P.R.GOVERNMENT COLLEGE (AUTONOMOUS)

KAKINADA

(Accredited by NAAC "A" Grade with 3.17 CGPA)

UG BOARD OF STUDIES : 2022 - 23

DEPARTMENT OF MATHEMATICS

Curriculum for the Academic Year 2022 - 23

INDEX

DEPARTMENT OF COLLEGIATE EDUCATION
GOVERNMENT OF ANDHRA PRADESH

PROCEEDINGS OF THE PRINCIPAL, PITHAPUR RAJAH's GOVT. COLLEGE[A]:: KAKINADA

Present: Dr. B.V. TIRUPANYAM, Ph.D.

Rc.No.12A/A.C/BOS/2022-23 **Dt.24Sept'2022**

Sub:P.R.G.C[A]–AcademicCell- **ConductofBOSMeetingsfortheAcademicYear 2022-23** – Guidelines issued -Regarding.

Ref: 1. Minutes of IQAC meeting dated 18 September 2022

2. Resolutions adopted in 22nd Staff Council Meeting held on 23 Sept 2022

PREAMBLE

The Autonomous colleges are, as per its vision, mission, stated objectives and core values, mandated to design and develop their own outcome -based curricula keeping in view the societal, local and global industry requirements, employability and industry – ready and transferable skills duly prescribing Course Outcomes (COs), Programme Outcomes (POs) and Programme Specific Outcomes (PSOs) and suitable learning outcome assessment management system through robust and transparent evaluation system to measure their attainment levels of the students.

The Sustained Developmental Goals (SDG-4)of UNEP recommended assurance of quality to students in HEIs promoting creativity, critical thinking and collaborative skills, while building curiosity, courage, resilience and gender equality among students.

Further, the NEP-2020 recommended that the HEIs shall equip students with such skills that translate them into leaders and potential entrepreneurs too besides credit transfer mechanism through ABC (Academic Bank of Credits).

The HEIs are also, as per the Revised Accreditation Framework [RAF] of NAAC, endowed with the responsibility of rolling out quality and holistic human resources to the modern Indian Economy by ingraining quality in teaching- learning process by facilitating the students experience a wide range of participative and experiential learning strategies including field trips, conferences, integration of technology, community service programmes, career guidance, certificate and value added courses, research and inquisition based teaching, exchange programmes, gender equity programmes, etc.

Besides, the students shall have social consciousness, regard for constitutional provisions, right perspective on environmental protection, awareness on gender equity, health and hygiene, Yoga and wellness, college social responsibility, culture and values, etc., to mention a few.

Further, the Ministry of India, GoI, through NIRF, prescribes quality research, infrastructure augmentation, enhanced placement and progression to higher education, equipment of employability skills leading to enhanced public perception about the college among the public.

Our institution has, from AY 2022-23, has devised its new vision and mission along with objectives and core values necessitating design and re-orientation of its academic administration in tune with them.

ORDER:

In the light of the above mandate and responsibilities prescribed by institutions vision and mission, SDG-4, NEP – 2020, NAAC, NIRF to the autonomous HEIs, need to customize, design and re-orient their academic and research administration in tune with the policies of above bodies, our insitution is no exception.

Hence, the Chairmen of U.G and P.G Boards of Studies of various Departments are requested to make necessary arrangements for the conduct of the meetings separately between **11 October 2022 and 15 October 2022**. They are further requested to prepare curricula and extracurricular activities and devise suitable evaluation system keeping in mind above recommendations to make students a wholesome personality and a 21st century student capable of facing challenges, adaptive to changes, creative and innovative.

Further, the Chairman of the each BOS, in association with the IQAC coordinator, preceding the BOS meeting, is requested to prescribe benchmarking, quality initiatives in pedagogy and learning; in design of curriculum (with 20% change) and optimum utilization of existing human, physical and ICT resources and adopt resolutions to the extent of benchmarks (As per SOP given in **Annexure – I)**. Further, as the regular attendance of students to the classes is a deciding factor in enhancement of quality in learning, a minimum attendance of 60% forI mid-term examination, 75% for II mid-term examination under CIA component shall be the benchmark for attendance and it shall be approved in the BOS. The Chairmen are also requested to approve the new programmes to be introduced for 2022-23, if any, number of certificate courses, their frequency, Bloom's- Taxonomy based evaluation system for effective learning outcomes as per the Annexure -I

