i) What is the ratio of the total sales of branch B_2 for both years to the total sales of branch B_4 forboth years?

(ii) Total sales of branch B_6 for both the years is what percent of the total sales of branches B_3 forboth the years?

(iii) What percent of the average sales of branches B_1, B_2 and B_3 in 2001 is the average sales ofbranches B_1, B_3 and B_6 in 2000?

(iv) What is the average sales of all the branches (in thousand numbers) for the year 2000?

(v) Total sales of branches B_1, B_3 and B_5 together for both the years (in thousand numbers) is?

P. R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUESTION BANK FOR ANALYTICAL SKILLS

UNIT-1 ARITHMETIC ABILITY

BODMASRULE AND SIMPLIFICATION

1. 12573+43495+23472=?
2. (8÷88)×8888088=?
3. The value of 1001÷11of 13 is?'
4. $20\frac{1}{2}+30\frac{1}{3}-15\frac{1}{6}$ =?
5. Simplify 2-$[2-\{2-2(2+2)\}]$=?
6. Simplify 18-$\left[5-\left\{6+2\left(7-\overline{8-5}\right)\right\}\right]$.
7. $(-5)(4)(2)(-\frac{1}{2})(\frac{3}{4})$ =?
8. Find the value of $\frac{(6+6+6+6)\div 6}{4+4+4+4\div 4}$
9. What is the value of $\frac{(P+Q)}{(P-Q)}$ if $\frac{P}{Q}$=7?

DECIMAL FRACTIONS

1. If 204 ÷ 12.75 = 16, then 2.04 ÷ 1.275 = ?
2. 0.03 × 0.0124 = ?
3. 7212 + 15.231 - ? = 6879
4. 4211.01 + 22.261 - ? = 2645.759
5. 0.004 × 0.5 = ?
6. 24.39 + 562.093 + 35.96 = ?
7. 926 + 9.026 + 0.926 + 9.0026 = ?
8. The expression (12.86 × 12.86 + 12.86 × p + 0.14 × 0.14) will be a perfect square for p equal to?

DIVISIBILITY RULE

1. If the the number 5*2 is divisible by 6 then *?

2. If the number 517*324 is completely divisible by 3 , then the smallest whole number in Place of * will be.

3. If the number 481*673 is completely divisible by 9, then the smallest whole number in Place of * will be.

4. If the number 97215*6 is completely divisible by 11, then the smallest whole number in Place of * will be.

5. If the number 91876*2 is completely divisible by 8, then the smallest whole number in Place of * will be.

7. Find the least value of * for which 7*5462 is divisible by 9

8. Find the least value of * for which 4832*18 is divisible by 11.

LCM and HCF:

1. What is the lowest common multiple of 12, 36 and 20?

2. Find the H.C.F of 108,288and 360.

3. Find the greatest common divisor of 24 and 16

4. Two numbers are in the ratio 2 : 3. If their L.C.M. is 48. what is sum of the numbers?

5. The ratio of two numbers is 4 : 5. If the HCF of these numbers is 6, what is their LCM?

6. The H.C.F. of two numbers is 5 and their L.C.M. is 150. If one of the numbers is 25, then the other is:

7. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, then find the other.

8. Find L.C.M of $\frac{2}{3}\frac{8}{9}\frac{16}{81}$ and $\frac{10}{27}$

VERBAL REASONING:

In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and replace the question mark in the given series.

1. 1,9,25,49,?,121

a)64 **b)81** c)91 d)100 ( )

2. 11,13,17,19,23,25,?

a)26 b)27 **c)29** d)37 ( )

3. 6,11,21,36,56,?

a)42 b)51 **c)81** d)91 ( )

4. 10,18,28,40,54.70,?

a)85 b)86 c)87 **d)88** ( )

5. 22,24,28,?,52,84

a)36 b)38 c)42 d)46 ( )

6. 28,33,31,36,?,39

a)32 **b)34** c)38 d)40 ( )

7. 6,17,39,72,?

a)83 b)94 **c)116** d)127 ( )

8. 325,259,204.160,127,105,?

a)94	b)96	c)98	d)100	( )

In each of the following questions, one term in the number series is wrong. Find out the wrong term

1.3,10,27,4,16,64,5,25,105

a)3	b)4	**c)10**	d)27	( )

2. 8,13,21,32,47,63,83

a)13	b)21	c)32	**d)47**	( )

3. 105,85,60,30,0,-45,-90

a)105	b)60	**c)0**	d)-45	( )
a)94	b)127	**c)202**	d)259	( )

5. 1,2,4,8,16,32,64,96

a)4	b)32	c)64	**d)96**	( )

6. 10,26,74,218,654,1946,5834

a)26	b)74	c)218	**d)654**	( )

7. 1,3,10,21,64,129,356,777

a)21	b)129	c)10	**d)356**	( )

8. 3,4,10,32,136,685,4116

a)10	**b)32**	c)136	d)4116	( )

BLOOD RELATIONS:

1. A is B's sister. C is B's mother. D is C's father. E is D's mother. Then, how is A related to D?
2. Pointing out to a lady, a girl said, "She is the daughter-in-law of the grandmother of my father's only son." How is the lady related to the girl ?
3. There are six persons A. B, C, D, E and F. C is the sister of F. B is the brother of E's husband. D is the father of A and grandfather of F. There are two fathers, three brothers and a mother in the group. Who is the mother ?
4. Pointing to a person, a man said to a woman, "His mother is the only daughter of your father. " How was the woman related to the person ?
5. A girl introduced a boy as the son of' the daughter of the father of her uncle. What is the relation between the boy and the girl ?

6. In a family, there are six members A, B, C, D, E and F.A and B are a married couple, A being the male member. D is the only son of C, who is the brother of A. E is the sister of D. B is the daughter-in-law of F, whose husband has died. How is E related to C ?
7. A woman introduces a man as the son of the brother of her mother. How is the man, related to the woman ?

CODING AND DECODING:

1. In a certain code 'MISSIONS' is written as 'MSIISNOS'.How is 'ONLINE' written in that code?
   a)**OLNNIE**
   b)ONILEN
   c)NOILEN
   d)ONNLIE
2. In certain code 'TIGER' is written as 'QDFHS'.How is 'FISH' written in that code?
   a)GERH
   b)**GRHE**
   c)GREH
   d)GHRE
3. In certain code 'FROZEN' is written as 'OFAPSG'.How is 'MOLTEN' is written in that code?
   a)OFPOMN
   b)OFSMPN
   c)**OFUMPN**
   d)OFUNPM
4. In certain code 'ROAR' is written as 'URDU'.How is 'URDU' written in that code?
   a)VXDQ
   b)**XUGX**
   c)ROAR
   d)VSOV
5. In certain code 'LIMCA' is written as 'HJLDZ'.Which of the following word is written as 'IFWJBP'?
   a)MERCURY
   b)**MEXICO**
   c)JAPAN
   d)HONDUS
6. In certain code 'HILTON' is written as 'HJLDZ'.How is 'BILLION' written in that code?
   a)IBLLION
   b)IBOILLN
   c)**IBLLOIN**
   d)IBLOILN
7. If in the English alphabet,every alternate letter from B onwards is written i small letters while others are written in capitals then how will the third day from Tuesday will be coded?
   a)WeDNeSdAY
   b)WEdnESdAY
   c)**FridAY**

d)THURSdAY

8. In a certain code 'CERTAIN' is coded as 'BFQUZJM'. How is 'MUNDANE' coded in that code?
   a)**LVMEZOD**
   b)NTCOMBF
   c)NTOCNBF
   d)LTMCZO

CLOCKS

1. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?
2. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions?
3. An accurate clock shows 8 o'clock in the morning. Through how may degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
4. A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?
5. At what time between 5 and 6 o' clock are the hands of a 3 minutes apart ?
6. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25?
7. At what angle the hands of a clock are inclined at 15 minutes past 5?
8. At what time between 2 and 3 o'clock will the hands of a clock be together?

CALENDAR

1. What was the day on 15th august 1947 ?
2. Today is Monday. After 61 days, it will be?
3. The last day of a century cannot be?
4. What was the day of the week on, 16th July, 1776?
5. It was Sunday on Jan 1, 2006. What was the day of the week Jan 1, 2010?
6. What was the day of the week on 28th May, 2006?
7. What will be the day of the week 15th August, 2010?
8. If 6^{th} March, 2005 is Monday, what was the day of the week on 6^{th} March, 2004?

UNIT-2 QUANTITATIVE APTITUDE

AVERAGE:

1. The average of 5, 10,15,20,25?
2. Find the average of first 40 natural numbers.
3. The average of five consecutive odd numbers is 61. What is the difference between the highest and least numbers.
4. The average of four consecutive odd numbers is 61 what is the difference between the highest and lowest numbers?

5. The average of 5 numbers is 15 and the average of first three numbers is 10. What is is the average of last two numbers?
6. The average age of 15 students of a class is 15 years. Out of these, the average age of 5 student is 14 year and that of the other 9students is 16 years. The age of the 15th student is?
7. The average of 5 numbers is 15 and the average of first three numbers is 10 and the average of last three numbers is 20. Then find the middle number?
8. The average of five numbers is 27. If one number is excluded, the average becomes 25. The excluded number is:

RATIO & PROPORTION:

1. If A : B = 2 : 3 B : C = 4 :7 then find A : B : C =?
2. If A : B = 2 : 3 B : C = 3 :4 then find A : B : C =?
3. If a:b=2:3 and b:c=3:5 then find a:c=?
4. If 2A=3B and 4B=5C, then A:C is
5. Find the mean proportional of 9 and 25
6. Find the third proportional to 16 and 4
7. If $\frac{A}{3} = \frac{B}{4} = \frac{c}{5}$, then A:B:C is
8. If $\frac{1}{5} : \frac{1}{x} :: \frac{1}{x} : \frac{1}{125}$, then the value of x is

PROBLEMS ON AGES:

1. A father said his son , " I was as old as you are at present at the time of your birth. " If the father age is 38 now, the son age 5 years back was :
2. The total age of A and B is 12 years more than the total age of B and C. C is how many years younger than A ?
3. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, the present age of B is ?
4. The age of a man is 4 times of his son. Five years ago, the man was nine times old as his son was at that time. The present age of man is?
5. The sum of the present ages of a father and his son is 60 years. five years ago, father's age was four times the age of the son. so now the son's age will be:
6. Six years ago Anita was P times as old as Ben was. If Anita is now 17 years old, how old is Ben now in terms of P ?
7. Sachin is younger than Rahul by 7 years. If the ratio of their ages is 7:9, find the age of Sachin.
8. The ratio of the present ages of P and Q is 3 : 4. Five years ago, the ratio of their ages was 5: 7. Find their present ages.

TIME - DISTANCE-SPEED

1. An athlete runs 200 metres race in 24 seconds. His speed is?
2. How many minutes does Aditya take to cover a distance of 400m, if he runs at a speed of 20 km/hr?
3. A car is running at speed of 108kmph. What distance will it cover in 15 seconds?
4. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

5. Peter can cover a certain distance in 1hr.24min. by covering two third of the distance at 4kmph and the rest at 5kmph. Find the total distance.
6. A and B are two stations 390km apart. A train starts form A at 10 a.m. and travels towards B at 65kmph. Another train starts form B at 11a.m.and travels towards A at 35 kmph. At what time do they meet?

PERCENTAGES

1. 8 ⅓ % expressed as fraction is **?**
2. 2 is what percent of 50?
3. What percent of $\frac{1}{2}$ is $\frac{1}{3}$?
4. X% of Y is Y% of ?
5. What is 25% of 25% equal to?
6. 30% of 140 = ? % of 840
7. 5 % of (50% of $\overline{\text{Rs 300)}}$ is?
8. 270 candidates appeared in an examination, of which 252 passed. The pass percentage is.

PROFIT AND LOSS

1. A man buys a cycle for Rs.1400 and sells it at a loss of 15%. What is the selling price of the cycle?
2. The CP of 21 articles is equal to SP of 18articles. Find the gain (or) loss percent
3. A man buys on article for Rs 27.50 and sells it for Rs 28.60. find his gain percent ?
4. An article is bought for RS. 450 and sold for Rs.400 .what is the loss%?
5. When a commodity is sold for Rs. 34.80 there is a loss of 25%, what is the cost price of commodity?
6. An article is sold at certain price. By selling it at $\frac{2}{3}$ of that price one loses 10%. Find the gain percent at original price..
7. Meena purchased two fans each at Rs.1200. She sold one fan at the loss of 5% and other at the gain 10%.Find the total gain or loss percent?

PARTNERSHIP

1. Dhilip and Manohar started a business by investing Rs.100000 and Rs.150000 respectively. Find the share of each out of a profit of Rs.24000?
2. Sanjay and Raju started a business and invested Rs.20000 and Rs.25000 respectively. After 4 months Raju left and Naresh joined by investing Rs.15000.At the end of the year there was a profit of Rs.4600. what is the share of Naresh?
3. Three partners A, B, C starts a business. Twice the investment of A is equal to thrice the capital of B and the capital of B is four times the capital of C. finds the share of each out of a profit of Rs.297000?
4. A, B, C hire meadow for Rs.2934.60. A puts in 10 oxen for 20 days; B 30 oxen for 8 days and C 16 oxen for 9 days. Find the rent paid by each?
5. A and B started a business in partnership by investing Rs.8000 and Rs.7000 respectively. If at the end of a year, a profit of Rs.22, 500 was earned. What is the share of A?
6. In partnership business, A has invested Rs.4200 while B has invested a certain amount. If out of the overall profit of Rs.600, A’s share is Rs.320, what is the amount invested by B (in Rs)?
7. Chetan and Suman started a business in partnership by investing Rs.15000 and Rs.18000 respectively. If at the end of the year, Chetan’s share in the profit was Rs.1200, what was the amount of total profit?

8. In a partnership business, A has invested 2000 for 5months, while B has invested Rs.3500 for a certain period. If out of the total annual profit of Rs.1440, B's share has been Rs.840. For how many months has he kept his investment in the business?

SIMPLE AND COMPOUND INTEREST

1. Find the simple interest on Rs 7500 in 4 years at 15% .
2. The simple interest on Rs. 6400 at $12\frac{1}{2}$ % per annum is Rs.2000, find the period
3. On what sum of money will the simple interest be Rs.2000 in 5 years 8% per annum?
4. A sum of Rs 1600 gives a simple interest of Rs252 in 2years and 4months. The rate of interest per annum is ?
5. Find the compound interest on Rs 8000 for 3years at 5% per annum
6. A sum of Rs. 3000 is lent for 3 years at 10% p.a compound interest. Find the amount
7. Find the amount on Rs 7500 at 4% per annum for 2 years compounded annually.
8. Find the compound interest on Rs.15,625 for 9 months at 16% per annum compounded Quarterly.

UNIT-3 DATA ANALYSIS

1. DIRECTIONS: Study the table carefully to answer the questions that follow: Maximum and minimum Temperature (in degree Celsius) recorded on first day of each month for five different cities.

Month	Temperature									
	Bhuj		Sydney		Ontario		Kabul		Beijing	
	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
1^{st}sep	24	14	12	2	5	1	34	23	12	9
1^{st}oct	35	21	5	-1	15	6	37	30	9	3
1^{st}nov	19	8	11	3	4	0	45	36	15	1
1^{st}dec	9	2	-5	-9	-11	-7	31	23	2	-3
1^{st}jan	-4	-7	-11	-13	-14	-19	20	11	5	-13

1. What is the difference between the max temperature of Ontario on 1^{st}nov and the min temperature of Bhuj on 1^{st}jan?

(1) 3 ℃ (2) 18 ˚C (3) 15 ℃ (4) 9 ℃ **(5) 11℃**

ANS: (5) Required difference = 4 - (-7) = 4 + 7 = 11

2: In which month respectively the max temperature of Kabul is 2^{nd} highest and min temperature of Sydney is highest?

(1) 1^{st}oct& 1^{st}jan (2) **1^{st}oct& 1^{st}nov** (3) 1^{st}dec& 1^{st}jan (4) 1^{st}sept& 1^{st}jan (5) 1^{st}dec& 1^{st} Sept

ANS: (1)

3: In which month on 1^{st} day is the difference between the max temperature & min temperature of Bhuj second highest?

(1) 1^{st}sept (2) 1^{st}oct **(3) 1^{st}nov** (4) 1^{st}dec (5) 1^{st}jan

ANS: (3) Temperature difference of Bhuj: 1^{st} Sept: 24-14=10℃, **1^{st} Nov: 19-8=11℃**,
1^{st} Oct: 35-21=14℃, 1^{st} Dec: 9-2=7℃, 1^{st} Jan -4+7=3℃

4: What is the average maximum temperature of Beijing over all the months together.

(1) 8.4℃ (2) 9.6℃ (3) 7.6℃ (4) 9.2℃ **(5) 8.6℃**

ANS: (5) Max temperature =12+9+15+2+5/5 = 43/5=8.6℃

5: What is the respective ratio between the min temperature of Beijing on 1st sept & the max temperature of Ontario on 1st Oct ?

(1) 3:4 **(2) 3:5** (3) 4:5 (4) 1:5 (5) 1:4

ANS: (2) required ratio = 9:15 = 3:5

2. Study the following table carefully answer the questions percentage of marks obtained by 6 students in 6 different subjects.

Sub/student	History (out of 50)	Geography (out of 50)	Maths(out 150)	Science(out 100)	English (out of 75)	Hindi (out of 75)
Amit	76	85	69	73	64	88
Bharat	84	80	85	78	73	92
Umesh	82	67	92	87	69	76
Nikhil	73	72	78	69	58	83
Pratiksha	68	79	64	91	66	65
Ritesh	79	87	88	93	82	72

1. What is the approximately the integral % of marks obtained by umesh in all the subjects?

(1) **80%** (2) 84% (3) 86% (4)78% (5) 77%

ANS: (1) Total marks obtained by Umesh = 41+33.5+92/100*150+87+69100*75+76/100*5

= 41+33.5+138+87+51.75+57=408.25

Required % = 408/500*100=80%

2. What is the avg % of marks obtained by all the students in hindi (approximated to two places of decimal)

(1) 77.45% **(2) 79.33%** (3) 75.52% (4) 73.52% (5) None of these

ANS: (2) Required avg of % in hindi =88+92+76+83+65+72/6=476/6=79.33%

3. What is the average marks of all the students in Mathematics?

(1) 128 (2) 112 **(3) 118** (4) 138 (5) 144

ANS: (3) avg. mark in mathematics = 15.

(69+85+92+78+64+88)/100*6 =150*476/100*6=119

4. What is the average marks obtained by all the students in geography?

(1)38.26 (2) 37.26 (3) 37.16 **(4) 39.16** (5) None of these

ANS: (4) Average marks in geography = 50(85+80+67+72+79+87)/6*1/100

= 50*470/6*1/100 = 39.16

5. What are the total marks obtained by pratiksha in all the subjects taken together?
   (1)401.75 (2) 410.75 (3) 402.75 (4) 420.75 **(5) None of these**

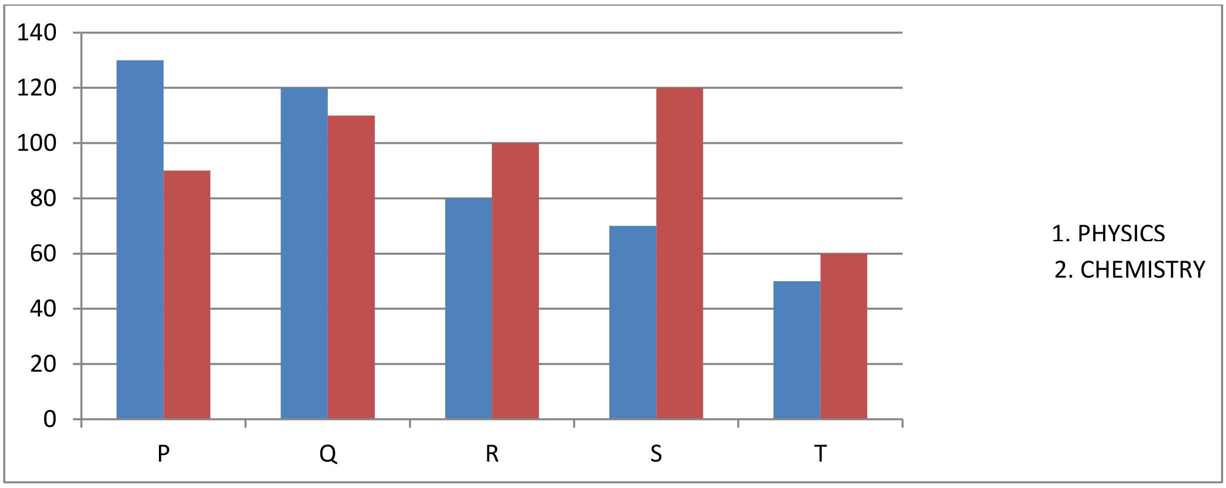
ANS: (5) marks obtained by

Ritesh = 50*68/100+50*79/100+150*64/100+91+75*66/100+50*86/100+75*65/100

=34+39.5+96+91+49.5+48.75 = 358.75

BAR GRAPHS

1. Study the following bar graphs carefully to answer these questions. Marks obtained by 5 students in physics & chemistry.



1. Marks obtained by S in chemistry is what percent of the total marks obtained by all the students in chemistry?
   (1) 25 (2) 28.5 (3) 35 (4) 31.5 (5) 22

ANS: (1) required %=120/90+110+100+120+60*100=120/480*100=25%

2. If the marks obtained by T in physics were increased by 14% of the original marks, what would be his new approximate % in physics if the max marks in physics were 140?
   (1)57 (2) 32 (3) 38 (4) 48 **(5) 41**

ANS: (5) increase in marks in physics of T=50*1.14=57. Required %= 57/140*100=40.7=41

3. What is the respective ratio between the total obtained by P in physics & chemistry together to the total marks obtained by T in physics & chemistry together?
   (1) 3:2 (2) 4:3 (3) 5:3 **(4) 2:1** (5)None of these

ANS: (4) required ratio = 130+90/60+50 = 220/110 = 2:1

4. What is the respective ratio between the total marks obtained by Q & S together in chemistry to the total marks obtained by P & R together in physics?