The Chairmen are, therefore, requested to

- Design curricula of Odd and even semesters for the A.Y 2022-23 both for U.G and P.G courses in tune with the stated vision, mission of the institution, RAF of NAAC, NEP-2020 and NIRF.
- Conduct meeting with employers, parents, alumni, shall take feedback on the existing curricula and invite suggestions and changes to be made.
- Invite the University nominee, subject experts, industrial nominees, student nominees, parents well in advance along with the date, venue, agenda, etc. A soft copy shall be communicated well in advance to the members to have an idea on the matters.
- Facilitate much room for intense deliberation on the design of the curricula, evaluation system, research component, enhancing learning experiences, resource utilization by staff and students, etc.,
- Each Department shall approve and recommend additional credits for additional modules, training programmes, N.S.S, N.C.C, participation in cultural programs, sports and games, environmental programs, blood donations camps, etc.
- All meetings shall be offline. Online attendance of members faculty will be permitted only in exceptional cases.
- The Chairmen shall submit minutes of the meeting in the prescribed format only (Annexure – II) in triplicate(hard copies) to the Academic cell for onward submission to the IQAC, Examination cell and library within three days from the completion of BOS meeting and besides hosting the soft copy in the college website within the period stipulated.
- Each Chairman of BOS, shall get the rough draft of the curricula verified and approved by the Principal, Academic Cell and IQAC before the actual BOS meetings to ensure uniformity and commensurate with the stated vision and mission of the college among the departments.
- The Academic Cell coordinator shall be the Chief Coordinator for the BOS meeting activity and IQAC coordinator will be the additional coordinator.
- The Academic Coordinator and IQAC coordinator shall conduct a meeting with the Chairmen, BOS between 28-29 September 2022 and explain the structure of curricula, uniformity other

modalities.

- The Controller of Examinations of the institution shall fund the BOS meetings from the available funds on the condition of reimbursement after receiving autonomous funds from UGC. Initially, he shall pay Rs. 5,000/- uniformly as an advance per Board to the respective Chairman (If BOS meetings for multiple Boards are to be held under one Chairmanship, he/ she shall be given advance amount equivalent to the number of Boards xRs.5000/-.
- TheChairmanofeachBOSshallapplytothePrincipalforadvanceamountformeetingtheBOSmeetings with head-wise expenditure in the prescribed format (Annexure-III).

Following contents shall be presented in the BOS document in order

1. Proceedings of the Principal pertaining to BOS
2. Composition of BOS
3. Vision and Mission of the college
4. Agenda: It shall include ATR on the previous BOS meeting first, resolutions, etc., later.
5. TableshowingtheAllocationofCreditsinthefollowingtableforboththeoryand Labin case of science subjects

S. No	Semester	Title of the Course (Paper)	Hrs./week	Max. Marks (SEE)	Marks in CIA	Credits
1	III	Abstract Algebra	6	50	50	4

6. Resolutionsadoptedinthemeetingwithdetaileddiscussionthattookplaceduringth e meeting (Activities and Benchmarking as per Annexure–I)
7. At the end of each theory paper, each topic shall be mapped as per the Blooms taxonomy and scope of that topic for skill/ employability/ entrepreneurship opportunities in the following table incorporated.

S. No	Subject	Semester	Title of the Course (Paper)	Topic	Parameter as per Blooms taxonomy (Knowledge/ Application/ Creativity/ Innovation	Experiential learning component	Scope (Skill/ employabil ity/ entreprenu ership)
1	Botany	III	Plant Physiology	Plant Cell	Knowledge	Shall be shown Microscope	
2	History	III	Tourism	Tourism management	Application	Apprenticeship	Employability

8. Each BOS Chairman shall, immediately after syllabus, tabulate the changes made in the syllabus/ paper along with justification, in the Proforma given in Annexure –I.
9. Attendance of Members present with signatures in the tabular form.
10. List of Examiners & Paper setters
11. Syllabus for each course (both theory & Practical in case of Science subjects) followedbymodelquestionpapers(theory&practical)andallocationofCIA(50 marks) for each course with structure.
12. CO-PO mapping /PO attainment data
13. Text & Reference Books
14. e-content links

PRINCIPAL
P.R. Govt. College (A)
KAKINADA

OFFICE OF THE DEAN, ACADEMIC AFFAIRS
ADIKAVI NANNAYA UNIVERSITY
RAJAMAHENDRAVARAM

No. ANUR/DAA/PR Govt. College (A)/Sub. Experts/2021 Date: 22-10-2021

PROCEEDINGS OF THE VICE-CHANCELLOR

Sub:- ANUR- DAA – Nominated University Subject Experts for BOS – PR Govt. College (A), Kakinada – Orders - Issued.