   (1) 23:25 **(2) 23:21** (3) 17:19 (4) 17:2 (5) None of these

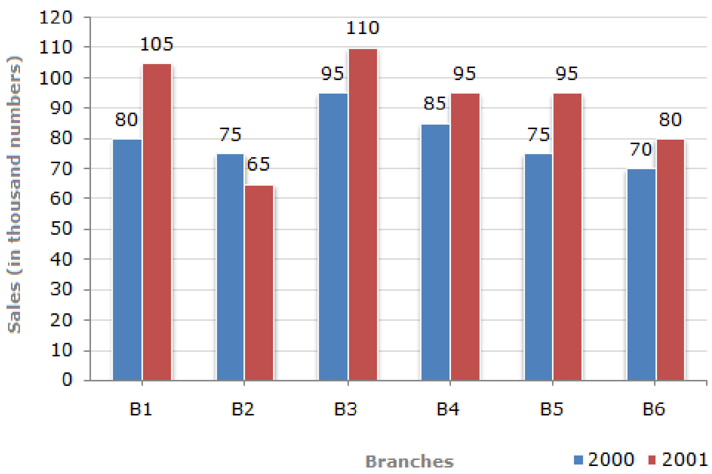
ANS: (2) Marks obtained by Q & S in chemistry=110+120=230.

Marks obtained by P & R in physics=130+80=210.

Required ratio=230/210=23:21.

2. The bar graph given below shows the sales of books (in thousand number) from six branches of a publishing company during two consecutive years 2000 and 2001.

Sales of Books (in thousand numbers) from Six Branches - B1, B2, B3, B4, B5 and B6 of a publishing Company in 2000 and 2001.



1. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?

A. 2:3 B. 3:5 C. 4:5 **D. 7:9**

Ans: D

Explanation: Required ratio $=\frac{(75+65)}{(85+95)} = \frac{140}{180} = \frac{7}{9} = 7:9$

2. Total sales of branch B6 for both the years is what percent of the total sales of branches B3 for both the years?

A. 68.54% B. 71.11% **C. 73.17%** D. 75.55%

Ans: 73.17%

Explanation: Required percentage $= \frac{70+80}{95+110} \times 100 = \frac{150}{205} \times 100 = 73.17$

3. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000?

A. 75% B.77.5% C. 82.5% **D.87.5%**

Ans: 87.5%

Explanation:

Average sales (in thousand number) of branches B1, B3 and B6 in 2000 $= \frac{1}{3} \times (80+95+70) = \frac{245}{3}$

Average sales (in thousand number) of branches B1, B2 and B3 in 2001$= \frac{1}{3} \times (105+65+110) = \frac{280}{3}$

Therefore, Required percentage = $= \frac{\frac{245}{3}}{\frac{280}{3}} \times 100 = \frac{245}{280} \times 100 = 87.5$

4. What is the average sales of all the branches (in thousand numbers) for the year 2000?

A. 73 **B. 80** C. 83 D. 88

Ans. 80

Explanation: Average sales of all the six branches (in thousand numbers) for the year 2000=

$$\frac{1}{6} \times (80+75+95+85+7+70) = 80$$

5. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is ?

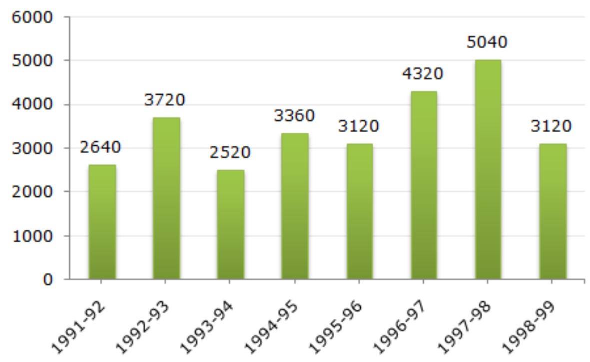
A. 250 B. 310 C. 435 **D. 560**

Explanation: Total sales of branches B1, B3 and B5 for both the years (in thousand numbers)

= (80 + 105) + (95 + 110) + (75 + 95) = 560.

3. The bar graph given below shows the foreign exchange reserves of a country (in million US $) from 1991 - 1992 to 1998 - 1999.

Foreign Exchange Reserves of a Country. (in million US $)



1. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves is?

   A. 2:6 B. 3:4 **C. 3:5** D. 4:4

   Ans: 3:5

   Explanation: Average foreign exchange reserves over the given period = 3480 million US $.

   The country had reserves above 3480 million US $ during the years 1992-93, 1996-97 and 1997-98, i.e., for 3 years and below 3480 million US $ during the years 1991-92, 1993-94, 1994-95, 1995-56 and 1998-99 i.e., for 5 years.

   Hence, required ratio = 3:5.

2. The foreign exchange reserves in 1997 - 98 was how many times that in 1994-95?

   A. 0.7 B. 1.2 C. 1.4 **D. 1.5**

   Ans:1.5

   Explanation: Required ratio=$\frac{5040}{3360} = 1.5$

3. For which year, the percent increase of foreign exchange reserves over the previous year, is the highest?

   A. 1992-93 B. 1993-94 C. 1994-95 D. 1996-97

   Ans:1992-93

   Explanation: There is an increase in foreign exchange reserves during the years 1992 - 1993, 1994 - 1995, 1996 - 1997, 1997 - 1998 as compared to previous year (as shown by bar-graph).

   The percentage increase in reserves during these years compared to previous year are:

   For 1992 – 1993 = $\frac{3720 - 2640}{2640} \times 100 = 40.91$

   For 1996 – 1997 = $\frac{4320 - 3120}{3120} \times 100 = 38.46$

   For 1997 – 1998 = $\frac{5040 - 4320}{4320} \times 100 = 16.67$

Clearly, the percentage increase over previous year is highest for 1992 - 1993.

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review?

A. 95% B. 110% C. 115% **D. 125%**

Ans: 125%

Explanation: Average foreign exchange reserves over the given period

$$= \frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)$$

$$= 3480 \text{ million US \$}$$

Foreign exchange reserves in 1996 - 1997 = 4320 million US $.

Required percentage= $\frac{4320}{3480} \times 100 = 124.14 \approx 125.$

5. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?

A. 100 B. 150 C. 200 D. 620

Ans:100%

Explanation: Foreign exchange reserves in 1997 - 1998 = 5040 million US $.

Foreign exchange reserves in 1993 - 1994 = 2520 million US $.

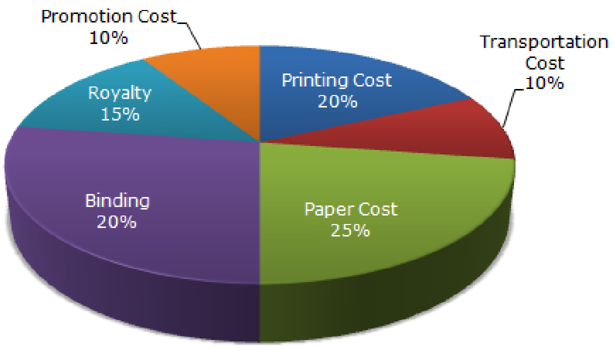
Increase = 5040 – 2520 = 2520 US $.

Percentage increase = $\frac{2520}{2520} \times 100 = 100$

PIE CHARTS

4. The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and the answer the questions based on it.

Expenditures (in percentage) Incurred in Publishing a Book



1. If for a certain quantity of books, the publisher has to pay Rs. 30, 600 as printing cost, then what will be amount of royalty to be paid for these books?

A. Rs.19,450 B. Rs.21,200 **C. Rs.22,950** D. Rs.26,150

Ans: Rs.22,950

Explanation: Let the amount of Royalty to be paid for these books be Rs. *r*.

Then, 20 : 15 = 30600 : *r* $\Rightarrow \frac{30600 \text{ x } 15}{20} = \text{Rs.} 22{,}950$

2. What is the central angle of the sector corresponding to the expenditure incurred on Royalty?

A. 15º B. 24º **C. 54º** D. 48º

Ans: 54º

Explanation: Central angle corresponding to Royalty = (15% of 360)º = $\left(\frac{15}{100} \times 360\right)^{o} = 54^{o}$

3. The price of the book is marked 20% above the C.P. If the marked price of the book is Rs. 180, then what is the cost of the paper used in a single copy of the book?

A. Rs.36 **B. Rs.37.50** C. Rs.42 D. Rs.44.25

Explanation:

Clearly, marked price of the book = 120% of C.P.

Also, cost of paper = 25% of C.P

Let the cost of paper for a single book be Rs. *n*.

Then, 120 : 25 = 180 : *n* ⇒ $n = \frac{25\ x}{120}$ = Rs. 37

4. If 5500 copies are published and the transportation cost on them amounts to Rs. 82500, then what should be the selling price of the book so that the publisher can earn a profit of 25%?

A. Rs.187.50 B. Rs.191.50 C. Rs.175 D. Rs.180

Ans: Rs.187.50

Explanation:

For the publisher to earn a profit of 25%, S.P. = 125% of C.P.

Also Transportation Cost = 10% of C.P.

Let the S.P. of 5500 books be Rs. *x*.

Then, $10:125 = 82500:x \Rightarrow x = \frac{125 \text{ x } 82500}{10} = 1031250$, Rs

S.P. of one book= Rs. $\frac{1031250}{5500} = 187.50$

5. Royalty on the book is less than the printing cost by:

A. 5% B. 33 1/5 % C. 20% **D. 25%**

Ans: 25%

Explanation:

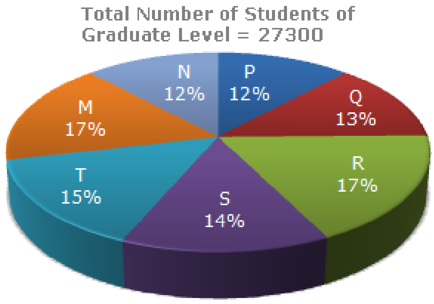
Printing Cost of book = 20% of C.P.

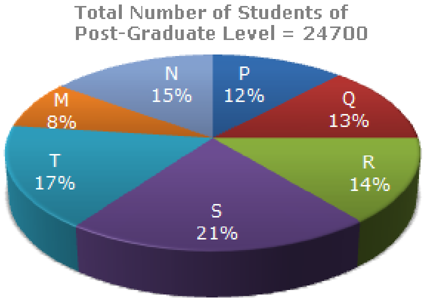
Royalty on book = 15% of C.P.

Difference = (20% of C.P.) - (15% of C.P) = 5% of C.P.

Percentage difference=$\frac{\text{difference}}{\text{printing cost}} \times 100 = \frac{\text{5\% of C.P.}}{\text{Printing Cost}} \times 100 = 25\%$

1. The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes in a town.





Distribution of students at graduate and post-graduate levels in seven institutes

1. What is the total number of graduate and post-graduate level students is institute R?

   A. 8320 B. 7916 C. 9116 **D. 8099**

   Ans: 8099

   Explanation: Required number = (17% of 27300) + (14% of 24700) = 4641+3458=8099.

2. What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?

   A. 14:19 B. 19:21 C. 17:21 **D. 19:14**

   Ans: 19:14

   Explanation: Required ratio $=\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{(21 \times 24700)}{14 \times 27300} = \frac{19}{14}$

3. How many students of institutes of M and S are studying at graduate level?

   A. 7516 **B. 8463** C. 9127 D. 9404

   Explanation: Students of institute M at graduate level= 17% of 27300 = 4641.
   Students of institute S at graduate level = 14% of 27300 = 3822.

   Total number of students at graduate in institutes M and S = (4641 + 3822) = 8463

4. What is the ratio between the number of students studying at post-graduate level from institutes S and the number of students studying at graduate level from institute Q?

   A. 13:19 B. 21:13 C. 13:8 **D. 19:13**

   Explanation: Required ratio = $\frac{21\% \text{ of } 24700}{13\% \text{ of } 27300} = \frac{21 \times 24700}{13 \times 27300} = \frac{19}{13}$

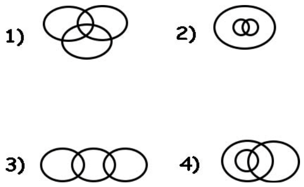
5. Total number of students studying at post-graduate level from institutes N and P is

   A. 5601 B. 5944 **C. 6669** D. 8372

Explanation: Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669

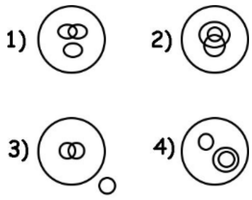
VENN DIAGRAM

1. Which of the following Venn- diagram correctly illustrates the relationship among the classes : Tennis fans, Cricket players, Students



Ans. 1

2. In a dinner party both fish and meat were served. Some took only fish and Some only meat. There were some vegetarians who did not accept either. The rest accepted both fish and meat. Which of the following Venn-diagrams correctly reflects this situation?



Ans. 1

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II year B.Sc., Degree Examinations - IV Semester
Mathematics Course-IV: REAL ANALYSIS
(w.e.f. 2020-21 Admitted Batch)

Total Hrs. of Teaching-Learning: 75 @ 6 hr/Week **Total credits: 04**

OBJECTIVES:

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.

This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

Course Outcomes:

After successful completion of this course, the student will be able to

- get clear idea about the real numbers and real valued functions.
- obtain the skills of analyzing the concepts and applying appropriate methods for testingconvergence of a sequence/ series.
- Test the continuity and differentiability and Riemann integration of a function.
- Know the geometrical interpretation of mean value theorems.

UNIT I (12 Hours)

Introduction of Real Numbers (No question is to be set from this portion)

Real Sequences: Sequences and their limits, Range and Boundedness of Sequences, Limit of a sequence and Convergent sequence. The Cauchy's criterion, properly divergent sequences, Monotone sequences, Necessary and Sufficient condition for Convergence of Monotone Sequence, Limit Point of Sequence, Subsequences , Cauchy Sequences – Cauchy's general principle of convergence theorem.

UNIT II: (12 Hours)

INFINITIE SERIES :

Series : Introduction to series, convergence of series. Cauchy's general principle of convergence forseries tests for convergence of series, Series of Non-Negative Terms.

1. P-test
2. Cauchy's n^{th} root test or Root Test.
3. D'-Alemberts' Test or Ratio Test.
4. Alternating Series – Leibnitz Test.

UNIT III: (12 Hours)

CONTINUITY:

Limits: Real valued Functions, Boundedness of a function, Limits of functions. Some extensions of the limitconcept, Infinite Limits. Limits at infinity. (No question is to be set from this portion).

Continuous functions: Continuous functions, Combinations of continuous functions, Continuous Functionson interval.

UNIT IV: **(12 Hours)**

DIFFERENTIATION AND MEAN VALUE THEOREMS: The derivability of a function, on an interval, at a point, Derivability and continuity of a function, Graphical meaning of the Derivative, Mean value Theorems; Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem

UNIT V: **(12 Hours)**

RIEMANN INTEGRATION : Riemann Integral, Riemann integral functions, Darboux theorem. Necessary and sufficient condition for R – integrability, Properties of integrable functions, Fundamental theorem of integral calculus, First mean value Theorem.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Real Analysis and its applications / Problem Solving.

TEXT BOOK:

1. Introduction to Real Analysis by Robert G.Bartle and Donlad R. Sherbert, published by JohnWiley.

REFERENCE BOOKS:

1. A Text Book of B.Sc Mathematics by B.V.S.S. Sarma and others, published by S. Chand &Company Pvt. Ltd., New Delhi.
2. Elements of Real Analysis as per UGC Syllabus by Shanthi Narayan and Dr. M.D. Raisinghania,published by S. Chand & Company Pvt. Ltd., New Delhi

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**Real Sequences**	2	2	30
II	**Infinite Series**	2	2	30
III	Continuity	2	2	30
IV	Differentiation And Mean value theorems	1	2	25
V	Riemann Integrations	1	2	25
Total		8	10	140

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 4 X 10 = 40

……………….

Total Marks = 60

………………..

P.R. Government College (Autonomous), Kakinada
II Year, B.Sc., Degree Examinations - IV Semester
Mathematics Course: Real Analysis
Paper–IV (Model Paper ((w.e.f. 2020-21 Admitted Batch)

Time: 2Hrs 30 min **Max. Marks: 60**

PART-I

Answer any FOUR questions. Each question carries FIVE marks. **4 X 5 M=20 M**

1. Show that $\lim_{n\to\infty}\left[\sqrt{\frac{1}{n^2+1}}+\sqrt{\frac{1}{n^2+2}}+\cdots+\sqrt{\frac{1}{n^2+n}}\right]=1.$
2. Prove that every convergent sequence is a Cauchy sequence
3. If $\sum u_n$ convergences absolutely then prove that $\sum u_n$ converges.
4. Test for the convergence of $\sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots2n}x^{n-1}\ (x>0)$
5. Examine for continuity the function f defined by $f(x)=\ |x|+|x-1|$ at 0 and 1
6. Prove that $f: R\to R$ given by $f(x)=x^2$ is a continuous function on R, but not uniformly continuous on R
7. Show that $f(x)=x\ sin(1/x), x\neq 0; f(x)=0, x=0$ is continuous but not derivable at $x=0$.
8. State and prove fundamental theorem of Integral Calculus.

PART - II

Answer ALL questions. Each question carries Eight marks. **5 X 8 M = 40 M**

9 (a) . State and Prove monotone convergence theorem.

(OR)

(b). Prove that the sequence $\{s_n\}$ defined by $s_n=1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}$ is convergent

10 (a) . State and prove Cauchy's n^{th} root test .

(OR)

(b) Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

11 (a) If f is a continuous function on $[a,b]$, then prove that it is uniformly continuous on $[a,b]$

(OR)

(b) Test the continuity of of $f(x)=\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\ if\ x\neq 0\ and\ f(0)=0$ at $x=0$.

12 (a) Show that $\frac{v-u}{1+v^2}<tan^{-1}v-\ tan^{-1}u<\frac{v-u}{1+u^2}\ for\ 0<u<v.$ Hence deduce that $\frac{\pi}{4}+\frac{3}{25}<tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}.$

(OR)

(b) State and Prove Cauchy's mean value theorem

13 (a) Show that $f(x)=3x+1$ is integrable on[1,2] and $\int_1^2(3x+1)dx=\frac{11}{2}$.

(OR)

(b) State and Prove first mean value theorem of integral calculus.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–IV: REAL ANALYSIS

Short Answer Questions

UNIT-I

1. By the definition of limit of a sequence show that $\lim \sqrt[n]{n} = 1$.
2. Show that $\lim_{n\to\infty}\left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \cdots + \sqrt{\frac{1}{n^2+n}}\right] = 1$.
3. State and prove sandwich theorem
4. Prove that every convergent sequence is a Cauchy sequence
5. Prove that if $\{S_n\}$ is a Cauchy sequence, then $\{S_n\}$ is bounded.
6. Show that the sequence $\{a_n\}$ defined by $a_n = \left(1+\frac{1}{n}\right)^n$ is convergent.

UNIT-II

7. State and prove Cauchy's general principle of convergence for series.
8. Test for convergence of $\sum \frac{1}{2^n+3^n}$
9. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1}-\sqrt{n^3}\right)$.
10. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1}-n\right)$.
11. Test the convergence of $\sum_{n=1}^{\infty}\frac{1}{n^3}\left(\frac{n+2}{n+3}\right)^n x^n,\ \forall\, x>0$
12. Test for the convergence of $\sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}x^{n-1}\ (x>0)$
13. If $\sum u_n$ convergences absolutely then prove that $\sum u_n$ converges

UNIT-III

14. Examine the continuity of the function defined by f(x) = $|x|$ +$|x-1|$ at x = 0, 1.
15. Find the points of the discontinuity of f(x)= $\frac{1}{2^n}$ for $\frac{1}{2^{n+1}} < x \le \frac{1}{2^n}$, where n =0, 1,2,…
    and f(0) =0
16. If $f:[a,b]\to R$ is continuous on $[a,b]$, then prove that f is bounded on $[a,b]$. Is the converse true and justify your answer
17. Define uniform continuity and prove that every uniform continuous function is continuous.
18. Prove that $f:R\to R$ given by $f(x)=x^2$ is a continuous function on R, but not uniformly continuous on R.
19. Prove that a function defined on $[a,b]$ is uniformly continuous on $[a,\ b]$ if it is continuous on $[a,\ b]$

UNIT-IV

20. If $f:[a,b] \to R$ is derivable at $c \in [a,b]$, then prove that f is continuous on $[a,b]$. Is its converse true? Justify your answer.
21. Test the continuity and differentiability of $f(x) = x\left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\right)$ $if\ x \neq 0$ and $f(0) = 0$ at $x = 0$.
22. Show that $f(x) = x\,sin(1/x), x \neq 0; f(x) = 0, x = 0$ is continuous but not derivable at $x = 0$.
23. State and Prove Rolle's theorem
24. Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in [a, b] where $0 < a < b$.
25. Examine the applicability of Rolle's theorem for $f(x) = 1 - (x-1)^{2/3}$ on [0, 2].
26. Verify Lagrange's Mean Value theorem for f(x) = (x-1) (x-2) (x-3) on [0,4].
27. Using Lagrange's Mean Value theorem prove that $1 + x < e^x < 1 + xe^x,\ \forall x > 0$

UNIT-V

28. If $f(x) = x^2\ \forall\ x \in [0,1]$ and P= {0,1/4 , 2/4, ¾, 1}, then find *U(P,f)* and *L(P,f)*.
29. If f is continuous on the $[a,b]$ then prove that f is Riemann Integrable on the $[a,b]$.
30. If f is monotonic on the $[a,b]$ then prove that f is Riemann Integrable on the $[a,b]$.
31. Show that the function f defined by $f(x) = \begin{cases} 1\ if & x\ is\ rational \\ -1 if & x\ is\ irrational \end{cases}$ for all *x* in [0,1] is not R-integrable on [0,1].
32. Evaluate $\int_0^{\pi/4} (sec^4 x - tan^4 x)dx$

Essay Questions

UNIT-I

1. Discuss the nature of the sequence $\{r^n\}$ for all $-1 < r \leq 1$.
2. Prove that the sequence $\{s_n\}$ defined by $s_1 = \sqrt{c} > 0, s_{n+1} = \sqrt{c + s_n}\ for\ all\ n \in Z^+$ converges to the positive root of $x^2 - x - c = 0$.
3. State and Prove monotone convergence theorem.
4. Prove that the sequence $\{s_n\}$ defined by $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ is convergent.
5. Show that the sequence $\{a_n\}\ defined\ by\ a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent**.**
6. Prove that every bounded sequence has at least one limit point
7. State and prove Cauchy's convergence theorem on sequences
8. Prove that a sequence is convergent iff it is bounded and has only one limit point

UNIT –II

9. State and prove Limit comparison test.
10. State and prove Cauchy's n^{th} root test .
11. Test for convergence of $\sum_{n=1}^{\infty} \sqrt[3]{n^3} + 1 - n$.
12. State and prove D'Alemberts ratio test.
13. State and prove Leibnitz test
14. Prove that $\sum_{n=1}^{\infty}(-1)^n \ \sin\frac{1}{n}$ is not absolutely convergent.
15. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$.

UNIT-III

16. If $f: [a,b] \to R$ is continuous on $[a,b]$, then f is bounded on $[a,b]$ and attains its bounds or infimum and supremum.
17. Let f: R→ R be such that f(x) = $\frac{sin(a+1)x + sinx}{x}$ for $x < 0, f(x) = c\ for\ x\ = 0$ and $f(x)$ =$\frac{(x+bx^2)-x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}$ for x >0. Determine the values of a, b, c which the function is continuous at x = 0.
18. If f is a continuous function on $[a,b]$, then prove that it is uniformly continuous on $[a,b]$.
19. Test the continuity of of $f(x) = \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\ if\ x \neq 0\ and\ f(0) = 0$ at $x = 0$.
20. If f is a continuous function on $[a,b]$, then prove that it is uniformly continuous on $[a,b]$.
21. State and Prove Intermediate value theorem.

UNIT-IV

22. State and Prove Lagrange's mean value theorem .
23. Discuss the applicability of Lagrange's mean value theorem for
    $f(x) = x(x-1)(x-2)$ on [0,1/2].
24. Using Lagrange's mean value theorem, show that
    $$x > log(1+x) > \frac{x}{1+x}\ if\ f(x) = log(1+x)\ \forall x > 0$$
25. Show that $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$, $\forall x > 0$
26. Show that $\frac{v-u}{1+v^2} < tan^{-1}v - \ tan^{-1}u < \frac{v-u}{1+u^2}\ for\ 0 < u < v.$ Hence deduce that $\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.
27. Prove that $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < sin^{-1}0.6 < \frac{\pi}{6} + \frac{1}{8}$
28. State and Prove Cauchy's mean value theorem.

UNIT-V

29. Show that $f(x) = 3x + 1$ is integrable on[1,2] and $\int_1^2 (3x+1)dx = \frac{11}{2}$.

30. Prove that $f(x) = x^2$ is integrable on $[0, a]$ and $\int_0^a x^2 dx = \frac{a^3}{3}$.

31. Prove that $f(x) = sinx$ is integrable on $[0, \frac{\pi}{2}]$ and $\int_0^{\pi/2} sinx\, dx = 1$.

32. State and prove Necessary and Sufficient condition for integrability.

33. If $f(x) = \frac{1}{2^n}$ when $\frac{1}{2^{n-1}} < x \leq \frac{1}{2^n}$; $(n = 0,1,2,3, \dots)$ and $f(0) = 0$ then prove that f is integrable on [0,1].

34. Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\, cosx}\, dx \leq \frac{\pi^3}{6}$.

35. Show that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$

36. State and prove fundamental theorem of Integral Calculus.

37. State and Prove first mean value theorem of integral calculus.

38. Using first mean value theorem of integral calculus show that

$$x > \log(1+x) > \frac{x}{1+x}, \quad x \geq 0$$

P.R. GOVERNMENT COLLEGE (A), KAKINADA
II B.Sc., MATHEMATICS – Semester IV (w.e.f. 2020-21)
Paper V : Linear algebra

Total Hrs. of Teaching Learning & Evaluation: 75 @ 6 h / Week Total Credits: 05

Objective:

- To improve the students ability of understanding the most application oriented topic in Mathematics that is Linear Algebra.
- To equip the skill of understanding the concepts and writing the proofs of the Theorems.

Unit - I: Vector Spaces – I (12 Hrs)

Vector spaces, General properties of vector spaces, n-dimensional vectors, Addition and scalar multiplication of vectors, Internal and external composition, Null Space, Vector Subspaces, Algebra of subspaces, Linear sum of two subspaces, Linear combination of vectors, Linear span, Linear dependence and linear independence of Vectors.

Unit - II: Vector spaces – II (12 Hrs)

Basis of vector space, Finite dimensional vector space, Basis extension, Co-ordinates, Dimension of vector space, Dimension of subspace, Quotient space and Dimension of Quotient space.

Unit - III: Linear transformations (12 Hrs)

Linear transformations, Linear operators, Properties of linear transformation, Sum and product of linear transformations, Algebra of Linear Operators, Range space and Null Space of LT, Rank and Nullity of a LT, Rank & Nullity theorem.

Unit - IV: Matrix (12 Hrs)

Linear Equations, Characteristic Values and Characteristic Vectors of square matrix – Cayley - Hamilton Theorem.

Unit - V: Inner Product Space (12 Hrs)

Inner Product spaces, Euclidean and Unitary spaces, Norm or length of a vector, Schwartz's inequality, Triangle Inequality, Parallelogram law, Orthogonality and orthonormal set, Complete orthonormal set, Gram-Schmidt Orthogonalisation Process, Bessel's inequlity and Parsvel's identity.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Additional Inputs: Diagonalization of a matrix.

Prescribed Text Books:

J.N. Sharma & A.R.Vasista, Linear Agebra, Krishna Prakasham Mandir, Meerut.

Books for Reference:

1. III year Mathematics Linear Algebra and Vector Calculus, Telugu Academy.
2. A Text Book of B.Sc. Mathematics, Vol-III, S. Chand & Co.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV PAPER-V

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**Vector Spaces – I**	2	2	30
II	**Vector Spaces – II**	2	2	30
III	**Linear transformations**	2	2	30
IV	**Matrix**	1	2	25
V	**Inner Product Space**	1	2	25
Total		8	10	140

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 4 X 10 = 40

……………….

Total Marks = 60

………………..

P.R. Government College (Autonomous), Kakinada
II Year, B.Sc., Degree Examinations - IV Semester
Mathematics Course: Linear Algebra
Paper–IV (Model Paper ((w.e.f. 2020-21 Admitted Batch)

Time: 2Hrs 30 min **Max. Marks: 60**

PART-I

Answer any FOUR questions. Each question carries FIVE marks. **4 X 5 M=20 M**

1. Determine whether the set of vector {(1, -2, 1), (2, 1, -1), (7, -4, 1)} is linearly dependent or Linearly independent.

2. Let p, q, r be the fixed elements of a field F.Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a vector space of $V_3(F)$.

3. Show that the set $\{(1,0,0), (1,10), (1,1,1)\}$ is a basis of $C^3(C)$. Hence find the coordinates of the vector (3+4i, 6i, 3+7i) in $C^3(C)$.

4. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is also finite dimensional and dim $W \le$ dim V.

5. Find a linear transformation $T: U \to V$ be such that whose basis and range are $\{(1,2,1), (2,1,0), (1,-1,-2)\}$ *and* $\{(1,0,0), (0,1,0), (1,1,1)\}$.

6. Find T(x, y, z) where T: $R^3 \to R$ is defined by T(1,1,1)=3, T(0,1,-2)=1, T(0,0,1)= -2.

7. Solve the following system of linear equations $2x - 3y + z = 0,\ x + 2y - 3z = 0,\ 4x - y - 2z = 0$.

8. State and prove Parallelogram Law

PART - II

Answer FOUR questions. Choose At lesat one from each section. Each question carries TEN marks. **4 X 10 M = 40 M**

SECTION – A

9. (a) Prove that a non empty subset W of a vector space $V(F)$ is a subspace of V if and only if $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.

(OR)

(b) Let V(F) be a vector space and S = { $\alpha_{1,}\alpha_{2,}...\alpha_n$ } is a finite subset of non-zero vectors of V(F). Then S is linearly dependent if and only if some vector $\alpha_k \in$ S, $2 \le k \le n$, can be expressed as a linear combination of its preceding vectors.

10. (a) State and prove the Basis extension theorem

(OR

(b) Let W be a sub space of a finite dimensional vector space V(F), then prove that

dim $(\frac{V}{W})$ = dimV – dimW.

11. (a) Find the null space ,rank, range ,and nullity of transformations $T: R^2 \rightarrow R^3$ defined by

$T(x, y) = (x + y, x - y, y)$

(OR)

(b) State and prove rank and nullity theorem.

12 (a) State and prove Cayley- Hamilton theorem

(OR)

(b) Find the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

13 (a) State and Prove Bessel's inequality

(OR)

(b) Given $\{(2,1,3), (1,2,3), (1,1,1)\}$ is a basis of R^3 construct an orthonormal basis

P.R. GOVERNMENT COLLEGE (A), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–V: LINEAR ALGEBRA

Short answers

UNIT-I

1. Let V be the set of all pairs $(a,\ b)$ of real numbers. Show that V is not a vector space with the operations defined by $(a_1,\ b_1) + (a_2,\ b_2) = (a_1 + a_2, 0)$, $c(a_1,\ b_1) = (ca_1,\ b_1)$.

2. Prove that the intersection of any two subspaces W_1 and W_2 of vector space V(F) is subspace of V(F).

3. Let p, q, r be the fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a vector space of $V_3(F)$.
4. Let R be the field of real numbers and $W = \{(x, y, z): x, y, z \text{ are rational numbers}\}$. Is W is a subspace of $V_3(R)$.

5. Show that the subset $W = \{(a, b, c)/a^2 + b^2 + c^2 \leq 1\}$ is not a subspace of $R^3(R)$.

6. Prove that the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.

7. If α, β, γ are linearly independent vectors of $V(R)$, then show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.

UNIT-II

8. Show that the set of vectors {(2, 1, 4), (1, -1, 2), (3, 1, -2)} form a basis for R^3.

9. Show that the set S={(1, 0, 0), (0, 1, 0), (1, 1, 0), (1, 1, 1)} is a spanning set of $R^3(R)$, but not a basis.

10. Show that the set $\{(1,0,0), (1,10), (1,1,1)\}$ is a basis of $C^3(C)$. Hence find the coordinates of the vector (3+4i, 6i, 3+7i) in $C^3(C)$.
11. Find the coordinate of (2, 3, 4, -1) with respect to the basis $B = \{(1,1,1,2), (1, -1,0,0), (0,0,1,1), (0,1,0,0)\}$ of $V_4(R)$.

12. Prove that every set of $(n + 1)$ or more vectors in an n-dimensional vector space is linearly dependent.

13. If U = {(1,2,1), (0,1,2)} and W = {(1,0,0), (0,1,0)} determine the dimension of U+W.

14. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is also finite dimensional and $\dim W \leq \dim V$.

15. Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a,b,c,d): b - 2c + d = 0\}$, $W_2 = \{(a,b,c,d): a = d, b = 2c\}$. Find the basis and dimension of (i) W_1, (ii) W_2, (iii) $W_1 \cap W_2$ and hence find $dim(W_1 + W_2)$.

16. Let W be a subspace of a finite dimensional vector space V(F), then prove that $dim\, V/W = \dim V - \dim W$.

UNIT-III

17. Define linear transformation and show that the function $T: R^3 \to R^3$defined by $T(x,y,z) = (x - y, 0, y + z)$ is a linear transformation.

18. Find a linear transformation $T: R^3 \to R$ such that $T(1,1,1) = 3, T(0,1,-2) = 1$, $T(0,0,1) = -2$.

19. Find a linear transformation $T: U \to V$ be such that whose basis and range are $\{(1,2,1), (2,1,0), (1,-1,-2)\}\ and\ \{(1,0,0), (0,1,0), (1,1,1)\}$.

20. Let $T: R^3 \to R^3$ be defined by $T(x,y,z) = (x - y + 2z,\ 2x + y - z,\ -x - 2y)$. Then verify Rank-nullity theorem.

21. Describe explicitly the linear transformation $T: R^3 \to R^3$ whose range space is spanned by {(1, 0, -1), (1, 2, 2)}.

22. Let $U(F)$ and $V(F)$ be two vector spaces such that T: U(F) → V(F) be a linear transformation. Then define range set of T and prove that the range set $R(T)$ is a subspace of $V(F)$.

UNIT-IV

23. Solve the system of linear equations $2x - 3y + z = 0$, $x + 2y - 3z = 0$, $4x - y - 2z = 0$ are consistent or not.

24. Is it the system of equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ are consistent.

25. Find the eigen values and eigen vectors of the square matrix A=$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 4 \end{pmatrix}$.

26. Verify Cayley-Hamilton theorem for the matrix A=$\begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

27. State and Prove Triangle-Inequality.

28. State and prove Parallelogram law in an inner product space V(F).

29. State and prove Parseval's inequality in an inner product space V(F).

30. Prove that every orthogonal set of non-zero vectors in an Inner Product Space V(F) is linearly independent.

31. Prove that every orthonormal set of vectors is linearly independent.

32. Prove that the set S = $\left\{\left(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\right),\left(\frac{2}{3},\frac{-1}{3},\frac{2}{3}\right),\left(\frac{2}{3},\frac{2}{3},\frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

Essay questions

UNIT-I

1. Let V(F) be a vector space and $W \subseteq V$. Then prove that the necessary and sufficient conditions for W to be a subspace of V are

   (i) $\alpha \in W,\ \beta \in W \Longrightarrow \alpha - \beta \in W\ \ (ii) a \in F,\ \alpha \in W \Longrightarrow a\alpha \in W$

2. State and prove the necessary and sufficient condition for a non-empty subset of a vector space to be a subspace.

3. Prove that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.

4. If $W_1$and W_2 are two subspaces of a vector space V(F) then prove that

   (i) $W_1 + W_2$ is a subspace of $V(F)$ and (ii) $W_1 \subseteq W_1 + W_2\ \ and\ \ W_2 \subseteq W_1 + W_2$.

5. If S and T are the subsets of a vector space $V(F)$ then prove that

   (i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ and (ii) $L(S \cup T) = L(S) + L(T)$.

6. Let $V(F)$ be a vector space and $S = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a finite subset of non-zero vectors of $V(F)$. Then prove that S is linearly dependent if and only if some vector $\alpha_k \in S, 2 \leq k \leq n$ can be expressed as a linear combination of its preceding vectors.

UNTI-II

7. Let W_1 and W_2 be two subspaces of a finite dimensional vector space V(F). Then prove that $dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

8. State and Prove Basis Existence theorem.

9. State and prove Basis Extension theorem.

10. Prove that any two bases of a finite dimensional vector space V(F) have the same number of elements.

11. Define finite dimensional vector space. If W is a subspace of a finite dimensional vector space V(F), then prove that W is finite dimensional and dim W$\leq n$.

12. Let W be a subspace of a finite dimensional vector space V(F) then prove that $\dim(V/W) = \dim\ V - \dim W$.

UNIT-III

13. Let $L(U,V)$ be the vector space of all linear transformations from $U(F)$ to $V(F)$ such that $dim\ U = n$ and $dim\ V = m$, then prove that $dim\ L(U,V) = mn$.

14. Let $U(F)$ and $V(F)$ be two vector spaces and $T: \text{U} \rightarrow \text{V}$ is a linear transformation. Then prove that the range space $R(T)$ is a subspace of $V(F)$ and null space $N(T)$ is a subspace of $U(F)$.

15. State and prove Rank - Nullity theorem.

16. Let $T: R^3 \rightarrow R^3$ be defined by $T(x, y, z) = (x - y + 2z,\ 2x + y - z,\ -x - 2y)$. Then verify Rank-nullity theorem.

UNIT-IV

17. Find the characteristic roots and the corresponding vectors of the square matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

18. Find the characteristic equation of A and hence find A^{-1}, where

$$A = \begin{pmatrix} 1 & 2 & -4 \\ 3 & -1 & 2 \\ 2 & 5 & 0 \end{pmatrix}.$$

19. State and prove Cayley-Hamilton theorem.

20. Verify Cayley -Hamilton theorem for the matrix $A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

21. State and prove Cauchy- Schwarz's inequality.

22. If u and v are two vectors in a complex inner product space V(F), then prove that $4\,\langle u, v\rangle = \|u + v\|^2 - \|u - v\|^2 + i\|u + iv\|^2 - \|u - iv\|^2$.

23. Given $\{(1, -1, 2), (0, 2, 1), (1, 2, 0)\}$ is a basis of R^3(R). Construct an orthonormal basis using Gram-Schmidt orthogonalisation process.

24. In a real inner product space, if u and v are two vectors such that $\|u\| = \|v\|$, then prove that $u - v$ and $u + v$ are orthogonal.

P. R. GOVERNMENT COLLEGE (A), KAKINADA
III B.Sc. MATHEMATICS - Semester V (w.e.f. 2018-2019)
Paper V: Ring Theory & Vector Calculus

Total Hrs. of Teaching Learning & Evaluation: 75 @ 6 h / Week **Total Credits: 05**

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OBJECTIVES:

- To impart knowledge on Ring Theory and its applications.
- To make awareness of the concepts of the transformation between line Integral, Surface Integral and Volume integral.
- To introduce the concepts of geometrical meaning of Gradient, Divergence and Curl

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RING THEORY

Unit – I: Rings – I **(14 Hrs)**

Definition of Ring and Basic Properties, Boolean Rings, Divisors of Zero and Cancellation Laws in Rings, Integral Domain, Division Ring and Fields, The Characteristic of a Ring – The Characteristic of an Integral Domain, the Characteristic of a Field, Sub Rings and Ideals.

Unit – II: Rings – II **(12 Hrs)**

Definition of Homomorphism – Homomorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental Theorem of Homomorphism – Maximal Ideals – Prime Ideals.

VECTOR CALCULUS

UNIT – III: Vector Differentiation **(12 Hrs)**

Vector differentiation –Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient, Divergence, Curl operators, Formulae involving these operators.

UNIT – IV: Vector Integration **(12 Hrs)**

Line Integral, Surface Integral, Volume Integrals with examples.

Unit – V: Vector Integration Applications **(10 Hrs)**

Gauss Divergence Theorem, Stokes theorem, Green's Theorem in plane and applications of these theorems**.**

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Additional Inputs: Euclidean Ring definition and Examples.

Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

Books for Reference:

1. Topics in Algebra by I. N. Herstine
2. Abstract Algebra by J. Fralieh, Published by Narosa Publishing house
3. Vector Calculus by Santhi Narayan, Published by S.C hand & Company Pvt. Ltd., New Delhi
4. Vector Calculus by R. Gupta, Published by Laxmi Publications.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-V, PAPER-V

Unit	TOPIC	V.S.A.Q	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Rings – I	01	01	02	22
II	Rings – II	01	01	02	22
III	Vector differentiation	01	01	02	22
IV	Vector integration	01	01	01	14
V	Vector Integration Applications	01	01	01	14
TOTAL		05	05	08	94

E.Q = Essay questions (8 marks)
S.A.Q = Short answer questions (5 marks)
V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 5 x 8 M = 40
Short answer questions : 3 x 5 M = 15
Very Short answer questions : 5 x 1 M = 05

Total Marks = 60

P.R. Government College (Autonomous), Kakinada
III year B.Sc., Degree Examinations - V Semester
Mathematics: Ring Theory & Vector Calculus
Paper – V (Model Paper w. e. f. 2019-2020)

Time: 2 Hrs 30 Min **Max. Marks: 60M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. 5 x 1 = 5 M

1. Define Boolean Ring.
2. Find Kernel of the Homomorphism $f: Z(\sqrt{2}) \to Z(\sqrt{2})$ defined by $f(m+n\sqrt{2}) = m - n\sqrt{2}, \forall\, m+n\sqrt{2} \in Z(\sqrt{2})$.
3. Find div f, where f = grad($x^3+y^3+z^3-3xyz$).
4. Evaluate $\int_0^1 (e^t \bar{\imath} + 2e^{-2t}\bar{\jmath})dt$.
5. State Green's Identities.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks. 3 x 5 = 15 M

6. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
7. Let R and R' be two rings and $f: R \to R'$ be a homomorphism. Then prove that the Kernel of f is an ideal of R.
8. Prove that div Curl $\overline{f}$ = 0.
9. If $\overline{F} = y\bar{i} + z\bar{j} + x\bar{k}$, find the circulation of F round the curve C, where C is the Circle $x^2 + y^2 = 1, z = 0$.
10. Evaluate $\oint_C (\cos x \,.\sin y - xy)\, dx + \sin x.\cos y\, dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. 5 X 8 = 40 M

SECTION – A

11. If E = {0, 1, 2, 3, 4}, then prove that (E, $+_5$, $\times_5$) form a field. Justify your answer.
12. Define the characteristic of a ring. Prove that the characteristic of an integral domain is either a prime or zero.
13. State and prove fundamental theorem of homomorphism of rings.
14. Show that an ideal U of a commutative ring R with unity is maximal if and only if the quotient ring R/U is a field.

SECTION – B

15. Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A$.

16. Find the directional derivative of f = $x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of 2i-j-2k.

17. If $\overline{F} = 4xz\bar{i} - y^2\overline{j} + yz\overline{k}$, evaluate $\int \overline{F}.N$ dS where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a.$

18. Verify Stokes theorem for $A=(2x-y)\bar{i}-yz^2\bar{j}-y^2z\bar{k}$, where **S** is the upper half surface of the sphere $x^2+y^2+z^2=1$ and **C** is its boundary.

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P.R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–V: RING THEORY AND VECTOR CALCULUS

Very Short Answer Questions

Unit-I

1. Define ring and give one example.
2. Define Boolean ring with example.
3. Write the set of zero divisors of $(Z_{12}, +_{12}, x_{12})$.
4. Define integral domain and give one example.
5. Write the units of the ring of Gaussian Integers.
6. Give an example of a division ring which is not a field.
7. Define field and give one example.
8. What is the characteristic of $(Z_{12}, +_{12}, x_{12})$.
9. Find all the associates of $(2 - i)$ in the ring of Gaussian integers.
10. Define sub ring and explain with one example.
11. Define subfield and explain with one example.
12. Define ideal and explain with one example.
13. Give an example of a sub ring which is not an ideal.
14. Define principal ideal and explain with one example.
15. Define principal ideal ring and explain with one example.
16. Define quotient ring.

Unit-II

17. Define homomorphism, homomorphic image, monomorphism, epimorphism.
18. Define isomorphism and automorphism.
19. Define kernel of a homomorphism.
20. Is the ring $2Z$ is isomorphic to the ring $3Z$.
21. Define Maximal ideal and Prime ideal with example.
22. Give an example of a prime ideal which is not a maximal.

Unit-III

23. Define scalar point function and vector point function
24. Prove that $\nabla r = \frac{\bar{r}}{r}$
25. Define divergence of a vector.
26. Define Solenoidal vector.
27. Define Laplacian operator.
28. If $\emptyset = x^2 - y^2$, then show that $\nabla^2 \emptyset = 0$.
29. Define curl of a vector
30. Define irrotational vector.
31. If $F = (x + 3y)i + (y - 2z)j + (x + pz)k$ is solenoidal, then find p?
32. Define scalar potential of an irrotational vector

Unit-IV

33. Define definite integral.
34. Find the value of $\bar{r}$ satisfying the equation $\frac{d^2 \bar{r}}{dt^2} = \bar{a}t + \bar{b}$, where $\bar{a}$, $\bar{b}$ are constant vectors.
35. Define smooth curve.
36. Define line integral.
37. Define circulation.
38. Define surface integral.