Ref:- 1. Lr. dated 15.09.2021, from the Principal, PR Govt. College (A), Kakinada
2.Proc. No: ANUR/PRG College (A), KKD/UG BoS/2019/09, dated 19.03.2019

Read:-Note for Orders of the Vice-Chancellor dated 21.10.2021

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ORDERS

Having consider the request cited in the ref. 1, the Vice-Chancellor is pleased to order that the following members be nominated as University Subject Experts for UG Board of Studies of ***PR Govt. College (A), Kakinada*** for a period of three years from the date of the proceedings issued.

S.No.	UG Courses	Name of the Subject Expert
1	English	Dr. Prasanthi Sree, AKNU MNS Campus, Kkd, Ph No: 9848297555, sathupathi.sri@gmail.com
2	Hindi	Dr. N Venkata Ramana, SKBR College, Amalapuram, Ph. No: 9849373773
3	Telugu	Dr. P. Nagaraju, GDC, Palakollu, Ph.No: 9052038569, raju00517@gmail.com
4	Sanskrit	Dr. TGY Acharyulu, SKR Womens College, Rajahmundry, Ph. No: 9848628812
5	Mathematics	Dr. V. Anantha Lakshmi, Principal, GDC Pithapuram, Ph. No : 9963786386, ananthamaths@rediffmail.com
6	Statistics & Actuarial Sciences	Dr. D V Ramana Murthy, HoD of Statistics, SKVT College, Rajamahendravaram, Ph.No: 9949135864, drdvrmurthy@gmail.com
7	Chemistry & Analytical Chemistry	Dr. K. Jhansi Lakshmi, Principal, Ideal College of Arts & Sciences, KKD, Ph.No: 9441236409, jhansikalisindi@gmail.com
8	Physics & Electronics	Dr. Paul Diwakar, Sri CRR College (A), Eluru, 9985050696
9	Petro Chemicals	Dr. M Trinadh, Lecturer in Chemistry, Govt. College (A), Rajahmundry, Ph. No: 8639551783
10	Bio-Chemistry	Dr. M Suvarchala, Lecturer in home science, ASD women's Degree College, KKD, Ph. No: 9346512694, suvarchakamallela@gmail.com
11	Food Science	
12	Botany	Dr. J. Sujatha, Leturer in Botany, GDC Rjy, Ph.No: 9441050910, drjsuneetha@gcrjy.ac.in
13	Microbiology	Dr. D Aruna, Lecturer in Micro-biology, ASD Women's College, Kakinada, Ph. No: 9182525872
14	Zoology	Dr. B. Tejo Murthy, Lecturer in Zoology, GDC Yeleswaram, Ph. No: 9703799970, drmtm2011@gmail.com
15	Bio Technology	Dr. B. Nageswari, Lecturer in Biotechnology, GDC Rjy, Ph. No: 986621955

16	Commercial Aquaculture	Dr. P Ramamohana Rao, Aquaculture Consultant, KKD, Ph. No: 9885144557, asreenivasulu@gmail.com
17	Computer Science & Computer Applications	Mr. N. Naga Subrahmanyesweri, Lecturer in Computer Science, ASD Women's College, KKD, Ph. No: 9948438376, yesweri.velugu@asddgcw.ac.in
18	Commerce	Dr. K. Ratna Manikyam, Govt. College (A), RJY, Ph. No: 8919230362, drkrm@gcrjy.ac.in
19	Economics	Dr. D. V. Nageshwara Rao, Lecturer, GDC, RJY, Ph. No: 9490919676
20	History	Dr. B. Anjani Kumari, Lecturer in charge, GDC (W), Ph. No: 891989337
21	Philosophy	Dr. V. Venkatarao, Lecturer in Philosophy, MR College, Vijayanagaram, Ph. No: 9440096609
22	Political Science	Dr. Seetha Mahalaxmi, Lecturer in Political Science, GDC, RJY Ph. No: 9491011844
23	Journalism & Mass Communication	Prof. DVR Murthy, Dept. of Journalism & Mass Communication, Andhra University, Vishakapatnam, Ph. No: 9985051793, 9440974092
24	Horticulture	Dr. J. Sujatha, Lecturer in Botany, GDC, Rjy, Ph. No: 9441050910, drjsuneetha@gcrjy.ac.in
25	Pharmaceutical Chemistry	Dr. K. Deepthi, Asst. Professor, Dept. of Chemistry, AKNU, Rjy, Ph. No: 9985469607, deepthikorabandi@gmail.com