39. Define flux.
40. Define volume integral

Unit-V

41. State Gauss divergence theorem.
42. Apply Gauss's theorem to prove that $\int_S \bar{r}.N\,ds = 3V$.
43. State Green's theorem.
44. State Stokes theorem.
45. Prove that $curl\ grad\ \emptyset = 0$ by using Stokes theorem.

Short Answer Questions

Unit-I

13. If R is a Boolean ring, then prove that
    iv. $a + a = 0, \forall\, a \in R$
    v. $a + b = 0 \Rightarrow a = b$
    vi. R is commutative under multiplication
14. Prove that, a ring R has no zero divisors iff the cancellation laws hold in R.
15. Prove that every field is an integral domain.
16. Prove that, the intersection of two sub rings of a ring R is a subring of R.
17. If R is a commutative ring and $a \in R$, then prove that $Ra = \{ra: r \in R\}$ is an ideal of R.
18. Prove that, the characteristic of a Boolean ring is 2.
19. Prove that the set of integers is a sub ring of the set of rational numbers w.r.to usual addition and multiplication.
20. Prove that, every sub ring is an ideal. Explain briefly.
21. If U is an ideal of a ring R with unity element and $1 \in U$, then $U = R$.
22. Prove that, the intersection of two ideals of a ring R is an ideal of R.
23. Prove that the characteristic of an integral domain is either a prime or zero.

Unit-II

24. If f is a homomorphism of a ring R into a ring R', then prove that f is an into isomorphism iff Ker $f = \{0\}$.
25. Prove that every quotient ring of a ring is a homomorphic image of the ring.
26. If U_1, U_2 are two ideals of a ring R, then prove that $U_1 + U_2 = \{x + y: x \in U_1, y \in U_2\}$ is also an ideal of R.
27. If R/U is the quotient ring, then prove that
    i. R/U is commutative, if R is commutative
    ii. R/U has unity element, if R has unity element
28. Prove that the homomorphic image of a ring is a ring.
29. If f is a homomorphism of a ring R into a ring R', then prove that Ker f is an ideal of R
30. If a function $f: C \longrightarrow R$ is defined by $f(m + in) = m,\ m + in \in C$ is a homomorphism.
31. If $f: C \longrightarrow M_2\,(R)$ is defined by $f(a + ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ then prove that f is an monomorphism, also find ker f.

Unit-III

32. $a = x + y + z$, $b = x^2 + y^2 + z^2$, $c = xy + yz + zx$ then prove that $[grad\ a,\ grad\ b,\ grad\ c] = 0$.
33. If $r = a\cos\omega\theta + b\sin\omega\theta$, where a, b are constant vectors and ω is constant scalar then prove that $\frac{d^2r}{dt^2} = -\omega^2 r$ & $r \times \frac{dr}{dt} = \omega(a \times b)$.
34. $If\ A = 5t^2 i + tj - t^3 k$ & $B = \sin ti - \cos tj$, then find the values of (i) $\frac{d}{dt}(A.B)$ and $(ii)\ \frac{d}{dt}(A \times B) at\ t = 0$.
35. $If\ r = t^2 i - tj + (2t + 1)k$, then find the values of $\frac{dr}{dt}, \frac{d^2r}{dt^2}$ $\left|\frac{dr}{dt}\right|, \left|\frac{d^2r}{dt^2}\right| at\ t = 0$.
36. If $\emptyset = 2xz^4 - x^2y$, then find the value of $\left|\frac{\partial\emptyset}{\partial x} i + \frac{\partial\emptyset}{\partial y} j + \frac{\partial\emptyset}{\partial z} k\right|$ at (2,-2,-1).
37. Prove that $\nabla\left(\frac{1}{r}\right) = \left(\frac{-\overline{r}}{r^3}\right)$.
38. Prove that $\nabla r = \frac{\bar{r}}{r}$, if $r = xi + yj + zk\ and\ r = |r|$.
39. Find the angle between the spheres $x^2 + y^2 + z^2 = 29$ & $x^2 + y^2 + z^2 - 4x - 6y - 8z - 47 = 0$ at (4,-3,2).
40. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9\ and\ x^2 + y^2 - z = 3\ at\ (2, -1, 2)$.
41. Find div F and curl F, where F= $(x^3 + y^3 + z^3 - 3xyz)$.
42. Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ in the direction of the vector $2i - j - 2k$ at $(1, -2, -1)$.
43. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at $P\ (1,\ 2,\ 3)$ in the direction of the line PQ, where $Q = (5,\ 0,\ 4)$.
44. If $\overline{r} = x\,i + y\,j + z\,k$ and $\overline{a}$ is a constant vector, then prove that

    $\frac{\partial}{\partial x}(\overline{a}.\overline{r})i + \frac{\partial}{\partial y}(\overline{a}.\overline{r})j + \frac{\partial}{\partial z}(\overline{a}.\overline{r})k = \overline{a}.$
45. Find the greatest value (maximum) of the directional derivative of the function $f = x^2yz^3$ at $(2,\ 1,\ -1)$

Unit-IV

46. Evaluate $\int_1^2 F(t).dt$, where $F(t) = (t - t^2)i + 2t^3 j - 3k$.
47. Find $\int_C F.d\overline{r}$, where $F = xyi + yzj + zxk$ and the curve C is $\bar{r} = ti + t^2 j + t^3 k$, t varying from -1 to 1.
48. Find $\int_C F.d\overline{r}$, where $F = 3x^2 i + (2xz - y)j + zk$ along the straight line C from $(0,0,0)$ to (2,1,3).
49. If $F = 3xyi - y^2 j$, evaluate $\int_C F.d\overline{r}$, where C is the curve $y = 2x^2$ in xy −plane from (0,0) to (1,2).
50. Evaluate $\oint_C F.N\ dS$, where $F = zi + xj - 3y^2zk$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.
51. If $F = 2xzi - xj + y^2 k$, evaluate $\int_V F.dV$, where V is the region bounded by the surfaces $x = 0, x = 2, y = 0, y = 6, z = x^2, z = 4$.

Unit-V

52. Show that $\int_S (axi + byj + czk).N\,dS = 4\frac{\pi}{3}(a + b + c)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.
53. Compute $\int_S (ax^2 + by^2 + cz^2)\,dS$ over the sphere $x^2 + y^2 + z^2 = 1$.

54. Apply divergence theorem to evaluate $\iint_S (x+y)\,dy\,dz+(y+z)\,dz\,dx+(x+y)\,dx\,dy$, where S is the surface of the sphere $x^2+y^2+z^2=4$.

55. Evaluate $\int_S F.N\,ds$, where $F=2x^2yi-y^2j+4xz^2k$ taken over the region in the first octant bounded by $y^2+z^2=9$ and $z=2$.

56. Evaluate $\int_C (\cos x \sin y - xy)\,dx+\sin x \cos y\,dy$ by Green's theorem where C is the circle $x^2+y^2=1$.

57. If $F=yi+(x-2xz)j-xyk$, evaluate $\int_S (\nabla\times F).N\,dS$, where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the $xy-$ plane.

58. Prove that $Curl\ \ grad\ \emptyset=0$ by using Stokes theorem.

Essay Questions

Unit-I

64. Let Z be the set of all integers, then prove that $(Z,*,o)$ is a commutative ring with unity, where the operations $*$ and o are defined by $a*b=a+b-1$ and $aob=a+b-ab$.
65. Prove that every finite integral domain is a field.
66. Prove that the set of even integers is a ring commutative without unity under usual addition & multiplication.

67. Prove that the set $Z(i)=\{a+ib : a,b\in Z, i^2=-1\}\ of$ Gaussion integers is an Integral domain with respect to addition & multiplication of numbers is a field.
68. Prove that $Q\sqrt{2}=\{a+b\sqrt{2} :\ a,b\in Q\}$ is a field w.r.to ordinary addition and multiplication of numbers.
69. Prove that the characteristic of a field is either a prime or zero.
70. If R is a division ring show that C(R) = $\{x\in R/\ xa=ax,\ \forall a\in R\}$ is a field.
71. Prove that every ideal of Z is a principal ideal.
72. If p is a prime, then prove that Z_p, the ring of residue classes modulo p is a field.
73. Let S be a non empty subset of a ring R. Then prove that S is a subring of R iff $a-b\in S$ and $ab\in S$, for all $a,b\in S$.
74. Prove that every field has no proper ideals.
75. Prove that, a commutative ring R with unity element is a field, if R has no proper ideals.
76. If U_1 and U_2 are two ideals of a ring R, then prove that $U_1\cup U_2$ is an ideal of R iff $U_1\subseteq U_2$ or $U_2\subseteq U_1$.
77. Prove that the ring of integers Z is a principal ideal ring.
78. If U is an ideal of the ring R, then prove that the set $R/U=\{x+U : x\in R\}$ is a ring w.r.t. the induced operations of addition(+) and multiplication(.) of cosets defined follows $(a+U)+(b+U)=(a+b)+U$ and $(a+U).(b+U)=(a.b)+U$.

Unit-II

79. In the ring Z of integers, prove that an ideal generated by prime integer is a maximal ideal.
80. If M is a maximum ideal of the ring of integers Z, then prove that M is generated by a prime number.
81. Prove that, an ideal U of a commutative ring R with unity is maximal iff the quotient ring R/U is a field.

82. Prove that, an ideal $U \neq R$ of a commutative ring R, is a prime ideal iff the quotient ring R/U is an integral domain.
84. Let $Z\sqrt{2} = \{m + n\sqrt{2} : m, n \in Z\}$ be a ring under addition and multiplication of numbers. Prove that $f: Z\sqrt{2} \longrightarrow Z\sqrt{2}$ defined by $f(m + n\sqrt{2}) = m - \sqrt{2}$, $\forall\, m + n\sqrt{2} \in Z\sqrt{2}$ is an automorphism. Also find $Ker\, f$.
85. Prove that, every maximal ideal of a commutative ring R with unity is a prime.
86. Prove that a field has no proper homomorphic image.

Unit-III

87. Find $div\, f$ and $curl\, f$, where $f = grad(x^3 + y^3 + z^3 - 3xyz)$ at $(1, -1, 1)$.
88. If $r = xi + yj + zk$ and a is a constant vector, then prove that

(i) $\frac{\partial}{\partial x}(a.r)i + \frac{\partial}{\partial y}(a.r)j + \frac{\partial}{\partial z}(a.r)k = a$

(i) $\frac{\partial}{\partial x}(a \times r).i + \frac{\partial}{\partial y}(a \times r).j + \frac{\partial}{\partial z}(a \times r)k = 0$

(ii) $\frac{\partial}{\partial x}(a \times r) \times i + \frac{\partial}{\partial y}(a \times r) \times j + \frac{\partial}{\partial z}(a \times r) \times k = -2a$

89. Prove that, 1. $div\, \bar{r} = 3$, 2. $curl\, \bar{r} = 0$, 3. $div\, (\bar{r} \times \bar{a}) = 0$, 4. $curl\, (\bar{r} \times \bar{a}) = -2\bar{a}$.
90. Prove that $grad(A.B) = (B.\nabla)A + (A.\nabla)B + B \times curl\, A + A \times curl\, B$.
91. If a is a constant vector prove that $curl\, \frac{a \times r}{r^3} = \frac{-a}{r^3} + \frac{3r}{r^5}\, (a.r)$.
92. Prove that $curl(A \times B) = AdivB - BdivA + (B.\nabla)A - (A.\nabla)B$.
93. Prove that $div\, (A.B) = B.curl\, A - A\, .curl\, B = (\nabla \times \mathrm{A}).B - (\nabla \times \mathrm{B}).A$.
94. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t,\ y = t^2,\ z = t^3$ at the point $(1,\ 1,\ 1)$.

Unit-IV

95. If $F = 4xzi - y^2j + yzk$, evaluate $\int_S F.N\, ds$ where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$.
C
96. If $F = (x + y^2)i - 2xj + 2yxk$, evaluate $\int_S F.N\, ds$, where S is the surface of the plane from $2x + y + 2z = 6$ in the first octant.
97. If F=2xz i – x j + y^2 k evaluate $\iiint_V F.dV$ where V is the region bounded by the surface x=0 , x=2 , y=0 , y=6 ,z=x^2 , z=4.
98. If F=(2x^2-3z)i – 2xy j -4x k, evaluate i) $\int \nabla.F\ dv$ and ii) $\int \nabla \mathrm{x} F\ dv$ wherte V is the closed region bounded by x = 0 , y = 0 , z = 0 , 2x +2y + z = 4.
99. If $\emptyset = 45x^2y$, evaluate $\iiint_V \emptyset\, dV$, where V is the closed region bounded by the plane $4x + 2y + z = 8, x = 0, y = 0, z = 0$.

Unit-V

100. State and prove Gauss's divergence theorem.
101. Verify Gauss's divergence theorem to evaluate $\int_S ((x^3 - yz)i - 2x^2yj + zk)\,.N\, dS$ over the surface of the cube bounded by the coordinate planes $x = y = z = a$.
102. State and prove Green's theorem.
103. Verify Green's theorem in the plane for $\int_C (xy + y^2)\,dx + x^2\,dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.
104. State and prove Stokes theorem.
105. Verify Stokes theorem for $F = (y - z + 2)i + (yz + 4)j - xzk$ where S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy −plane.

P.R. GOVERNMENT COLLEGE (A), KAKINADA
III B.Sc., MATHEMATICS – Semester V (w.e.f. 2018-19)
Paper VI : Linear algebra

Total Hrs. of Teaching Learning & Evaluation: 75 @ 6 h / Week Total Credits: 05

Objective:

- To improve the students ability of understanding the most application oriented topic in Mathematics that is Linear Algebra.
- To equip the skill of understanding the concepts and writing the proofs of the Theorems.

Unit - I: Vector Spaces – I **(12 Hrs)**

Vector spaces, General properties of vector spaces, n-dimensional vectors, Addition and scalar multiplication of vectors, Internal and external composition, Null Space, Vector Subspaces, Algebra of subspaces, Linear sum of two subspaces, Linear combination of vectors, Linear span, Linear dependence and linear independence of Vectors.

Unit - II: Vector spaces – II **(12 Hrs)**

Basis of vector space, Finite dimensional vector space, Basis extension, Co-ordinates, Dimension of vector space, Dimension of subspace, Quotient space and Dimension of Quotient space.

Unit - III: Linear transformations **(12 Hrs)**

Linear transformations, Linear operators, Properties of linear transformation, Sum and product of linear transformations, Algebra of Linear Operators, Range space and Null Space of LT, Rank and Nullity of a LT, Rank & Nullity theorem.

Unit - IV: Matrix **(12 Hrs)**

Linear Equations, Characteristic Values and Characteristic Vectors of square matrix – Cayley - Hamilton Theorem.

Unit - V: Inner Product Space (**12 Hrs**)

Inner Product spaces, Euclidean and Unitary spaces, Norm or length of a vector, Schwartz's inequality, Triangle Inequality, Parallelogram law, Orthogonality and orthonormal set, Complete orthonormal set, Gram-Schmidt Orthogonalisation Process, Bessel's inequlity and Parsvel's identity.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Additional Inputs: Diagonalization of a matrix.

Prescribed Text Books:

J.N. Sharma & A.R.Vasista, Linear Agebra, Krishna Prakasham Mandir, Meerut.

Books for Reference:

9 III year Mathematics Linear Algebra and Vector Calculus, Telugu Academy.

10 A Text Book of B.Sc. Mathematics, Vol-III, S. Chand & Co.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-V, PAPER VI

Unit	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Vector spaces - I	01	01	02	22
II	Vector spaces - II	01	01	02	22
III	Linear Transformation	01	01	01	14
IV	Char. values and char. vectors	01	01	01	14
V	Inner product spaces	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P. R. Government College (A), Kakinada
III year B.Sc. Degree Examinations, – V Semester
Mathematics Course: Linear Algebra
Paper-VI (Model Paper w.e.f. 2019-20)

Time: 2 Hrs 30 Min **Max. Marks: 60 M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Define linear combination of vectors.
2. Write the standard basis of $V_2(R)$.
3. Find the null space of the transformation $T: R^2 \to R^3$ defined by $T(x, y) = (x+y, x-y, y)$.
4. Find the Eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
5. Write Bessel's inequality.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.

3 x 5 = 15 M

6. Determine whether the set of vector {(1, -2, 1), (2, 1, -1), (7, -4, 1)} is linearly dependent or Linearly independent.
7. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is also finite dimensional and dim $W \leq$ dim V.
8. Find T(x, y, z) where T: $R^3 \to R$ is defined by T(1,1,1)=3, T(0,1,-2)=1, T(0,0,1)= -2.
9. Solve the following system of linear equations

$$2x - 3y + z = 0,\ x + 2y - 3z = 0,\ 4x - y - 2z = 0.$$

10. State and prove Parsevel's identity.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11 Prove that a non empty subset W of a vector space $V(F)$ is a subspace of V if and only if $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.

12 Let V(F) be a vector space and S = { $\alpha_{1,}\alpha_{2,}...\alpha_n$ } is a finite subset of non-zero vectors of V(F). Then S is linearly dependent if and only if some vector $\alpha_k \in$ S, $2 \le k \le n$, can be expressed as a linear combination of its preceding vectors.

13 Show that the set of vectors { (2, 1, 4), (1, -1, 2), (3, 1, -2)} form a basis for R^3.

14 Let W be a sub space of a finite dimensional vector space V(F), then prove that dim V/W = dimV – dimW.

SECTION – B

15 State and prove rank and nullity theorem.

16 Discuss for all values of λ, the system of equations $x + y + 4z = 6, x + 2y - 2z = 6, \lambda x + y + z = 6$ as regards existence and nature of solutions.

17. State and prove Cauchy-Schwarz's inequality.

18. Applying Gram-Schmidt process, obtain an orthonormal basis of $R^3(R)$ from the basis {(2, 0, 1), (3, -1, 5), (0, 4, 2) }.

11.

12.

P.R. GOVERNMENT COLLEGE (A), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank
PAPER–VI: LINEAR ALGEBRA

Very short answers

UNIT-I

1. Define addition of vectors and scalar multiplication of vectors.
2. Define internal composition and external composition.
3. Define vector space and null space.
4. Define linear combination of vectors.
5. Define linear span of a set.
6. Define linear sum of two subspaces.
7. Define linear independent set and linear dependent set.
8. A single non-zero vector forms a ____.
9. If S and T are the subspaces of vector space V(F) then L(SUT) = ______.
10. Show that the vectors (2, -3), (6, -9) are linearly dependent in V_2 (R).

UNIT-II

11. Define basis of a vector space.
12. Define finite dimensional vector space.
13. Write the standard basis of $V_3(R)$.
14. Define Coordinates.
15. Every set of (n+1) or more vectors in an n-dimensional vector space is______.
16. Define Quotient space.
17. Show that the vectors (1, -1), (-2, -3) form a basis of V_2 (R).
18. Extend S={$(1, 1, 1)$} into a basis for V_3 (R).

UNIT-III

19. Define linear transformation.
20. Define null space and range space.
21. Show that the mapping $T: V_1 \rightarrow V_3$ defined by $T(x) = (x, 2x, 3x)$ is a Linear transformation.
22. Find a Linear Transformation $T: R^2 \rightarrow R^2$ such that $T(1,0) = (1,1)$ and $T(0,1) = (-1,2)$

23. Find the null space of the linear transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x + y, x - y, y)$

24. Test whether the mapping $T: R^2 \to R^2$ defined by $T(a,b) = (2a + 3b, 3a - 4b)$ is a linear transformation or not.

25. Test whether the mapping $T: R^2 \to R^2$ defined by $T(a,b) = (|a|, 0)$ is a linear transformation or not.

26. Find $T: R^3 \to R^4$ is a linear transformation whose range is spanned by (1,-1,2,3) and (2,3,-1,0).

UNIT-IV

27. Define symmetric and skew symmetric matrices with examples.

28. Define diagonal matrix with an example.

29. Define Hermitian and skew Hermitian matrices with examples.

30. Define Eigen values and Eigen vectors of a square matrix.

31. Find the Eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

32. Define Characteristic equation with an example.

33. Find the characteristic polynomial of a square matrix A=$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 6 \\ 3 & 1 & 4 \end{bmatrix}$.

34. State Cauchy-Hamilton theorem.

UNIT-V

35. Define Inner product space.

36. Define Unitary space.

37. Define Euclidean space.

38. State Parallelogram law.

39. State Triangle inequality.

40. State Cauchy-Schwartz’s inequality.

41. State Bessel’s inequality.

42. State Parseval’s inequality.

43. Every orthogonal set of non-zero vectors in an inner product space V(F) is ____ set.

44. Find a unit vector which is orthogonal to (1, 2, 1) in $R^3(R)$ with the standard inner product.

Short answers

UNIT-I

33. Let V be the set of all pairs $(a,\ b)$ of real numbers. Show that V is not a vector space with the operations defined by $(a_1,\ b_1) + (a_2,\ b_2) = (a_1 + a_2, 0),\ c(a_1,\ b_1) = (ca_1,\ b_1)$.

34. Prove that the intersection of any two subspaces W_1 and W_2 of vector space V(F) is subspace of V(F).

35. Let p, q, r be the fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a vector space of $V_3(F)$.

36. Let R be the field of real numbers and $W = \{(x, y, z): x, y, z \text{ are rational numbers}\}$. Is W is a subspace of $V_3(R)$.

37. Show that the subset $W = \{(a, b, c)/a^2 + b^2 + c^2 \leq 1\}$ is not a subspace of $R^3(R)$.

38. Prove that the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$.

39. If α, β, γ are linearly independent vectors of $V(R)$, then show that $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ are also linearly independent.

UNIT-II

40. Show that the set of vectors {(2, 1, 4), (1, -1, 2), (3, 1, -2)} form a basis for R^3.

41. Show that the set S={(1, 0, 0), (0, 1, 0), (1, 1, 0), (1, 1, 1)} is a spanning set of $R^3(R)$, but not a basis.

42. Show that the set $\{(1,0,0), (1,10), (1,1,1)\}$ is a basis of $C^3(C)$. Hence find the coordinates of the vector (3+4i, 6i, 3+7i) in $C^3(C)$.

43. Find the coordinate of (2, 3, 4, -1) with respect to the basis $B = \{(1,1,1,2),$ $(1, -1,0,0), (0,0,1,1), (0,1,0,0)\}$ of $V_4(R)$.

44. Prove that every set of $(n + 1)$ or more vectors in an n-dimensional vector space is linearly dependent.

45. If U = {(1,2,1), (0,1,2)} and W = {(1,0,0), (0,1,0)} determine the dimension of U+W.

46. Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a,b,c,d): b - 2c + d = 0\}$, $W_2 = \{(a,b,c,d): a = d, b = 2c\}$. Find the basis and dimension of (i) W_1, (ii) W_2, (iii) $W_1 \cap W_2$ and hence find $dim(W_1 + W_2)$.

47. Let W be a subspace of a finite dimensional vector space V(F), then prove that $dim\, V/W = \dim V - \dim W$.

UNIT-III

48. Define linear transformation and show that the function $T: R^3 \to R^3$ defined by $T(x,y,z) = (x - y, 0, y + z)$ is a linear transformation.

49. Find a linear transformation $T: R^3 \to R$ such that $T(1,1,1) = 3, T(0,1,-2) = 1$, $T(0,0,1) = -2$.

50. Find a linear transformation $T: U \to V$ be such that whose basis and range are $\{(1,2,1), (2,1,0), (1,-1,-2)\}\ and\ \{(1,0,0), (0,1,0), (1,1,1)\}$.

51. Let $T: R^3 \to R^3$ be defined by $T(x,y,z) = (x - y + 2z,\ 2x + y - z,\ -x - 2y)$. Then verify Rank-nullity theorem.

52. Describe explicitly the linear transformation $T: R^3 \to R^3$ whose range space is spanned by {(1, 0, -1), (1, 2, 2)}.

53. Let $U(F)$ and $V(F)$ be two vector spaces such that T: U(F) → V(F) be a linear transformation. Then define range set of T and prove that the range set $R(T)$ is a subspace of $V(F)$.

UNIT-IV

54. Solve the system of linear equations $2x - 3y + z = 0$, $x + 2y - 3z = 0$, $4x - y - 2z = 0$ are consistent or not.

55. Is it the system of equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ are consistent.

56. Find the eigen values and eigen vectors of the square matrix A=$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 4 \end{pmatrix}$.

57. Verify Cayley-Hamilton theorem for the matrix A=$\begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

58. State and Prove Triangle-Inequality.

59. State and prove Parallelogram law in an inner product space V(F).

60. State and prove Parseval’s inequality in an inner product space V(F).

61. Prove that every orthogonal set of non-zero vectors in an Inner Product Space V(F) is linearly independent.

62. Prove that every orthonormal set of vectors is linearly independent.

63. Prove that the set S = $\left\{\left(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\right),\left(\frac{2}{3},\frac{-1}{3},\frac{2}{3}\right),\left(\frac{2}{3},\frac{2}{3},\frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

Essay questions

UNIT-I

25. Let V(F) be a vector space and $W \subseteq V$. Then prove that the necessary and sufficient conditions for W to be a subspace of V are

(i) $\alpha \in W,\ \beta \in W \Longrightarrow \alpha - \beta \in W\ \ (ii) a \in F,\ \alpha \in W \Longrightarrow a\alpha \in W$

26. State and prove the necessary and sufficient condition for a non-empty subset of a vector space to be a subspace.

27. Prove that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.

28. If $W_1$and W_2 are two subspaces of a vector space V(F) then prove that

(i) $W_1 + W_2$ is a subspace of $V(F)$ and (ii) $W_1 \subseteq W_1 + W_2\ \ and\ \ W_2 \subseteq W_1 + W_2$.

29. If S and T are the subsets of a vector space $V(F)$ then prove that

(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ and (ii) $L(S \cup T) = L(S) + L(T)$.

30. Let $V(F)$ be a vector space and $S = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a finite subset of non-zero vectors of $V(F)$. Then prove that S is linearly dependent if and only if some vector $\alpha_k \in S, 2 \leq k \leq n$ can be expressed as a linear combination of its preceding vectors.

UNTI-II

31. Let W_1 and W_2 be two subspaces of a finite dimensional vector space V(F). Then prove that $dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

32. State and Prove Basis Existence theorem.

33. State and prove Basis Extension theorem.

34. Prove that any two bases of a finite dimensional vector space V(F) have the same number of elements.

35. Define finite dimensional vector space. If W is a subspace of a finite dimensional vector space V(F), then prove that W is finite dimensional and dim W$\leq n$.

36. Let W be a subspace of a finite dimensional vector space V(F) then prove that $\dim(V/W) = \dim\ V - \dim W$.

UNIT-III

37. Let $L(U,V)$ be the vector space of all linear transformations from $U(F)$ to $V(F)$ such that $dim\ U = n$ and $dim\ V = m$, then prove that $dim\ L(U,V) = mn$.

38. Let $U(F)$ and $V(F)$ be two vector spaces and $T: \mathrm{U} \to \mathrm{V}$ is a linear transformation. Then prove that the range space $R(T)$ is a subspace of $V(F)$ and null space $N(T)$ is a subspace of $U(F)$.

39. State and prove Rank - Nullity theorem.

40. Let $T: R^3 \to R^3$ be defined by $T(x, y, z) = (x - y + 2z,\ 2x + y - z,\ -x - 2y)$. Then verify Rank-nullity theorem.

UNIT-IV

41. Find the characteristic roots and the corresponding vectors of the square matrix

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

42. Find the characteristic equation of A and hence find A^{-1}, where

$$A = \begin{pmatrix} 1 & 2 & -4 \\ 3 & -1 & 2 \\ 2 & 5 & 0 \end{pmatrix}.$$

43. State and prove Cayley-Hamilton theorem.

44. Verify Cayley -Hamilton theorem for the matrix $A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

45. State and prove Cauchy- Schwarz's inequality.

46. If u and v are two vectors in a complex inner product space V(F), then prove that $4\langle u, v\rangle = \|u + v\|^2 - \|u - v\|^2 + i\|u + iv\|^2 - \|u - iv\|^2$.

47. Given $\{(1, -1, 2), (0, 2, 1), (1, 2, 0)\}$ is a basis of R^3(R). Construct an orthonormal basis using Gram-Schmidt orthogonalisation process.

48. In a real inner product space, if u and v are two vectors such that $\|u\| = \|v\|$, then prove that $u - v$ and $u + v$ are orthogonal.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III B.Sc. MATHEMATICS – Semester VI (w.e.f. 2018-19)

Course (Elective VII (A)) : LAPLACE TRANSFORMS

Total Hrs. of Teaching Learning & Evaluation: 75 @ 6 h / Week Total Credits: 05

Objectives:

- To understand the concepts of Laplace Transform and Inverse Laplace Transform.
- To find the Laplace transform of some functions.

UNIT – I : Laplace Transform I (12 hrs)

Definition of Integral Transform – Laplace Transform Linearity, Piecewise continuous Functions, Existence of Laplace Transform, Functions of Exponential order, and of Class A.

UNIT – II : Laplace Transform II (12 hrs)

First Shifting Theorem, Second Shifting Theorem, Change of Scale Property, Laplace Transform of the derivative of f(t), Initial Value theorem and Final Value Theorem.

UNIT – III : Laplace Transform III (12 hrs)

Laplace Transform of Integrals – Multiplication by t, Multiplication by t^n – Division by t, Laplace Transform of Bessel's function, Laplace Transform of error function, Laplace Transform of Sine and cosine integrals.

UNIT – IV : Inverse Laplace Transform I (12 hrs)

Definition of Inverse Laplace Transform. Linearity, First Shifting Theorem, Second Shifting Theorem, Change of Scale property, use of partial fractions, Examples.

UNIT – V : Inverse Laplace Transform II (12 hrs)

Inverse Laplace transforms of Derivatives – Inverse Laplace Transforms of Integrals – Multiplication by spowers of p – Division powers of 'p'- Convolution definition - Convolution Theorem – proof and Applications – Heaviside's Expansion theorem and its Applications.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Prescribed Text book:

Integral Transforms by A.R.Vasishta and R.K. Gupta, Krishnaprakasan media Pvt. Ltd. Meerat.

Reference Books:

1. Integral Transforms by Dr. J. K. Goyal and K. P. Gupta, Pragati Prakashan.
2. M. D. Raisinghania Integral Transform, S. Chand & Co., New Delhi.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-VI
PAPER –VII, Elective VII (A)

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Laplace Transforms - I	01	01	01	14
II	Laplace Transforms - II	01	01	02	22
III	Laplace Transforms - III	01	01	01	14
IV	Inverse Laplace Transforms - 1	01	01	02	22
V	Inverse Laplace Transforms - 2	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : _ = 60

P. R. Government College (Autonomous), Kakinada
III Year B.Sc. Degree Examinations, VI Semester - Mathematics
Course (Elective VII (A)) - LAPLACE TRANSFORMS
PAPER - VII : MODEL PAPER (W.e.f. 2019-20)

Time: 2 Hrs 30 Min **Max. Marks : 60 M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Define Laplace Transform.
2. Find $L[t^3 e^{-3t}]$.
3. What is the Laplace transform of $L\{\frac{sint}{t}\}$
4. Write the Inverse of Laplace Transform of $\frac{a}{p^2+a^2}$.
5. If $L^{-1}\{f(p)\} = F(t)$ then what is the inverse Laplace transform of $f^{(n)}(p)$?

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.
3 x 5 = 15 M

6. Find $L\{t^n\}, n$ is a positive integer.
7. State and Prove first shifting theorem in Laplace Transforms.
8. Find $L\{t(3sin2t - 2cos2t)\}$
9. Find $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$.
10. Find $L^{-1}\left\{\log\left(1+\frac{1}{p^2}\right)\right\}$.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. Find $L\{F(t)\}$, where $F(t) = \begin{cases} 0 & \text{when } 0 < t < 1 \\ t & \text{when } 1 < t < 2 \\ 0 & \text{when } t > 2 \end{cases}$
12. Find $L\{sin\sqrt{t}\}$.
13. If $L\{F(t)\} = f(p) then\ prove\ that\ L\{F(at)\} = \frac{1}{a} f\left(\frac{p}{a}\right)$.
14. Find $L\{C_i(t)\}$

SECTION – B

15. Prove that $L^{-1}\left\{\frac{4p+5}{(p-1)^2(p+2)}\right\} = 3te^t + \frac{1}{3}e^t - \frac{1}{3}e^{-2t}$
16. Show that $L^{-1}\left\{\frac{p^2}{(p^4+4a^4)}\right\} = \frac{1}{2a}(coshat.sinat + sinhat.cosat)$.
17. Apply convolution theorem to find the inverse Laplace transform of the function $\frac{1}{(p-2)(p^2+1)}$.
18. Find$L^{-1}\left\{\frac{p}{(p^2+a^2)^2}\right\}$.

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA

III B.SC MATHEMATICS – Semester VI (w.e.f. 2018-19)

Course (Elective - VII (B)): Numerical Analysis

Total Hrs. of Teaching, Learning and Evaluation: 75 @ 5 h / Week ________

Total Credits: 05

Objective:

- To find the different types of errors in computation and then to reduce the errors.
- To find the approximate Polynomial for the given data when the data is even or uneven by using interpolation, also we can find the differentiation even if the function is not known explicitly.
- To find the solution of Algebraic and Transcendental equations using Bisection, Falsi Position, Iteration and Newton Raphson methods.

Unit - I: Errors in Numerical Computation **(12 hrs)**

Errors and their accuracy, Mathematical preliminaries, Errors and their analysis, Absolute, Relative and Percentage errors, A general error formula.

Unit - II: Solutions of Algebraic and transcendental equations **(12 hrs)**

Bisection Method, Iteration Method, Method of false position, Newton Raphson Method, Generalised Newton Raphson method, Muller's method.

Unit - III: Interpolation – I **(12 hrs)**

Errors in polynomial interpolation, Finite Differences, Forward, Backward and central difference operators, Shift and average difference operators, Symbolic relation between the operators, Detection of errors by use of difference tables, Differences of a polynomial.

Unit - IV: Interpolation - II **(12 hrs)**

Interpolation for equal intervals: Newton's forward, backward, Gauss forward, Backward, Strilling's, Bessel's and Everette's formulae.

Unit - V: Interpolation – III **(12 hrs)**

Interpolation for uneven intervals: Lagrange's interpolation formula, Error in Lagrange's Interpolation, Divided differences and their properties, Relation between divided differences formula, Forward, Backward and central difference operators, Newton's divided differences, Inverse Interpolation.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Prescribed Text books:

Numerical Analysis by S. Ranganatham, MVSSN Prasad, Dr. V. Ramesh Babu, S. Chand & Company.

Reference books:

1. Numerical Analysis by S. S. Sastry, Prentice Hall, NewDelhi.
2. Numerical Analysis by Kamali Surya Narayana, S. Chand & co, New Delhi.
3. Numerical Analysis by Gupta & Malik, Krishna Prakashan media (P) Ltd, Meerut.S

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SEMESTER-VI
PAPER – VII : ELECTIVE VII (B)

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Errors in Numerical Computations	01	01	01	14
II	Solutions of Algebraic and transcendental equations	01	01	02	22
III	Interpolation – I	01	01	02	22
IV	Interpolation – II	01	01	02	22
V	Interpolation – III	01	01	01	14
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : _60__

P.R. Government College (Autonomous), Kakinada
III year B.Sc. Degree Examinations : VI Semester - Mathematics
Course (Elective VII (B)) : Numerical Analysis
Paper VII: MODEL PAPER (w.e.f. 2019-20)

Time: 2 Hrs 30 Min **Max. Marks : 60 M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. 5 x 1 = 5 M

1. Estimate 1/3 to three significant digits and find its absolute error.
2. Write the formula for Newton - Raphson method to find the approximate root.
3. Define Shift operator.
4. Write the Bessel's Formula for interpolation
5. Write the divided difference of $f(x) = x^2 - 5$ for the arguments 2 and 4.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.
3 x 5 = 15 M

6. Define absolute, relative and percentage errors and give an example.
7. Find a root of the equation x^3-2x-5=0 by using Newton-Raphson method.
8. Find the missing term in the following data given below.

X	0	1	2	3	4
Y	1	3	9	-	81

9. Derive Newton's backward interpolation formula.
10. Using the inverse Lagrange's interpolation formula, if $y_1 = 4$, $y_3 = 12$, $y_4 = 19$, $y_x = 7$, then find the value of x.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. 5 X 8 = 40 M

SECTION – A

11. If u = $4x^2y^3/z^4$ and errors in x, y, z be 0.001, compute the relative maximum error in u, when x = y = z = 1.
12. Find the real root of the equation $x^3 - 9x + 1 = 0$ by using Regula Falsi Method.
13. Find the root of the equation $f(x) = e^x - 3x = 0$ by using Newton-Raphson method.
14. (1) Prove that $\mu^2 = 1 + \frac{\delta^2}{2}$ $and\ \Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$

    (2) Find the missing term in the following table:

x	1	2	3	4	5	6	7	8
y	1	8	-	64	-	216	343	512

SECTION – B

15. Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$, (ii) $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$

(iii) $\nabla = 1 - e^{-hD}$ and (iv) $\Delta - \nabla = \delta^2$.

16. Using Newton's Forward interpolation formula, find the value of f(x) when x = 1.4.

X	1.1	1.3	1.5	1.7	1.9
Y	0.21	0.69	1.25	1.89	2.61

17. Apply Stirling's folmula to find the value of f(1.22) from the following table.

x	0	0.5	1.0	1.5	2.0
f(x)	0	0.191	0.341	0.433	0.477

18. By means of Newton's divided difference formula, find the value f(8) and f(15) from the following table.

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank for
PAPER–VII: NUMERICAL ANALYSIS

Very Short Answer Questions

Unit-I

1. Define absolute, relative, percentage errors.
2. Find the relative error, if $\frac{1}{3}$ is approximated to 0.333.
3. Find the percentage error, if 625.483 is approximated to 3 significant figures.
4. Round off the numbers 37.46235 to four significant digits and compute E_A, E_R, E_P.
5. Round off the numbers 865250 to four significant digits and compute E_A, E_R, E_P.
6. Find the percentage error if 625.483 is approximated to 3 significant digits.
7. Write general error formula.

Unit-II

8. Find the interval in which $\sqrt{29}$ lies______.
9. Write the formula for Newton - Raphson method to find the approximate root.
10. Find the interval in which a root of the equation $x^3 - 4x - 9 = 0$ lies.
11. Write the condition for the convergence of iterative method.
12. Transcendental equation consists of _________ functions.
13. If $f(a) < 0, f(b) > 0$ then by bisection method the first approximation for the root of $f(x) = 0$ is
14. Rate of convergence of Bisection method is _________.
15. Rate of convergence of Newton-Raphson method is _______.
16. Oldest method for finding root of $f(x) = 0$ is _________.

Unit-III

17. Prove that $\Delta = E - 1$.
18. Evaluate $\Delta Tan^{-1}x$.
19. Define shift operator.
20. Define forward difference operator, backward difference operator.
21. Define averaging operator.
22. Define enlargement operator.
23. Define inverse operator
24. Prove that $\nabla = \Delta E^{-1}$.
25. Evaluate $\Delta \frac{2^x}{x!}$ if h=1.
26. Construct the difference table for the following data.

x	3	3.1	3.2	3.3
y	0.3333	0.32258	0.31250	0.30303

27. Prove that $E - 1 = \Delta$

28. Evaluate $\Delta tan^{-1}ax$.

29. Evaluate Δe^{x} if h=1.

30. Evaluate $\Delta tan^{-1}x$

31. Evaluate $\Delta logf(x)$.

32. Evaluate $(\frac{\Delta^2}{E})x^3$.

33. Construct the difference table for the following data:

x	1	2	3	4	5
y	12	16	15	18	20

34. Show that the first forward difference of a constant is zero.

Unit-IV

35. Write the Newton's forward interpolation formula.

36. Write the Newton's backward interpolation formula.

37. Write the Gauss forward interpolation formula.

38. Write the Gauss backward interpolation formula.

39. Write the Bessel's formula for interpplation.

40. Write the Strilling's interpolation formula

41. Write the Everette's interpolation formula.

Unit-V

42. Write the Lagrange's interpolation formula.

43. Write the relation between divided difference and forward differences.

44. Write the relation between divided difference and backward differences.

45. Define inverse interpolation.

46. Write the divided difference of $f(x) = x^2 - 5$ for the arguements $2, 4$.

Short Answer Questions

Unit-I

1. Evaluate the sum $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to four significant digits and find its absolute and relative errors.
2. Define absolute, relative, percentage errors and give an example.
3. Derive general error formula.
4. When the numbers $x = 4.488$, $y = 1.321$ are rounded to two decimal places ,find the value of $q = \frac{x}{y}$ and $E_R(q)$.

Unit-II

5. Explain bisection method.
6. Find the root of the equation $x^3 - 4x - 9 = 0$ using bisection method in 4 stages.
7. Explain iterative method.
8. Solve the equation $sinx = 5x - 2$ by using iteration method.
9. Explain Regular Falsi method.
10. By using Regula - Falsi method, find an approximate root of the equation $x^4 - x - 10 = 0$ that lies in between 1.8 and 2. Carry out three approximations.
11. Explain Newton-Raphson method.
12. Find the root of the equation $x^3 - 2x - 5 = 0$ by using Newton Raphson method
13. Find by Newton's method, the real root of the equation $xe^x - 2 = 0$ correct to 3 decimal places.
14. Explain Muller's method.

Unit-III

15. Prove that (i) $\Delta f(x)g(x)$, (ii) $\Delta \frac{f(x)}{g(x)}$ and (iii) $E = e^{hD}$.
16. Find the missing term in the following data.

x	0	1	2	3	4
y	1	3	9	__	81

17. Find the missing term in the following data.

x	1	2	3	4	5	6	7
y	2	4	8	---	32	64	128

18. The following table gives a set of values of x and the corresponding values of $y = f(x)$.

x	10	15	20	25	30	35
y	19.97	21.51	22.47	23.52	24.65	25.89

Form the forward difference table and write down the values of $\Delta f(10)$, $\Delta^2 f(10)$, $\Delta^3 f(15)$ and $\Delta^4 f(15)$.

Unit- IV

19. Derive Newton's forward interpolation formula.
20. Find the polynomial which satisfies the data $y(0) = 1,\ y(1) = 0,\ y(2) = 1,\ y(3) = 10$ and hence find $y(4)$.
21. Find sin 52° from the data.

x	45°	50°	55°	60°
$sinx$	0.7071	0.7660	0.8192	0.8660

22. Find the value of $f(3)$ by using forward difference table

X	0	1	2	4	5
F(x)	1	14	15	5	6

23. Find the Newton's forward difference interpolating polynomial for the data

X	0	1	2	3
f(x)	1	3	7	13

24. Derive Newton's backward interpolation formula.
25. Find $f(7)$ from the following table.

x	0	2	4	6	8
$f(x)$	7	13	43	45	367

26. Derive Gauss forward interpolation formula.
27. Find $f(32)$ from the following table by Using Gauss forward interpolation formula.

x	25	30	35	40
$f(x)$	0.2707	0.3027	0.3386	0.3940

28. Derive Gauss backward interpolation formula.
29. Estimate the sales of the year 1936 from the following table

Year	1901	1911	1921	1931	1941	1951
Sales (in lakhs)	12	15	20	27	39	52

30. Apply Strilling's formula to find the value of $f(2.2)$ from the following data.

x	0	0.5	1.0	1.5	2.0
$f(x)$	0	0.191	0.341	0.433	0.477

31. Apply Bessel's formula to find a polynomial of degree three or less which takes the following values of the function u_x

x	4	6	8	10
u_x	1	3	8	20

Unit-V

32. Derive lagrange's interpolation formula.

33. Derive Newton's divided difference interpolation formula.

34. Find y at $x = 10$ from the data using lagranes's interpolation formula

x	5	6	9	11
$f(x)$	12	13	14	16

35. Find u_x in the power of $(x-1)$ from the data $u_0 = 8, u_1 = 11,\ u_4 = 68, u_{5=}123$.

Essay Questions

Unit-I

1. If $u = \frac{4x^2y^3}{z^4}$ and errors in$x,\ y,\ z$ be 0.001, compute the relative maximum error in u, when $x = y = z = 1$.
2. If $u = \frac{5xy^2}{z^3}$ and errors in $x,\ y,\ z$ be 0.001, compute the relative maximum error in u when $x = y = z = 1$.

Unit-II

3. Find the real root of the equation $f(x) = x^3 - 18 = 0$ by using Bisection method
4. Find the real root of $x = e^{-x}$ by iteration method.
5. Find the approximate value of real root of $x^3 - x - 1 = 0$ by bisection method.
6. Find out the roots of $x^3 - x - 4 = 0$ using false position method.
7. Evaluate value of $\sqrt{12}$ by iterative method.
8. Find the square root of 24 using Newton Raphson method.