(BY ORDERS)

22/10/21

Dean
ACADEMIC AFFAIRS

To
The Principal, PR Govt. College (A), Kkd
PA to R
PS to VC,
OOF

Proceedings of the Principal, PITHAPUR RAJAH'S GOVERENMENT COLLEGE(A): Kkainada

Present : Dr.B.V.Tirupanyam,Ph.D

Rc.No.12A/A.C/BOS/2022-23, Dated: 24 Sept 2022

Sub: P.R.Government College (A), Kakinada-Board of Studies(BOS)-nomination of Members-orders Isued.

Ref: UGC Guidelines for Autonomous Colleges – 2018

ORDER:

The Principal, P.R.Govt.College(A), Kakinada is pleased to constitute Board of Studies in MATHEMATICS for framing the syllabi in Mathematics subject for all semesters duly following the norms of the UGC Autonomous guidelines.

S.No	Name with Designation and Address	Designation
1	Smt. M.Surekha I/C of Mathematics P. R. Govt. College (A), Kakinada	Chair Person
2	Dr. V.Ananatha Lakshmi Principal, A.S.D.Govt degree college for women (A) , . Kakinada	University Nominee
3	i) Dr. P. Subhashini, Principal Government Degree College, Pithapuram . ii) Sri. K. Chittibabu, Lecturer in Mathematics, Government Degree College, Ramachandrapuram.	Subject expert
4	Sri. P. S. R. Subrahmanyam, Rtd. HOD of Mathematics, Ideal College of Arts & Science (A), Kakinada	Alumni Member
5	Dr.B.V.Tirupanyam	Principal
6	Sri. G .Syam Prasad Reddy	Faculty of the Department
7	Sri. G. Prasada Rao	Faculty of the Department
8	Smt. K.S.I.Priyadarshini	Faculty of the Department
9	Smt. L.S.B.R.Bhanu	Faculty of the Department
10	Smt. K. Samrajyam	Faculty of the Department
11	Smt. V. Haripriya	Faculty of the Department
12	Smt. A. Geetha Sowjanya	Faculty of the Department

13	N.S.S.Nagadevi	Faculty of the Department
14	U.S.K.Mahalakshmi	Student Member II B.Sc (M.P.C)-EM-I
15	N.Venkatesh	Student Member II B.Sc- M.S.Cs
16	V.Dora babu	Student Member III B.Sc – M.C.Cs
17	A.Rani	Student Member III B.Sc – M.P.Cs

The above members are requested attend the BOS meetings and share their valuable views , suggestions on the following functionaries:

a) Prepare syllabi for the subject keeping in view the objectives of the college, interest of the stake holders and National requirement for consideration and approval of the Academic Council.

b) Suggest methodologies for innovative teaching and evaluation techniques .

c) Suggest panel of names to the Academic council for appointment of examiners.

d) Coordinate research, teaching, extension and other activities in the department of the college.

The term of the members will be Three years from the date of the nomination. The Chairman of the BOS(HoD / Lecturer In-Charge of the department) is directed to coordinate with the Principal of the college and conduct BoS meeting as and when necessary, but at least twice a year.

B.V.Tirupanyam 17-Oct-2022

PRINCIPAL
P.R.GOVERNMENT COLLEGE(A)
KAKINADA

PRINCIPAL
P.R.Govt. College (A)
KAKINADA

Copy to:
1. The above individuals
2. File

P. R. GOVERNMENT COLLEGE (A), KAKINADA

Department of Mathematics

The Board of Studies meeting for **Mathematics** subject during the academic year 2022-2023 is conducted at the Dept. of Mathematics on 02.11.2022 at 10:00 AM with Smt.M.Surekha , Lecturer In-charge in Mathematics the chair along with the following members.