9. Find the reciprocal of 18 using Newton Raphson method.
10. Find the square root of 25, given $x_0 = 2$ & $x_1 = 7$ using bisection method.
11. Find the root of the equation $xlog_{10}(x)$= 1.2 using false position method

Unit-III

12. Prove that (i) $1 + \Delta = E$, (ii) $\nabla = 1 - E^{-1}$, (iii) E=e^{hD}, (iv) $\nabla = \Delta E^{-1}$,
    (v) $(1 + \Delta)(1 - \nabla) = 1$ and (vi) $\Delta\nabla = \nabla\Delta = \Delta - \nabla$
13. Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$, (ii) $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$
    (iii) $\nabla = 1 - e^{-hD}$ and (iv) $\Delta - \nabla = \delta^2$.
14. Find the missing term in the following table.

x	1	2	3	4	5	6	7	8
y	1	8	--	64	--	216	343	512

Unit-IV

15. Find $\log 58.75$ from the following data.

x	40	45	50	55	60	65
$logx$	1.60206	1.65321	1.69897	1.74036	1.77815	1.81291

16. Estimate the population increase in between 1955-1961.

$year$	1921	1931	1941	1951	1961	1971
$Population\ in\ lakhs$	20	24	29	36	46	51

17. The following are marks obtained by 492 students in certain examination:

$marks\ not\ more\ than$	40	45	50	55	60	65
$no.of\ students$	210	253	307	381	413	492

Find the number of students who got more than 48 marks but not more than 50 marks.

18. Estimate the population of the year 1925.

$year$	1891	1901	1911	1921	1931
$Population\ (thousands)$	46	66	81	93	101

19. Using Newton's forward interpolation formula, find the value of $f(x)$ when $x = 1.4$

x	1.1	1.3	1.5	1.7	1.9
y	0.21	0.69	1.25	1.89	2.61

20. Interpolate by means of Gauss backward interpolation formula the sales for the concern for the year 1976, given that

$year$	1940	1950	1960	1970	1980	1990
$sales(in\ lakhs)$	17	20	27	32	36	38

21. Find $f(3.75)$ using Gauss forward interpolation formula, given that

x	2.5	3.0	3.5	4.0	4.5	5.0
$f(x)$	24.145	22.043	20.225	18.644	17.262	16.047

22. Given that $\sqrt{6500} = 80.6223$, $\sqrt{6510} = 80.6846$, $\sqrt{6520} = 80.7456$, $\sqrt{6530} = 80.8084$, find $\sqrt{6526}$ by using Gauss backward interpolation formula.

23. Apply Strilling's formula to find y_{28} given that y_{20} =49225, $y_{25} = 48316$, $y_{30} = 47236$, $y_{35} = 45926$, $y_{40} = 44300$.

24. Apply Laplace-Everette's formula to obtain y_{25} given that $y_{20} = 2854$, $y_{24} = 3162,\ y_{28} = 3544,\ y_{32} = 3992.$

Unit-V

25. Using Lagrange's interpolation formula, calculate $f(3)$ for the following table.

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

26. By means of Newton's divided difference formula, find the values of $f(8), f(15)$ from the following table.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

27. Evaluate $f(10)$, given $f(x) = 168,\ 192,\ 336$ at$x = 1,\ 7,\ 15$, respectively by using Lagranges's interpolation formula.

28. Using divided difference table, find $f(x)$ which takes the values 1, 4, 40, 85 as $x = 0$, $1,\ 3,\ 4.$

P. R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - Semester VI (w.e.f. 2018-19)
Course (Elective VII (C)) : NUMBER THEORY

Total Hours of Teaching, Learning and Evaluation: 75 @ 5 h/week   Total credits: 05

Objectives:

- To understand the concepts of Number Theory.
- To know about the applications of Number Theory

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UNIT-I : Divisibility **(12 hrs)**

Divisibility – Greatest Common Divisor – Euclidean Algorithm – The Fundamental Theorem of Arithmetic.

UNIT-II : Congruences **(12 hrs)**

Congruences – Special Divisibility Tests - Chinese Remainder Theorem- Fermat's Little Theorem – Wilson's Theorem – Residue Classes and Reduced Residue Classes – Solutions of Congruences

UNIT-III : Number Theory from an Algebraic Viewpoint **(12 hrs)**

Number Theory from an Algebraic Viewpoint – Multiplicative Groups, Rings and Fields

UNIT-IV : Quadratic Residues **(12 hrs)**

Quadratic Residues - Quadratic Reciprocity – The Jacobi Symbol

UNIT-V Greatest Integer Function **(12 hrs)**

Greatest Integer Function – Arithmetic Functions – The Mobius Inversion Formula

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Reference Books:

1. 'Introduction to the Theory of Numbers' by Niven, Zuckerman & Montgomery (John Wiley & Sons).
2. 'Elementary Number Theory' by David M. Burton.
3. 'Elementary Number Theory' by David, M. Burton, 2 nd Edition, UBS Publishers.
4. ' Introduction to Theory of Numbers' by Davenport H., Higher Arithmetic, 5th Edition (John Wiley & Sons Publishers), Niven, Zuckerman & Montgomery.(Camb, Univ, Press)
5. 'Number Theory' by Hardy & Wright published by Oxford Univ, Press.
6. 'Elements of the Theory of Numbers' by Dence, J. B & Dence T.P published by Academic Press.

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SEMESTER-VI

PAPER –VII, Elective VII (C)

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Divisibility	01	01	01	14
II	Congruence	01	01	02	22
III	Number Theory from an Algebraic view point	01	01	01	14
IV	Quadratic Residues	01	01	02	22
V	Greatest Integer Function	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III Year B.Sc. Degree Examinations, VI Semester - MATHEMATICS

Course (Elective VII (C)) - NUMBER THEORY

PAPER VII, MODEL PAPER (w.e.f. 2019-20)

Time: 2 Hours 30 Min **Max. marks: 60 M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Find the GCD of 18, 24.
2. Define 'a is congruent to b mod m'.
3. Define multiplicative group.
4. Define quadratic residues.
5. Define arithmetic function.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.
3 x 5 = 15 M

6. If (a, b) = 1 then prove that (a+b, a-b) is either 1 or 2.
7. Solve the congruence 25x ≡ 15 (mod 120)
8. If $a \equiv b \pmod{m}, c \equiv d \pmod{m}$, then prove that $ac \equiv bd \pmod{m}$
9. If G is finite and $a \in G$ then prove that there is a positive integer $n < |G|$ such that $a^n = e$.
10. Prove that any complete residue system modulo m forms a group under addition modulo m.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. State and prove Fundamental theorem of arithmetic.
12. State and prove Fermat's little theorem.
13. If c/ab and (b, c) =1 then prove that c/a.
14. Let G=(a) be finite group of order n and let G' be a sub group of order m. Prove that m/n.

SECTION – B

15. Determine whether 219 is a quadratic residue or non residue mod 383.
16. Let p be an odd prime the prove that for all n, $\left(\frac{n}{p}\right) \equiv n^{p-1/2} (mod\ p)$
17. State and Prove mobius inversion formula.
18. Find all primes p such that $\left(\frac{10}{p}\right) = 1$.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - Semester VI (w.e.f. 2018-19)
Course (Elective VII (D)): GRAPH THEORY

Total Hours of Teaching, Learning and Ealuation: 75 @ 5 h/week Total credits: 05

Objectives:

- To introduce the most application oriented field in Mathematics i.e., Graph Theory.
- To impart the awareness on applications of Graph Theory

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UNIT –I : Graphs and Subgraphs (12 hrs)

Graphs, Simple Graph, Graph isomorphism, The incidence and adjacency matrices, Subgraphs, Vertex degree, Hand shaking theorem, Paths and connection, Cycles.

UNIT -II : Applications, Trees (12 hrs)

Applications, The shortest path problem, Sperner's lemma, Trees, Cut edges and bonds, Cut vertices, Cayley's formula.

UNIT -III : Applications of Trees, Connectivity (12 hrs)

Applications of Trees – The connector problem.
Connectivity: Connectivity, Blocks and Applications, Construction of reliable communication Networks.

UNIT- IV : Euler Tours & Hamilton Cycles (12 hrs)

Euler tours, Euler Trail, Hamilton path, Hamilton cycles, Dodecahedron graph, Peterson graph, Hamiltonian graph, Closure of a graph.

UNIT- V: Applications of Eulerian Graphs (12 hrs)

Applications of Eulerian graphs, The Chinese postman problem, Fleury's Algorithm – The travelling salesman problem.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

PRESCRIBED BOOK: Graph Theory with Applications by J.A. Bondy and U .S.R. Murthy, published by Mac. Millan Press.

REFERENCE BOOKS :

1. Graph theory with Applications by J.A.Bondy and U.S.R. Murthy published by Mac.Millan Press.
2. Introduction to Graph theory by S.Arumugham and S.Ramachandran, published by Scitech Publications, Chennai-17.
3. Graph Theory and Combinations by H.S.Govinda Rao published by Galgotia Publications.

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SEMESTER-VI

PAPER –VII, Elective VII D

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Graphs and Subgraphs	01	01	01	14
II	Applications, Trees	01	01	01	14
III	Applications of trees, Connective	01	01	02	22
IV	Euler tours and Hamilton Cycles	01	01	02	22
V	Applications of Eulerian graphs	01	0	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1mark)
S.A.Q.= Short answer questions (5 marks)
E.Q .= Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P.R. Government College (Autonomous), Kakinada
III Year B.Sc. Degree Examinations VI Semester MATHEMATICS
Course (Elective VII (D) GRAPH THEORY
PAPER - VII : MODEL PAPER (w.e.f. 2019-20)

Time: 2 Hours 30 Min **Max. marks: 60 M**

PART – I

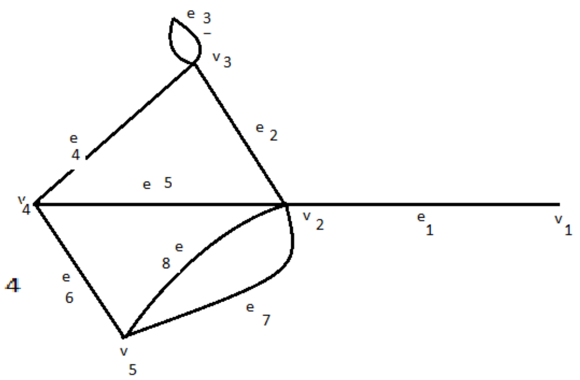
Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Define a simple graph.
2. How many edges have a tree with n vertices.
3. Define edge connectivity.
4. If a graph has an Euler trail then how many vertices have odd degree.
5. For tracing which type of graph Fleury's algorithm.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.
3 x 5 = 15 M

6. Write the vertex set, edge set and degree of every vertex of the graph.



7. Prove that in a tree, any two vertices are connected by a unique path.
8. Define the vertex cut and edge cut of a graph G(V,E) and give examples.
9. Define Eulerian graph and give an example.
10. Explain the travelling sales man problem.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. Define the degree of a vertex in a graph G and prove that the number of vertices of odd degree is even.

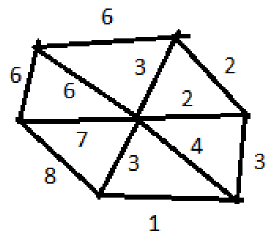
12. Write the Dijkastras algorithm for finding the shortest path between two vertices in a graph and give an example.

13. Prove that a connected graph is a tree if and only if every edge is a cut edge.

14. If G is connected then prove that any two vertices of G lie on a common cycle.

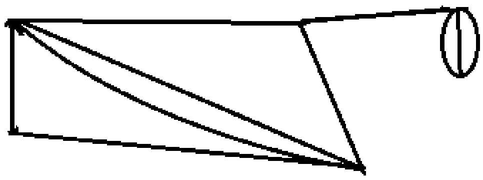
SECTION – B

15. Write the Kruskal's Algorithm for finding a minimal spanning tree and find a minimal spanning tree of the following graph.



16. Draw the Dodecahedran graph and a Hamilton cycle of it.

17. Define blocks of a graph and draw the blocks of the following graph.



18. Write the Fleury's algorithm and give an example.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - VI Semester, (w.e.f. 2018-19)
Course (Cluster VIII A -1) INTEGRAL TRANSFORMS

Total Hours of Teaching, Learning and Evaluation: 75 @ 5 h/week   Total credits: 05

Objectives:

- To be able to apply Laplace transform and inverse laplacetransform to find the solution of Ordinary Linear Differential Equations and Integral Equations.
- To understand the concepts of Infinite and Finite Fourier Transforms.
- To be able to find the Fourier transform of some functions.

UNIT – I : Application of Laplace Transform to solutions of Differential Equations (12 hrs)

Solutions of Differential Equations with Constants Co-efficient, Solutions of Differential Equations with Variable Co-efficient.

UNIT – II : Application of Laplace Transform (12 hrs)

Solution of Simultaneous Ordinary Differential Equations, Solutions of Partial Dif ferential Equations.

UNIT – III : Application of Laplace Transforms to Integral Equations (12 hrs)

Definitions: Integral Equations - Abel's, Integral Equation - Integral Equation of Co nvolution Type, Integro Differential Equations - Application of L.T. to Integral Equations.

UNIT – IV : Fourier Transforms – I (12 hrs)

Definition of Fourier Transform – Fourier's Inverse Transform – Fourier Cosine Transform – Linear Property of Fourier Transform – Change of Scale Property for Fourier Transform – Sine Transform and Cosine Transform Shifting Property – Modulation Theorem.

UNIT – V : Fourier Transform - II (12 hrs)

Convolution Definition – Convolution Theorem for Fourier Transform – Parseval's Indentify – Relationship Between Fourier and Laplace Transforms – Problems.
related to Integral Equations.

Finte Fourier Transforms :

Finite Fourier Sine Transform – Finite Fourier Cosine Transform – Inversion Formula for Sine and Cosine Transforms only statement and related problems.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Prescribed Text book:

Integral Transforms by A.R. Vasishta and R.K. Gupta, Krishnaprakasan media Pvt. Ltd. Meerat.

Reference Books:

Integral Transforms by Dr. J. K. Goyal and K. P. Gupta, Pragati Prakashan.
M. D. Raisinghania Integral Transform, S. Chand & Co., New Delhi.

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-VI
PAPER VIII (A) 1, CLUSTER VIII (A) 1

UNIT	TOPIC	V.S. A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Application of Laplace Transform to solutions of Differential Equations	01	01	02	22
II	Application of Laplace Transforms	01	01	01	14
III	Application of Laplace Transforms to Integral Equations	01	01	01	14
IV	Fourier Transforms-I	01	01	02	22
V	Fourier Transform-II	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60
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P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

III Year B.Sc. Degree Examinations VI Semester Mathematics

Course (Cluster – VIII (A) -1) INTEGRAL TRANSFORMS

PAPER VIII (A) 1, MODEL PAPER (w.e.f. 2019-20)

Time: 2 hrs 30 Min **Max. Marks : 60M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Write the formula of L$\{y''\}$.
2. Find $L\left(\frac{\partial y}{\partial x}\right)$.
3. Write the Integral equation of convolution type.
4. Write the Fourier Sine Transform of F(x).
5. Find the cosine transform of $2e^{-5x}$.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.

3 x 5 = 15 M

6. Solve $\frac{d^2y}{dx^2} + y = 0$ under the conditions that $y = 1, \frac{dy}{dx} = 0$ when $t = 0$.
7. Solve $(D^2 - 3)x - 4y = 0, x + (D^2 + 1)y = 0 \quad t > 0$
   $If\ x = y = Dy = 0, Dx = 2\ when\ t = 0.$
8. Solve the integral equation $F(t) = e^{-t} - 2\int_0^t \cos(t-u)\,F(u)du$
9. Find the Fourier Transform of $F(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$.
10. Solve the integral equation $\int_0^\infty f(x)cos\lambda x\ dx = e^{-\lambda}$.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. Solve $(D+1)^2 y = t$, give that $y = -3$, when $t = 0$ and $y = -1$, when $t = 1$.
12. Solve $\frac{\partial y}{\partial t} = 2\frac{\partial^2 y}{\partial x^2}$, where $y(0,t) = 0 = y(5,t)$ and $y(x,0) = 10\sin 4\pi x$.
13. Solve the integral equation $\int_0^1 \frac{F(u)du.}{(t-u)^{\frac{1}{3}}} = t(1+t)$.
14. Solve the integral equation $\int_0^t F(u)\,\mathrm{F}(t-u)\,du = 16\,sin4t$.

SECTION – B

15. Find the Fourier Cosine Transform of e^{-x^2}.
16. State and Prove Parsvel's identity for Fourier Transforms.
17. Find the finite cosine transform of $f(x)$ if $f(x) = -\frac{\cos k(\pi - x)}{k \sin k\pi}$.
18. Find the finite cosine transform of $(1 - \frac{x}{\pi})^2$.

P.R. GOVERNMENT COLLEGE (A), KAKINADA
III B.Sc. MATHEMATICS - VI Semester (w.e.f. 2018-19)
Course (Cluster VIII B-1) Advanced Numerical Analysis

Total hours of teaching, learning and Evaluation: 75 @ 5 hours/ week Total Credits: 05

Objective:

- To find the integration and solution for ordinary differential equations using numerical methods.
- To find the best fitted curve for the given data.

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Unit – I : Curve Fitting **(12 hrs)**

Least – Squares Curve Fitting Procedures, Fitting of a Straight Line, Nonlinear Curve Fitting, Curve Fitting by a Sum of Exponentials.

UNIT - II : Numerical Differentiation **(12 hrs)**

Derivatives using Newton's forward difference formula, Newton's backward difference formula, Derivatives using central difference formula, Stirling's interpolation formula, Newton's divided difference formula, Maximum and minimum values of a tabulated function.

UNIT- III : Numerical Integration **(12 hrs)**

General quadrature formula on errors, Trapozoidal rule, Simpson's 1/3 – rule, Simpson's 3/8 – rule and Weddle's rules, Euler – Maclaurin Formula of Summation and Quadrature, The Euler Transformation.

UNIT – IV : Solutions of Simultaneous Linear Systems of Equations **(12 hrs)**

Solution of linear systems – Direct methods: Matrix inversion method, Gaussian elimination methods, Gauss-Jordan Method, Method of factorization, Solution of Tridiagonal Systems, Iterative methods: Jacobi's method, Gauss- Seidel method.

UNIT – V : Numerical solution of ordinary differential equations **(12 hrs)**

Introduction, Solution by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge – Kutta methods.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Reference Books :

1. Numerical Analysis by S.S. Sastry, Published by Prentice Hall India (Latest Edition).
2. Numerical Analysis by G. Sankar Rao, published by New Age International Publishers, New Hyderabad.
3. Finite Differences and Numerical Analysis by H.C. Saxena published by S.Chand and Company, Pvt. Ltd., New Delhi.
4. Numerical methods for scientific and engineering computation by M.K.Jain, S.R.K.Iyengar, R.K. Jain.

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SEMESTER-VI

PAPER - VIII B - I, (CLUSTER – I)

UNIT	TOPIC	V.S.A.Q	**S.A.Q**(including choice)	**E.Q**(including choice)	Total Marks
I	Curve Fitting	**01**	**01**	**01**	**14**
II	Numerical Differentiation	**01**	**01**	**01**	**14**
III	Numerical Integration	**01**	**01**	**02**	**22**
IV	Solution of Linear System of Equations	**01**	**01**	**02**	**22**
V	Numerical Solutions for ODE	**01**	**01**	**02**	**22**
TOTAL		**05**	**05**	**08**	**94**

E.Q = Essay questions (8 marks)

S.A.Q = Short answer questions (5 marks)

V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 5 x 1 M = 05

Short answer questions : 3 x 5 M = 15

Very Short answer questions : 5 x 8 M = 40

Total Marks : = 60

P.R. Government College (Autonomous), Kakinada

III B.Sc. Examination - VI Semester

Mathematics (Cluster – I) : Advanced Numerical Analysis

PAPER - VIII B-I : MODEL PAPER (W.e.f. 2019-20)

Time: 2 hrs 30 Min **Max. Marks : 60 M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. 5 x 1 = 5 M

1. Write the normal equations for fitting of a straight line.
2. Write the formula for $\frac{dy}{dx}$ at $x = x_1$.
3. Write Simpson's $\frac{3}{8}^{th}$ - rule.
4. Write the formula of A^{-1} for a non singular matrix A.
5. Write Euler's formula for y_n.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks. 3x 5=15 M

6. Find the least square line y = a + bx for the data.

x_i	1	2	3	4	5
y_i	14	27	40	55	68

7. From the following table, find x correct to 4 decimal places for which y is minimum and find this value of y.

X	0.60	0.65	0.70	0.75
Y	0.6221	0.6155	0.6138	0.6170

8. Evaluate $\int_0^1 x^3\, dx$ with five sub-intervals by using Trapezoidal rule.
9. Solve the equations x + y + z = 6; 3x + 3y + 4z = 20; 2x + y + 3z = 13 by using Gaussian elimination method.
10. Solve $\frac{dy}{dx} = x + y, y(0) = 1$ by using Picard's method upto 3 approximations.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. 5 X 8 = 40 M

SECTION – A

11. Fit a second degree polynomial to the following data by the method of least squares.

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

12. Form the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for x = 1.5.

X	1.5	2.0	2.5	3.0	3.5	4.0
Y	3.375	7.0	13.625	24.0	38.875	59.0

13. Evaluate $I = \int_0^1 \frac{1}{1+x}\, dx$ correct to three decimal places by using Simpson's $\frac{1}{3}^{\text{rd}}$ – rule with $h = 0.125$.

14. Derive Newton's general quadrature formula.

SECTION – B

15. Solve the equations 2x + 3y + z = 9; x + 2y + 3z = 6; 3x + y + 2z = 8 by using factorization method.

16. Solve the following equations by using Gauss-Seidel method.
$8x - 3y + 2z = 20$, $4x + 11y - z = 33$, $6x + 3y + 12z = 35$.

17. Given $\frac{dy}{dx} = -xy^2$, $y(0) = 2$. Compute y(0.2) in steps of 0.1 by using modified Euler's method.

18. Obtain the values of y at $x = 0.1$ and 0.2 by using Runge - Kutta method of fourth order for the differential equation $y' + y = 0$, $y(0) = 1$.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

Question Bank

PAPER – VIII B-I: ADVANCED NUMERICAL ANALYSIS

Very Short Answer Questions

Unit-I

1. Define method of least squares.
2. Write the normal equations for fitting of a straight line.
3. Write the normal equations for fitting of a second degree parabola.
4. If $y = a_0 + a_1 x^2$, then write the third normal equation to fit the curve.
5. If $y = ax^b$, then write the first normal equation to fit the curve.
6. If $y = ae^{bx}$, then write the first normal equation to fit the curve.

Unit-II

7. Write the first derivative of Newton's Forward Difference Formula.
8. Write the second derivative of Newton's Backward Difference Formula.
9. Write the first derivative of Stirling's Formula.
10. Write the second derivative of Stirling's Formula.
11. Write the second divided difference for the arguments x_0, x_1 and x_2.

Unit-III

12. Define Numerical Integration.
13. Write the Newton-Cote's quadrature formula.
14. Write Trapezoidal rule.
15. Write Simpson's $\frac{1}{3}^{\text{rd}}$ – rule.
16. For what values of n, Simpson's $\frac{1}{3}^{\text{rd}}$ – rule can be applied.
17. For what values of n, Simpson's $\frac{3}{8}^{\text{th}}$ – rule can be applied.
18. Write Simpson's $\frac{3}{8}^{\text{th}}$ – rule.
19. Write Weddle's rule.

Unit-IV

20. Define linear equation in n unknowns.
21. The solution for the system of linear equations $AX = B$ exists by using the matrix inversion method, if ______.
22. Gauss elimination method reduces the system of equations to an equivalent _____ system.
23. In the method of factorization, the square matrix A can be written as $A = LU$, where $L =$____.
24. In the method of factorization, the square matrix A can be written as $A = LU$, where $U =$___.

25. The solution of tridiagonal system uses ________ method.

26. The number of iterations required to Gauss-Seidel method ______ than the number of iterations required to Gauss-Jacobi's method.

Unit-V

27. Write Taylor's series for $f(x)$ at $x = x_0$.
28. Write Taylor's series for $f(x) = \log(1 + x)$ is ______.
29. Write the iterative formula for Picard's method of successive approximations.
30. Write the disadvantage of Picard's method.
31. Runge – Kutta method is better than Taylor's series method, because _______.
32. Write Euler's formula.
33. Write Modified Euler's formula.
34. Write Runge - Kutta formula for first order.
35. Write Runge - Kutta formula for second order.
36. Write Runge - Kutta formula for third order.
37. Write Runge - Kutta formula for fourth order.

Short Answer Questions

Unit-I

1. Derive the normal equations for fitting of a straight line.
2. By the method of least squares, find the straight line that best fits the following data.

x_i	1	2	3	4	5
y_i	14	27	40	55	68

3. Derive the normal equations of a polynomial of second degree.

4. Fit a parabola to the data given below.

X	0.0	1.0	2.0
Y	1.0	6.0	17.0

5. Explain the procedure to fit a power function for the given data by the method of least squares.

6. Explain the procedure to an exponential function for the given data by the method of least squares.

Unit-II

7. Derive the first and second order derivatives of Newton's forward difference formula at $x = x_0$.

8. Derive the first and second order derivatives of Newton's backward difference formula at $x = x_n$.
9. Derive the first and second order derivatives of Stirling's interpolation formula at $x = x_0$.

10. Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0$ for the following data.

X	0	2	4	6	8	10

f(x)	0	12	248	1284	4080	9980

11. Find the first two derivatives of $f(x)$ at $x = 1$ from the following table.

X	-2	-1	0	1	2	3	4
f(x)	104	17	0	-1	8	69	272

12. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 0.4$.

X	0.0	0.1	0.2	0.3	0.4
Y	1.0000	0.9975	0.9900	0.9776	0.9604

13. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$.

X	1	2	3	4	5
Y	7.4036	7.7815	8.1291	8.4510	8.7506

Unit-III

14. Derive the Trapezoidal rule from general formula for numerical integration.
15. Using Simpson's $\frac{3}{8}^{\text{th}}$ − rule, evaluate $\int_0^6 \frac{1}{1+x^2}\, dx$ by dividing the range into 6 equal parts.
16. Derive Simpson's $\frac{3}{8}^{\text{th}}$ − rule from general formula for numerical integration.
17. Derive Simpson's $\frac{1}{3}^{\text{rd}}$ − rule from general formula for numerical integration.
18. Use the Euler-Maclaurin formula to prove $\sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$
19. Derive Newton-Cote's quadrature formula.

Unit-IV

20. Solve the system of equations $x + y + z = 7$, $x + 2y + 3z = 16$, $x + 3y + 4z = 22$ by using matrix inversion method.
21. Solve the system of equations $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ by using matrix inversion method.
22. Solve the System of equations $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$ by using Gauss-Elimination method.
23. Solve the system of equations $x + y + z = 6$, $3x + 3y + 4z = 20$, $2x + y + 3z = 13$ by using Gauss-Elimination method.
24. Using Gauss-Jordan method, solve the system $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$.
25. Solve the system of equations $2x + y + z = 2$, $x + 3y + 2z = 2$, $3x + y + 2z = 2$ by using LU Decomposition.

26. Solve the system of equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$ by using Gauss-Jacobi's method.

27. Solve the system of equations $x + 10y + z = 6$, $10x + y + z = 6$, $x + y + 10z = 6$ by using Gauss- Seidel iterative method.

28. Solve the system of equations $10x + 2y + z = 9$, $2x + 20y - 2z = -44$, $-2x + 3y + 10z = 22$ by using Gauss-Seidel iterative method

Unit-V

29. Explain Taylor's series method.

30. Using Taylor's series method, find $y(0.1)$ correct to four decimal places if $y' = x - y^2$ and $y(0) = 1$.

31. Using Taylor's series method, solve the equation $\frac{dy}{dx} = x^2 + y^2$ for $x = 0.4$, given that $y = 0$ when $x = 0$.

32. Find an approximate value of y for $x = 0.1$ and $x = 0.2$, if $\frac{dy}{dx} = x + y$ and $y = 1$ at $x = 1$ by using Picard's method. Check your answer with the exact particular solution.

33. Solve the equation $y' = x + y^2$, subject to the condition $y = 1$ when $x = 0$ by using Picard's method of successive approximations.

34. Find the value of y at $x = 0.1$ by Picard's method, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.

35. Using Euler's method, compute $y(0.3)$ with $h = 0.1$ from $y' = x + y$, $y(0) = 1$. Compute the result obtained by this method with the result obtained by analytical method.

36. Using Euler's method, compute $y(0.6)$ with $h = 0.2$ from $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.

37. Solve the differential equation $y' = -y$ with the condition $y(0) = 1$ at $x = 1$ by using Euler's method with $h = 0.1$.

38. Find $y(0.1)$ and $y(0.2)$ by using Euler's modified formula, given that $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$.

39. Use Euler's modified method, find y for $x = 0.2$. Given that $\frac{dy}{dx} = \log(x + y)$ with the initial condition that $y = 1$ when $x = 0$.

40. Explain Runge-Kutta method and use it to solve $y' = xy$ for $x = 1.4$, given that $x = 1$, $y = 2$.

<u>Essay Questions</u>

Unit-I

1. The table given below gives the temperature T (in °C) and lengths l (in mm) of a heated rod. If $l = a_0 + a_1 T$, find the best values for a_0 and a_1.

T(in °C)	20	30	40	50	60	70
l (in mm)	800.3	800.4	800.6	800.7	800.9	801.0

2. By the method of least squares fit a parabola of the form $y = a + bx + cx^2$ for the following data.

X	2	4	6	8	10
Y	3.07	12.85	31.47	57.38	91.29

3. Determine the constants a and b by the method of least squares such that $y = ae^{bx}$ fits the following data.

X	2	4	6	8	10
Y	4.077	11.084	30.128	81.897	222.62

4. Fit a function of the form $y = ax^b$ to the following data.

X	2	4	7	10	20	40	60	80
Y	43	25	18	13	8	5	3	2

5. Fit a curve $y = ax^b$ to the following data.

X	1	2	3	4	5	6
Y	2.98	4.26	5.21	6.10	6.80	7.50

Unit-II

6. Find the first and second derivatives of the function tabulated below at the point $x = 1.5$.

X	1.5	2.0	2.5	3.0	3.5	4.0
Y	3.375	7.0	13.625	24.0	38.875	59.0

7. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$.

X	1.0	1.2	1.4	1.6	1.8	2.0	2.2
Y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

8. Use Stirling's formula to find $f'(1.22)$ from the following table.

X	1.0	1.1	1.2	1.3	1.4
f(x)	0.84147	1.89121	0.93204	0.96356	0.98545

9. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$.

X	0	1	2	3	4	5	6
Y	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

10. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.6$.

X	1.2	1.3	1.4	1.5	1.6
Y	0.9320	0.9636	0.9855	0.9975	0.9996

11. Compute $f'(4)$ and $f'(5)$ for the following data.

X	1	2	4	8	10
f(x)	0	1	5	21	27

12. From the following table, find the values of x for which y is maximum and find this value of y.

X	1.2	1.3	1.4	1.5	1.6
Y	0.9320	0.9636	0.9855	0.9975	0.9996

Unit-III

13. Derive Trapezoidal rule and use it to evaluate $\int_0^{\frac{\pi}{2}} \sin x \; dx$. Also compare with its exact value.

14. Find the area bounded by the curve and the X-axis from $x = 7.47$ to $x = 7.52$ for the following data by using Trapezoidal rule.

X	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

15. Find the value of $\int_3^7 x^2 \log x \; dx$ by taking four strips using Trapezoidal rule.
16. Evaluate $\int_0^1 e^x \, dx$ approximately in steps of 0.05 by using Simpson's $\frac{1}{3}^{rd}$ $-$ rule.
17. Estimate the value of the integral $\int_1^3 \frac{1}{x} \, dx$ by using Simpson's $\frac{3}{8}$ th - rule.
18. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ th - rule taking $h = \frac{1}{6}$. Hence obtain an approximate value of π.
19. Evaluate the integral $\int_4^{5.2} \log x \; dx$ using Weddle's rule.
20. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's rule and compare with its exact value.

Unit-IV

21. Solve $2x + 2y + z = 12$, $3x + 2y + 2z = 8$, $5x + 10y - 8z = 10$ by Gauss-Elimination method.

22. By the method of factorization, solve $2x + 3y + z = 9$, $x + 2y + 3z = 6$, $3x + y + 2z = 8$.

23. Solve the tridiagonal system $x + 2y = 7$, $x - 3y - z = 4$, $4y + 3z = 5$.

24. Solve the system of equations $10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$ by using Gauss-Jacobi's iteration method.

25. Solve the system of equations $10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$ by using Gauss-Seidel method.

Unit-V

26. Given differential equation $y'' - xy' - y = 0$ with the conditions $y(0) = 1$ and $y'(0) = 0$, determine the value of $y(0.1)$ by using Taylor's series method.

27. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2+1}$ with the initial conditions $y = 0$ when $x = 0$, obtain the value of y for $x = 0.25$ by using Picard's method of successive approximations.

28. Solve $\frac{dy}{dx} = 1 + 2xy$ and $y(0) = 0$ by using Picard's method.

29. Given $\frac{dy}{dx} = -xy^2, y(0) = 2,$ compute y(0.2) in steps of 0.1 by using modified Euler's method.

30. Given $\frac{dy}{dx} = x^2 - y$ and $y(0) = 1$. Determine $y(0.02)$, $y(0.04)$ and $y(0.06)$ by using Euler's modified method.

31. Apply the fourth order Runge-Kutta method to find an approximate value of y when $x = 1.2$ in steps of 0.1, given that $y' = x^2 + y^2$ and $y(1) = 1.5$.

32. Use Runge-kutta fourth order method, find an approximate values of y when $x = 0.1$, $x = 0.2$ and $x = 0.3$, given that $\frac{dy}{dx} = x + y$ and $y(0) = 1$.

33. Obtain the values of y at $x = 0.1,\ 0.2$ by using Runge-kutta method of fourth order for the differential equation $y' + y = 0,\ y(0) = 1$.

P. R. GOVERNMENT COLLEGE (A), KAKINADA
III YEAR B.Sc. MATHEMATICS - SEMESTER – VI (w.e.f. 2018-19)
Course : CLUSTER VIII- (C)-1 PRINCIPLES OF MECHANICS

No. Hours: 75 hrs **Credits : 05**

Objectives:

- To understand the concepts of Mechanics which have applications in Physical problems.
- To Know the spherical co ordinate system.

Unit – I: **(12 hrs)**

D'Alembert's Principle and Lagrange's Equations: Some definitions – Lagrange's equations for a Holonomic system – Lagrange's Equations of motion for conservative, nonholonomic system.

Unit – II: **(12 hrs)**

Variational Principle and Lagrange's Equations: Variatonal Principle – Hamilton's Principle – Derivation of Hamilton's Principle from Lagrange's Equations – Derivation of Lagrange's Equations from Hamilton's Principle – Extension of Hamilton's Principle – Hamilton's Principle for Non-conservative, Non-holonomic system – Generalised Force in Dynamic System – Hamilton's Principle for Conservative, Non-holonomic system – Lagrange's Equations for Non- conservative, Holonomic system - Cyclic or Ignorable Coordinates.

Unit – III: **(12 hrs)**

Conservation Theorem, Conservation of Linear Momentum in Lagrangian Formulation – Conservation of angular Momentum – conservation of Energy in Lagrangian formulation.

Unit – IV: **(12 hrs)**

Hamilton's Equations of Motion: Derivation of Hamilton's Equations of motion – Routh's procedure – equations of motion – Derivation of Hamilton's equations from Hamilton's Principle – Principle of Least Action – Distinction between Hamilton's Principle and Principle of Least Action.

Unit – V: **(12 hrs)**

Canonical Transformation: Canonical coordinates and canonical transformations – The necessary and sufficient condition for a transformation to be canonical – examples of canonical transformations – properties of canonical transformation – Lagrange's bracket is canonical invariant – poisson's bracket is canonical invariant - poisson's bracket is invariant under canonical transformation – Hamilton's Equations of motion in poisson's bracket – Jacobi's identity for poisson's brackets.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Reference Text Books :

1. Classical Mechanics by C.R.Mondal Published by Prentice Hall of India, New Delhi.

2. A Text Book of Fluid Dynamics by F. Charlton Published by CBS Publications, New Delhi.

3. Classical Mechanics by Herbert Goldstein, published by Narosa Publications, New Delhi.

4. Fluid Mechanics by T. Allen and I.L. Ditsworth Published by (McGraw Hill, 1972)

5. Fundamentals of Mechanics of fluids by I.G. Currie Published by (CRC, 2002)

6. Fluid Mechanics : An Introduction to the theory, by Chia-shun Yeh Published by (McGraw Hill, 1974).

7. Introduction to Fluid Mechanics by R.W Fox, A.T Mc Donald and P.J. Pritchard Published by

(John Wiley and Sons Pvt. Ltd., 2003)

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS - VI Semester (w.e.f. 2018-19)
Course (Elective VIII (D)-1) : APPLIED GRAPH THEORY

Total Hours of Teaching-Learning : 75 @ 5 h/week **Total credits:05**

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Objectives:

- To introduce the most application oriented field in Mathematics i.e., Graph Theory.
- To impart the awareness on applications of Graph Theory

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Unit – I : Matchings **(12 hrs)**

Matchings – Alternating Path, Augmenting Path – Matchings and coverings in Bipartite graphs, Marriage Theorem, Minimum Coverings.

Unit – II : Perfect Matchings **(12 hrs)**

Perfect matchings, Tutte's Theorem, Applications, The Personal Assignment problem – The optimal assignment problem, Kuhn-Munkres Theorem

Unit – III : Edge Colorings **(12 hrs)**

Edge Chromatic Number, Edge Coloring in Bipartite Graphs – Vizing's theorem.

Unit – IV : Applications of Matchings, Independent sets and Cliques **(12 hrs)**

Applications of Matchings, The timetable problem, Independent sets, Covering number, Edge Independence Number, Edge Covering Number - Ramsey's theorem.

Unit – V : Ramsey's Number **(12 hrs)**

Determination of Ramsey's Numbers – Erdos Theorem, Turan's Theorem and Applications, Sehur's theorem, A geometry problem.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

PRESCRIBED BOOK: Graph Theory with Applications by J.A.Bondy and U .S.R. Murthy published by Mac. Millan Press.

REFERENCE BOOKS :

1. Graph theory with ApplicationsbyJ.A.Bondy and U.S.R.Murthy published by Mac.Millan Press
2. 2. Introduction to Graph theory by S.Arumugham and S.Ramachandran, published by Scitech Publications, Chennai-17
3. Graph theory and combinations by H.S.Govinda Rao published by Galgotia Publications
4.

BLUE PRINT FOR QUESTION PAPER PATTERN,

SEMESTER-VI

PAPER –VIII (D)-1, Cluster VIII D - 1

UNIT	TOPIC	V.S.A.Q	S.A.Q (including choice)	E.Q (including choice)	Marks Allotted
I	Matchings	01	01	02	22
II	Perfect matchings	01	01	01	14
III	Edge Colorings	01	01	01	14
IV	Applications of Matchings, Independent sets and Cliques	01	01	02	22
V	Ramsey's Number	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : _____= 60

P. R. Government College (A), Kakinada
III year B.Sc. Examination -Semester VI - Mathematics
Cluster -VIII (D) 1: Applied Graph Theory

PAPER VIII (D) 1 (MODEL PAPER w.e.f. 2019-20)

Time: 2 hours 30 Min **Max. Marks: 60**

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PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Define bipartite graph.
2. Define Perfect Matching.
3. State Vizing's theorem.
4. Define an independent set.
5. State the Erdos Theorem.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.
3 x 5 = 15M

6. Define and give example of maximum and Perfect matching in graphs.
7. Prove that every 3-regular graph with out cut edges has a perfect matching.
8. Define an M-alternating tree and give an example.
9. Explain the time table problem with an example.
10. Give an example of (3,5) – Ramsey graph.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. In a bipartite graph, prove that the number of edges in a maximum matching is equal to the number of vertices in a minimum covering.
12. Define and give example of M-Alternating Path and Covering of a graph.
13. Write the Kuhn – Munkers Algorithm.
14. If G is bipartite, then prove that $X' = \Delta$.

SECTION – B

15. A set $S \subseteq V$ is an independent set of G if and only if V/S is a covering of G

16. If $\delta > 0, then\ prove\ that\ \alpha' + \beta' = V$.

17. Prove that $r(k, l) \leq \binom{k+l-2}{k-1}$.

18. If a simple graph G contains no K_{m-1}, then prove that G is degree majorised by some complete m-partite graph H.

P. R. GOVERNMENT COLLEGE (A), KAKINADA

III YEAR B.Sc. MATHEMATICS - SEMESTER – VI (w.e.f. 2017-18)

Course: CLUSTER VIII- (A,B)-2 SPECIAL FUNCTIONS

No. Hours: 75 hrs **Credits: 05**

Objectives:

- To understand the concepts of special functions which have applications in Physical Sciences.
- To learn finding power series solutions to some special types of differential equations.

UNIT - I : HERMITE POLYNOMIAL **(12 hrs)**

Hermite Differential Equations, Solution of Hermite Equation, Hermite's Polynomials, Generating function, Other forms for Hermite Polynomial, To find first few Hermite Polynomials, Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials. (CHAPTER: 6.1 to 6.8)

UNIT - II : LAGUERRE POLYNOMIALS **(12 hrs)**

Laguerre's Differential equation, Solution of Laguerre's equation, Laguerre Polynomials, Generating function, Other forms for the Laguerre Poynomials, To find first few Laguerre Polynomials, Orthogonal property of the Laguerre Polynomials, Recurrence formula for Laguerre Polynomials, Associated Laguerre Equation. (CHAPTER: 7.1 to 7.9)

UNIT – III : LEGENDRE'S EQUATION **(12 hrs)**

Definition, Solution of Legendre's Equation, Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation(derivationis not required), To show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1-2xh+h^2)^{1/2}$, Orthogonal properties of Legendre's Equation, Recurrence formulae, Rodrigue's formula. (CHAPTER: 2.1 to 2.8, 2.12)

UNIT - IV : BESSEL'S EQUATION **(12 hrs)**

Definition, Solution of Bessel's General Differential Equations, General solution of Bessel's Equation, Integration of Bessel's equation in series for n=0, Definition of $J_n(x)$, Recurrence formulae for $J_n(x)$, Generating function for $J_n(x)$. (CHAPTER: 5.1 to 5.7)

UNIT - V : BETA AND GAMMA FUNCTIONS **(12 hrs)**Euler's Integrals - Beta and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions,Another form of Beta Function, Relation between Beta and Gamma Functions, Other Transformations. (CHAPTER: 2.9 to 2.15)

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Prescribed text book: Special Functions by J.N. Sharma and Dr. R.K. Gupta.

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SEMESTER-VI, CLUSTER VIII –A, B, D-2

SPECIAL FUNCTIONS

UNIT	TOPIC	V.S.A.Q 1 M	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	Hermit Polynomial	01	01	01	14
II	Laguerre Polynomial	01	01	01	14
III	Legendre’s Equation	01	01	02	22
IV	Bessel’s Equation	01	01	02	22
V	Beta And Gama Functions	01	01	02	22
Total		05	05	08	94

V.S.A.Q. = Very Short answer questions (1 mark)
S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Very Short answer questions : 5 x 1 M = 05
Short answer questions : 3 x 5 M = 15
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P.R. Government College (A), Kakinada
III B.Sc. Degree Examinations: Semester-VI, Mathematics
COURSE (Cluster VIII (A,B) 2) Special Functions
PAPER-VIII (A,B)-2 (MODEL PAPER w.e.f. 2019-2020)

Time: 2 hrs 30 Min **Max. Marks : 60 M**

PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Write the generating function of Hermit's polynomial.
2. Show that $L_1(x) = 1 - x$.
3. Define Legendre's equation.
4. Write $J_0(x)$.
5. Show that $\Gamma(1) = 1$.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.
3 x 5 = 15M

6. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)dx$.
7. Show that $L_2(x) = \frac{1}{2!}(2 - 4x + x^2)$.
8. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.
9. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.
10. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}$ dx.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. State and Prove Rodrigue's formula for $H_n(x)$.
12. Prove that $xL_n''(x) + (1 - x)L_n'(x) + nL_n(x) = 0$.
13. Prove that $(2n + 1)xP_n = (n + 1)P_{n+1} + nP_{n-1}$.
14. Show that $\int_{-1}^{1} P_m(x).P_n(x)\, dx = 0\ if\ m \neq n$.

SECTION – B

15. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.
16. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.
17. When n is a positive integer, prove that $\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-1)^n 2^n\sqrt{\pi}}{1.3.5.....(2n-1)}$
18. Prove that $B(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$

PR GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
QUESTION BANK
PAPER–VIII B-2: SPECIAL FUNCTIONS

Very Short Answer Questions

Unit - I

1. Write the Hermite's Differential Equation.
2. Write the Hermite's Polynomial.
3. Write the generating function of Hermit's polynomial.
4. Write the Rodrigue's formula for $H_n(x)$.
5. Find the value of $H_0(x)$.
6. Write the orthogonal properties of Hermite's polynomial.