Name with Designation and Address		Signature
Chair Person Smt. M.Surekha I/C of Mathematics P. R. Govt. College (A), Kakinada	Chair Person	M.Surekha 2/11/22
University Nominee Dr. V.Ananatha Lakshmi Principal A.S.D.Govt degree college for women (A) , Kakinada	University Nominee	V.Anantha Lakshmi 2/11/22
Members Nominated by Executive Committee of the College i) Dr. P. Subhashini, Principal Government Degree College, Pithapuram . ii) Sri. K. Chittibabu, Lecturer in Mathematics, Government Degree College, Ramachandrapuram.	Subject expert	[signature] K.Chittibabu 2/11/22
From Alumni Sri. P. S. R. Subrahmanyam, Rtd. HOD of Mathematics, Ideal College of Arts & Science (A), Kakinada	Alumni Member	[signature] 2/11/22
Dr.B.V.Tirupanyam	Principal	
Sri. G .Syam Prasad Reddy	Faculty of the Department	[signature]
Sri. G. Prasada Rao	Faculty of the Department	[signature]
Smt. K.S.I.Priyadarshini	Faculty of the Department	K.S.I. P[illegible]
Smt. L.S.B.R.Bhanu	Faculty of the Department	LSBRBhanu
Smt. K. Samrajyam	Faculty of the Department	K. Sa[illegible]
Smt. V. Haripriya	Faculty of the Department	V. Haripriya
Smt. A. Geetha Sowjanya	Faculty of the Department	
N.S.S.Nagadevi	Faculty of the Department	[signature]

Student Members		
U.S.K.Mahalakshmi	Student Member II B.Sc (M.P.C)-EM-I	U.S.K.Mahalakshmi 9492728167
N.Venkatesh	Student Member II B.Sc –M.S.Cs	N.Venkatesh
V.Dora Babu	Student Member III B.Sc –M.C.Cs	V.Dorababu. 6305025739
A.Rani	Student Member III B.Sc –M.P.Cs	M.Rani

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

Meeting of the Board of studies is held at 11AM on 02-11-2022 in the Department of Mathematics, P.R.Govt. College (A), Kakinada with the following agenda.

Agenda

1. a)To approve the curriculum, blue print and model paper for 1st year B.Sc Course under CBCS based as per the directions of the APSCHE for the admitted batch 2022 -23(I & II Semesters).

b)To approve the curriculum, blue print and model paper of practical examinations for 1st year B.Sc Course under CBCS based as per the directions of the AKNU for the admitted batch 2022 -23.(I & II Semesters).

2. To approve the curriculum, blue print and model paper for 2nd year B.Sc Course under CBCS based as per the directions of the APSCHE for the admitted batch 2021 -22 (III & IV Semesters)

3. To approve the curriculum, blue print and model paper for 3rd year B.Sc Course under CBCS based as per the directions of the APSCHE for the admitted batch 2020 -21(V & VI Semesters)

4. To approve the Two Certificate Courses one for Non-Mathematics(Basic Mathematics)and another for Mathematics (Commutative Mathematics) students were introduced in this academic year

5. To approve the incorporation of additional inputs to various courses (where ever it is felt necessary) for enhancing students understanding over the concerned course and this shall not be considered for evaluation purpose.

6. To approve the Examination procedure for the courses for I, II, III years of B.Sc (2022 – 23, 2021-22& 2020-21 admitted batches).

a) Each theory subject is evaluated for 100 Marks (I, II&III Years) out of which 50 Marks through semester end examination for I & II year, 60 marks through semester end examination for III year and internal assessment would be for 50 Marks for I & II year and 40 marks for III year.

b) The minimum pass mark for both internal and external examinations is 18 marks (36%), but as a whole student is subjected to get 40% marks (40 out of total 100 marks) to pass the subject. (I, II&III Years)

c) Internal assessment for 50 Marks is as follows: (I, II&III Years)

Paper I, II, III, IV, V :(First and Second Year)

Weight age for Internal Assessment is 50 marks.

For Mid Semester Examinations - 25 marks

For Continuous Assessment - 25 marks

Two Mid Semester Examinations will be conducted for 50 marks (1 hours15 min) in the following.

Question Paper pattern:

Short answer Questions (5mark) : 03 out of 05 : 3 x 5 = 15 marks

Essay answer question (10 marks) : 01 out of 02 : 1 x 10 =10 marks

= 25 marks

The average of two mid examination marks are to be taken for 25 marks.