Unit –II

7. Define Laguerre's differential equation.
8. Write the Laguerre's polynomial.
9. Write the generating function of Laguerre's polynomial.
10. Write the Rodrigue's formula for Laguerre's polynomial.
11. Show that $L_1(x) = 1 - x$.
12. Write the orthogonal properties of $L_n(x)$.

Unit - III

13. Define Legendre's equation.
14. Write the Legendre's polynomial.
15. Write the general solution of Legendre's equation.
16. Write the generating function of Legendre's polynomial.
17. Write the Laplace's first integral for $P_n(x)$.
18. Write the Laplace's second integral for $P_n(x)$.
19. Write the orthogonal properties of $P_n(x)$.
20. Write the Rodrigue's formula for $P_n(x)$.
21. Show that $P_n(1) = 1$.

Unit - IV

22. Define Bessel's equation.
23. Write the Bessel's function of the first kind of order n.
24. Write the general solution of Bessel's equation.
25. Write $J_0(x)$.
26. When n is a positive integer, show that $J_{-n}(x) = (-1)^n J_n(x)$.

Unit - V

27. Define Gamma function.
28. Show that $\Gamma(1) = 1$.
29. Find $\Gamma\left(\frac{3}{2}\right)$.
30. Evaluate $\int_0^\infty x^4 . e^{-x}\, dx$.
31. Define Beta function.
32. Write the relation between Beta and Gamma functions.
33. Write the legendre's duplication formula.

Short Answer questions

Unit - I

1. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}\ and\ H_{2n+1}(0) = 0$.
2. Find Hermite Polynomials for n=0, 1, 2.
3. Prove that $H_n'' = 4n(n-1)H_{n-2}$
4. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$
5. Prove that, if $m < n$, $\frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^m n!}{(n-m)!} H_{n-m}(x)$.
6. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)$.

Unit - II

7. Show that $L_n(x) = \frac{e^x}{n!}\frac{d^n(x^n e^{-x})}{dx^n}$.
8. Prove that $L_n'(x) = -\sum_{r=0}^{n-1} L_n(x)$.
9. Show that $L_2(x) = \frac{1}{2!}(2 - 4x + x^2)$.

Unit - III

10. Show that $P_n(x)$ is the coefficient of h^n in the expansion in ascending powers of $(1 - 2xh + h^2)^{-1/2}$.
11. Show that $P_n(1) = 1\ and\ P_n(-x) = (-1)^n P_n(x)$.
12. Prove that $(2n+1)P_n = P_{n+1}' - P_{n-1}'$.
13. Prove that $(n+1)P_n = P_{n+1}' - xP_n'$.
14. Prove that $(1-x^2)P_n' = n(P_n - xP_n)$.
15. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

Unit - IV

16. Prove that, when n is a positive integer $J_{-n}(x) = (-1)^n J_n(x)$.
17. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$
18. Show that $J_n(-x) = (-1)^n J_n(x)$ for positive or negative integers.

Unit - V

19. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$
20. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}$ dx.
21. Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\ dx$.
22. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}} dy\ \ and\ hence\ show\ that\ \ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
23. Evaluate $\int_0^2 x(8 - x^3)^{1/3} dx$.
24. Evaluate $\int_0^2 \frac{x^2}{\sqrt{2-x}}\ dx$
25. Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\ dx$.
26. Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

Essay Questions
Unit –I

1. State and Prove generating function of the Hermit's polynomial.
2. State and Prove Rodrigues formula for $H_n(x)$.
3. Find Hermite Polynomials for n=0, 1, 2, 3, 4 and 5.
4. State and Prove Orthogonal Properties of Hermite Polynomials.
5. Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.
6. Prove that $H_n'(x) = 2nH_{n-1}(x) \quad n \geq 1$.

Unit – II

7. Prove that $\frac{1}{1-t} e^{-tx/(1-t)} = \sum_{n=0}^{\infty} t^n L_n(x)$
8. Find $L_0(x), L_1(x), L_2(x), L_3(x)$.
9. Prove that $\int_0^{\infty} e^{-x} L_n(x) L_m(x)\, dx = \begin{cases} 0 & if\ m \neq n \\ 1 & if\ m = n \end{cases}$
10. $xL_n'(x) = nL_n(x) - nL_{n-1}(x)$
11. Prove that $xL_n''(x) + (1-x)L_n'(x) + nL_n(x) = 0$ and hence deduce that $L_n'(0) = -n$.
12. Prove that $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$

Unit – III

13. When n is a positive integer, prove that $P_n(x) = \frac{1}{\pi}\int_0^{\pi}\left[x \pm \sqrt{x^2-1}\cos\varphi\right]^n d\varphi$.
14. When n is a positive integer, prove that $P_n(x) = \frac{1}{\pi}\int_0^{\pi}\left[x \pm \sqrt{x^2-1}\cos\varphi\right]^n \frac{d\varphi}{\left[x \pm \sqrt{x^2-1}\cos\varphi\right]^{n+1}}$
15. Prove that $\int_{-1}^{1}[P_n(x)]^2 dx = \frac{2}{2n+1}$.
16. Prove that $\int_{-1}^{1} P_m(x).P_n(x)dx = 0 \ \ if\ m \neq n$.
17. Prove that $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$
18. Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's Polynomials.
19. Prove that $P_n(x) = \frac{1}{n!2^n}.\frac{d^n}{dx^n}(x^2-1)^n$.
20. Prove that $\int_{-1}^{1}(x^2-1)P_{n+1}P_n'\, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$.
21. Prove that $\int_{-1}^{1}x^2 P_{n+1}P_{n-1}\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.

Unit – IV

22. Prove that when n is a positive integer, $J_n(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x(z-\frac{1}{z})}{2}}$ in ascending and descending powers of z.
23. Prove that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

24. Prove that $xJ_n'(x) = -nJ_n(x) + xJ_{n-1}(x)$.

25. Prove that $\frac{d}{dx}(xJ_nJ_{n+1}) = x(J_n^2 - J_{n+1}^2)$.

26. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

27. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

28. Prove that $\sqrt{\frac{\pi x}{2}}J_{3/2}(x) = \frac{1}{x}\sin x - \cos x$.

29. Show that $\cos x = J_0 - 2J_2 + 2J_4 - \cdots$ .... and
$\sin x = 2J_1 - 2J_3 + 2J_5 - \cdots$ ...

Unit V

30. Show that $\Gamma\left(n+\frac{1}{2}\right) = \frac{1.3.5....(2n-1)\sqrt{\pi}}{2^n}$, when n is a positive integer.

31. When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-1)^n 2^n\sqrt{\pi}}{1.3.5.....(2n-1)}$

32. Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

33. Show that $\Gamma\left(\frac{3}{2}-x\right)\Gamma\left(\frac{3}{2}+x\right) = \left(\frac{1}{4}-x^2\right)\pi\sec\pi x$ provided $-1 < 2x < 1$.

34. Show that (i) $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \pi\sqrt{2}$ (ii) $\Gamma(x)\Gamma(-x) = -\frac{\pi}{x\sin\pi x}$

35. Prove that $\int_0^{\pi/2} sin^{2l-1}\theta.cos^{2m-1}\theta\, d\theta = \frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

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P.R. GOVERNMENT COLLEGE (A), KAKINADA

III YEAR B.Sc. MATHEMATICS - SEMESTER – VI (w.e.f. 2018-19)
Course : CLUSTER VIII- (C)-2 FLUID MECHANICS

No. Hours: 75 hrs **Credits : 05**

Objectives:

- To understand the concepts of Fluid Mechanics which have applications in water flow, blood flow etc.
- To Know the recent advances in fluid dynamics.

Unit - I: **(12 hrs)**

Kinematics of Fluids in Motion - Real fluids and Ideal fluids – Velocity of a Fluid at a point – Streamlines and pthlines – steady and Unsteady flows – the velocity potential – The Vorticity vector – Local and Particle Rates of Change – The equation of Continuity – Acceleration of a fluid – Conditions at a rigid boundary – General Analysis of fluid motion.

Unit – II: **(12 hrs)**

Equations of motion of a fluid- Pressure at a point in fluid at rest – Pressure at a point in a moving fluid – Conditions at a boundary of two inviscid immiscible fluids – Euler’s equations of motion – Bernoulli’s equation – Worked examples.

Unit – III: **(12 hrs)**

Discussion of the case of steady motion under conservative body forces - Some flows involving axial symmetry – Some special two-dimensional flows – Impulsive motion – Some further aspects of vortex motion.

Unit – IV: **(12 hrs)**

Some Two – dimensional Flows, Meaning of two-dimensional flow – Use of Cylindrical polar coordinates – The stream function – The complex potential for two-dimensional, Irrotational, Incompressible flow – Uniform Stream – The Milne-Thomson Circle theorem – the theorem of Blasius.

Unit – V: **(12 hrs)**

Viscous flow,Stress components in a real fluid – Relations between Cartesian components of stress – Translational motion of fluid element – Thes rate of strain quadric and principal stresses – Some further properties of the rate of strain quadric – Stress analysis in fluid motion – Relations between stress and rate of strain – the coefficient of viscosity and laminar flow - The Navier- Stokes equations of motion of a viscous fluid.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(12 Hrs)**

Reference Text Books :

1. A Text Book of Fluid Dynamics by F. Charlton Published by CBS Publications, New Delhi.

2. Classical Mechanics by Herbert Goldstein, published by Narosa Publications, New Delhi.

3. Fluid Mechanics by T. Allen and I.L. Ditsworth published by (McGraw Hill, 1972)

4. Fundamentals of Mechanics of fluids by I.G. Currie published by (CRC, 2002)

5. Fluid Mechanics, An Introduction to the theory by Chia-shun Yeh published by (McGraw Hill, 1974)

6. Fluids Mechanics by F.M White published by (McGraw Hill, 2003)

7. Introduction to Fluid Mechanics by R.W Fox, A.T Mc Donald and P.J. Pritchard published by (John Jiley and Sons Pvt. Ltd., 2003).

P. R. GOVERNMENT COLLEGE (A), KAKINADA

B.Sc. THIRD YEAR MATHEMATICS - SEMESTER – VI (w.e.f. 2018-19)
Course : CLUSTER VIII- (D) 2 DISCRETE MATHEMATICS

No. Hours: 75 hrs **Credits : 05**

Objectives:

- To understand the concepts of Discrete Mathematics.
- To know the applications of Discrete Mathematics in Computer Science.

Unit - I: **(12 hrs)**

Normal Forms – Disjunctive - Conjunctive Principal Disjunctive, Principal Conjunctive Normal Forms – Ordering and Uniqueness of Normal Forms.

Unit – II: **(12 hrs)**

The theory of Inference for the statement Calculus - Rules of inferences - Consistency of Premises - Automatic Theorem proving.

Unit – III: **(12 hrs)**

The predicate calculus - Inference Theory of the Predicate Calculus.

Unit – IV: **(12 hrs)**

Lattices as partially Ordered sets - Lattices as Algebraic Systems - Boolean Algebra - Boolean Functions- Minimization.

Unit – V: **(12 hrs)**

Finite - State Machines - Basic Concepts of Graph Theory - Basic Definitions – Paths -Reach ability and Connectedness - Matrix Representation of Graphs - Trees.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Reference Text Books:

1. Scope and Standard as in the book " Discrete Mathematical Structures With Applications To Computer Science" by Tremblay, J.P. & Manohar, R - Published by McGraw-Hill International Edition -1987 Edition. 2. "Discrete Mathematics & Graph Theory" by Bhavanari Satyanarana & Kuncham Syam Prasad, PHI Publications, New Delhi, Second Edition, 2014.

3. "Mathematical Foundation of Computer Science" by Bhavanari Satyanarayana, TV Pradeep Kumar, SK. Mohiddin Shaw, BS Publications, Hyderabad.2016.

P.R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS

Massive Open Online Course (MOOCS) CERTIFICATE COURSE

Additional Credits: Achieved Credits

Guidelines of this course:

After completion of the course the student is able to get 2 additional credits through the examination cell under the following conditions.

- Completed the course through the online platforms Swayam, UGC, CEC, NPTEL, AICTE, NCERT, etc.
- Course related to any Mathematical subject or interdisciplinary with mathematics one of the subject.
- Course contains at least a minimum of 4 weeks.
- Course completion certificate must be submitted to the Examination cell through the department.

For more details about online courses go through the following links:

- http://www.apcce.gov.in/SwC
- https://swayam.gov.in/
- http://free.aicte-india.org/
- https://ugcmoocs.inflibnet.ac.in/
- https://swayam.gov.in/nc_details/CEC
- https://swayam.gov.in/nc_details/NCERT

P.R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS

CERTIFICATE COURSE ON “CRITICAL THINKING”

Additional Credits: 2

Objectives of the Course:

Critical thinking is the ability to think clearly and rationally about what to do or what to believe. It includes the ability to engage in reflective and independent thinking. This course is designed for all second year degree students.

The objectives of this course are:

- To understand the components of critical thinking.
- To utilize non-linear thinking
- To use logical thinking.

Modules of the course:

1.	Critical Reasoning	- 15 hours
2.	Facts, Inference and Judgment	- 6 hours
3.	Probably/Definitely True or False	- 6 hours
4.	Cause and Effect	- 6 hours
5.	Course of Action	- 6 hours
6.	Syllogisms	- 6 hours
	Total Hours	**- 45 hours**

Examination Pattern: Multiple choice questions 50. Each question carries 1 mark.

Outcome of the course:

After the completion of the course the student is able

- To analyze and evaluate arguments
- To separate facts from opinions
- To think logically and arrive at sound decisions based on logical conclusions.
- To understand the logical connections between ideas.

LIST OF EXAMINERS & PAPER SETTERS IN MATHEMATICS

S.No.	Name of the Lecturer	Address
1	Smt. Gayatri	Lecturer in Mathematics, Government College (A), Rajamahendravaram.
2	Dr. D. Sai Baba	Lecturer in Mathematics, Sri.ASNM Govt college (A),Palkolu
3	Dr. Ch. Srinivas	Lecturer in Mathematics, Government College (A), Rajamahendravaram
4	Sri. K. Chitti Babu	Lecturer in Mathematics, Government Degree College, Ramachandrapuram.
5	Dr. V.S. Patnayak	Lecturer in Mathematics, M.R College, Vizaianagaram
6	Sri. K. Kameswara Rao	Lecturer in Mathematics, Government Degree College,Pithapuram
7	Ms. Y. Padmaja	Lecturer in Mathematics, Government Degree College, Ramachandrapuram
8	Sri. T. Srinivas Reddy	Lecturer in Mathematics, Government Degree College, Ramachandrapuram
9	Sri N. Kiran Kumar	Lecturer in Mathematics, Government Degree College, Mandapeta
10	Dr. SK. Sajana	Lecturer in Mathematics, S.R.R. Government Degree College, Vijayawada.
11	M. Madhavi	Lecturer in Mathematics, Government Degree College, Tuni.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2021-2022 (ODD SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **235** (actual)

No. of Permanent posts sanctioned : 05

No. of Permanent staff working : nil

No. of Contract faculty : 05

No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC EM	6	-	-	-	6	
2	I MPE	6	-	-	-	6	
3	I MCAc	6	-	-	-	6	
4	I MCPc	6	-	-	-	6	
5	I MSAs	6	-	-	-	6	
6	I MPCs	6	-	-	-	6	
7	I MECs	6	-	-	-	6	
8	I MSCs	6	-	-	-	6	
9	I MCCs	6	-	-	-	6	
10	I MEIOT	6	-	-	-	6	
11	I BVOC	6	-	-	-	6	
12	II MPC TM	6	-	-	-	6	
13	II MPC EM	6	-	-	-	6	
14	II MPE	6	-	-	-	6	
15	II MCAc	6	-	-	-	6	
16	II MCPc	6	-	-	-	6	
17	II MSAs	6	-	-	-	6	
18	II MPCs	6	-	-	-	- 6	
19	II MECs	6	-	-	-	6	
20	II MSCs	6	-	-	-	6	
21	II MCCs	6	-	-	-	6	
22	II BVOC	6	-	-	-	6	
23	III MPC TM	5+5	-	-	-	10	
24	III MPC EM	5+5	-	-	-	10	
25	III MPE	5+5	-	-	-	10	
26	III MCAc	5+5	-	-	-	10	
27	III MCPc	5+5	-	-	-	10	
28	III MSAs	5+5	-	-	-	10	
29	III MPCS	5+5	-	-	-	10	
30	III MECs	5+5	-	-	-	10	
31	III MSCs	5+5	-	-	-	10	
32	III MCCs	5+5	-	-	-	10	
33	III BVOC	3	-	-	-	3	
Total Work load for the subject Mathematics						**235**	

GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2021-2022 (ODD SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **141** (adjusted)

No. of Permanent posts sanctioned : 05

No. of Permanent staff working : NIL

No. of Contract faculty : 05

No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1.	I MPC EM, MPE,	6	-	-	-	6	
2.	I MCAc	6	-	-	-	6	
3.	I MCPc, MSAs	6	-	-	-	6	
4.	I MPCs, MECs, MEIOT	6	-	-	-	6	
5.	I MSCs, MCCs	6	-	-	-	6	
6.	I BVOC	6	-	-	-	6	
7.	II MPC TM	6	-	-	-	6	
8.	II MPC EM, MPE,	6	-	-	-	6	
9.	II MCAc	6	-	-	-	6	
10.	II MCPc, MSAs	6	-	-	-	6	
11.	II MPCs, MECs	6	-	-	-	6	
12.	II MSCs, MCCs	6	-	-	-	6	
13.	II BVOC	6	-	-	-	6	
14.	III MPC TM	5+5	-	-	-	10	
15.	III MPC EM, MPE	5+5	-	-	-	10	
16.	III MCAc	5+5	-	-	-	10	
17.	III MCPc, MSAs	5+5	-	-	-	10	
18.	III MPCS, MECs	5+5	-	-	-	10	
19.	III MSCs, MCCs	5+5	-	-	-	10	
20.	III BVOC	3	-	-	-	3	
Total Work load for the subject Mathematics						**141**	

In addition to these hours there are activity hours @ 2 hours for each class for 1st and 2nd years and 1 hour for 3rd year.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2021-2022 (EVEN SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **224** (actual)
No. of Permanent Postssanctioned : 05
No. of Permanent staff working : NIL
No. of Contract faculty : 05
No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1.	I MPC TM	6	-	-	-	6	
2.	I MPC EM	6	-	-	-	6	
3.	I MPE	6	-	-	-	6	
4.	I MCAc	6	-	-	-	6	
5.	I MCPc	6		-	-	6	
6.	I MSAs	6	-	-	-	6	
7.	I MPCs	6	-	-	-	6	
8.	I MECs	6	-	-	-	6	
9.	I MEIOT	6	-	-	-	6	
10.	I MSCs	6	-	-	-	6	
11.	I MCCs	6	-	-	-	6	
12.	IBVOC	6	-	-	-	6	
13.	II MPC TM	6	-	-	-	6	
14.	II MPC EM	6	-	-	-	6	
15.	II MPE	6	-	-	-	6	
16.	II MCAc	6	-	-	-	6	
17.	II MCPc	6	-	-	-	6	
18.	II MSAs	6	-	-	-	6	
19.	II MPCS	6	-	-	-	6	
20.	II MECs	6	-	-	-	6	
21.	II MSCs	6	-	-	-	6	
22.	II MCCs	6	-	-	-	6	
23.	III MPC TM	5	-	-	-	5	
24.	III MPC EM	5	-	-	-	5	
25.	III MPE	5	-	-	-	5	
26.	III MCAc	5	-	-	-	5	
27.	III MCPc	5	-	-	-	5	
28.	III MSAs	5	-	-	-	5	
29.	III MPCS	5	-	-	-	5	
30.	III MECs	5	-	-	-	5	
31.	III MSCs	5	-	-	-	5	
32.	III MCCs	5	-	-	-	5	
33.	III BVOC	3	-	-	-	3	
34.	Maths cluster	5+5	-	-	-	10	
35.	Project work	5	-	-	-	5 for each project	
36.	Analytical Skills	24	-	-	-	24	
Total Work load for the department of Mathematics						**224**	

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2021-2022 (EVEN SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **138** (adjusted)
No. of Permanent Postssanctioned : 05
No. of Permanent staff working : NIL
No. of Contract faculty : 05
No. of Part – Time Faculty : 03

S. No.	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1.	I MPC (TM)	6	-	-	-	6	
2.	I MPC EM, MPE	6	-	-	-	6	
3.	I MCAc	6	-	-	-	6	
4.	I MCPc, MSAs	6	-	-	-	6	
5.	I MPCs, MECs, MEIOT	6	-	-	-	6	
6.	I MSCs, MCCs	6	-	-	-	6	
7.	IBVOC	6	-	-	-	6	
8.	II MPC (TM)	6	-	-	-	6	
9.	II MPC EM, MPE	6	-	-	-	6	
10.	II MCAc	6	-	-	-	6	
11.	II MCPc, MSAs	6	-	-	-	6	
12.	II MPCS, MECs	6	-	-	-	6	
13.	II MSCs, MCCs	6	-	-	-	6	
14.	III MPC (TM))	5	-	-	-	5	
15.	III MPC EM, MPE	5	-	-	-	5	
16.	III MCAc	5	-	-	-	5	
17.	III MCPc, MSAs	5	-	-	-	5	
18.	III MPCS, MECs	5	-	-	-	5	
19.	III MSCs, MCCs	5	-	-	-	5	
20.	III BVOC	3	-	-	-	3	
21.	Maths cluster	5+5	-	-	-	10	
22.	Project work	5	-	-	-	5 for each project	
23.	Analytical Skills	24	-	-	-	12	
Total Work load for the department of Mathematics						**138**	

In addition to these hours there are activity hours @ 2 hours for each class for 1st and 2nd years and 1 hour for 3rd year

P. R. GOVERNMENT COLLEGE (A), KAKINADA

ACTION PLAN FOR THE ACADEMIC YEAR 2021-2022

DEPARTMENT OF MATHEMATICS

S.No.	Month	Week	Item as approved in BOS and to be incorporated in AC Meeting agenda as Institution Plan	Outcome of the activity
1	November 2021	IV	Mathematics Extension Lecture	Students Knowledge will be updated
2	November 2021	II	General Quiz Competitions	The competitive spirit will be improved among the students
3	December, 2021	II	Competitions to school children	The children may be inspired and put more concentration on Mathematics
4	December 2021	III	Celebration of Mathematics Day on 22nd Dec -2021	The students will be motivated to pursue higher education in Mathematics
5	February, 2022	IV	Science day celebrations	Students will get more interest to do projects and there is a scope to know the applicability of all subjects
6	March, 2022	II	π Day celebrations	To import the knowledge on the significance of π