For continuous assessment – 25 marks distributed in the following way:

Student Project / Assignment - 10 marks (**Assignment)**

Seminar - 10 marks

Viva voce exam - 05 marks

Paper VI-A ; VII-A : (Final Year)

Weight age for Internal Assessment is 40 marks.

For Mid Semester Examinations - 20 marks

For Continuous Assessment - 20 marks

Two Mid Semester Examinations will be conducted for 40 marks (1 hours15 min) in the following.

Question Paper pattern:

Short answer Questions (5mark) : 04 out of 06 : 4 x 5 = 20 marks

Essay answer question (10 marks) : 02 out of 04 : 2 x 10 =20 marks

= 40 marks

The average of two mid examination marks are to be taken for 20 marks.

For continuous assessment – 20 marks distributed in the following way:

Student Project / Assignment - 10 marks (**Assignment)**

Seminar - 05 marks

Viva voce exam - 05 marks

d) Internal assessment for 50 Marks is as follows: (For Certificate Courses)

vii) Study Project : 20 Marks

viii) Student Seminar : 10 Marks

ix) Viva-voce : 10 Marks

x) Assignment : 10 Marks

7. Scheme of Valuation for Practical's

- Record - 10 Marks
- Viva voce - 10 Marks
- Test - 30 Marks
- Total - 50 marks

Answer any 5questions. At least 2 questions from each section. Each question carries 6 marks.

8. To award two extra credit to students who have registered and completed SWAYAM course successfully.

9. To award 4 credits for each first and second phases of Apprenticeship between 1st and 2nd year and 2nd and 3rd year (two summer vacations).

10. To implement pedagogical strategies to enrich teaching and learning process.

11. To approve the proposed departmental activities for 2022-23.

12. To approve the list of examiners and paper setters for the academic year 2022-23.

13. Any other item with the permission of the chair.

M. Sureka

CHAIRMAN
BOARD OF STUDIES

Resolutions taken :

The following resolutions are approved by university nominee and all the members of BOS

After reviewing the existing titles and contents of class I,II,III,IV and V framed by APSHE, the board come out with the following resolutions.

Resolution – I

It is resolved to approve the following changes of course I,II,III,IV and V of Mathematics as it is given by APSCHE.

COURSE-I

1. Paper-I model can be changed

   Short answer questions : 5 x 4 = 20M

   Essay answer questions : 5 x 6 = 30M

2. Practical exams will be included for the students joining the academic year 2022-23.
3. Change of variables topic is deleted in Unit-I
4. Equations that do not contain x or y and Equations homogeneous in x and y; topics are deleted in Unit-II
5. Linear differential equations with non-constant coefficients is deleted in Unit-V.
6. Legendre's linear equations is added in Unit – V.

COURSE-II

1. Paper-II model can be changed

   Short answer questions : 5 x 4 = 20M

   Essay answer questions : 5 x 6 = 30M

2. Simplified form of the equations of two spheres topic is deleted in Unit-IV.

COURSE-III

1. Homomorphism topic is shifted from Unit-IV to Unit-III.
2. Analytical Skills Paper model can be changed.

Multiple choice questions	: 30 x 1 = 30M
Short answer questions	: 4 x 5 = 20 M
Total	: 50 M

COURSE – IV

1. Bolzano –Weierstras theorem topic is deleted in Unit-I
2. Absolute convergence and conditional convergence topics are deleted in Unit-II
3. Uniform continuity topic is deleted in Unit-III

COURSE – V

1. Matrices , elementary properties , Inverse matrix, Rank of a matrix are deleted in Unit-IV.

COURSE – VI

1. It is resolved to approved the curriculum, blue print and model paper for 3rd year B.Sc Course under CBCS based as per the directions of the APSCHE for the admitted batch 2020 -21. (V&VI Semesters)

Resolution – II

1. It is resolved to approved the incorporation of additional inputs to various courses (where ever it is felt necessary) for enhancing students understanding over the concerned course and this shall not be considered for evaluation purpose.

2. Resolved to adopt Community Service Project for all the students at the end of Sem –II.
3. Resolved to send all the final year Mathematics students for on job training apprenticeship in connection with industries for off-site Project in the end of Sem V/VI with the industries in accordance with their interest of study.
4. It is resolved to approve the proposed departmental activities for 2022-23.
5. It is resolved to approve the list of examiners and paper setters for the academic year 2022-23.
6. **S**treamlining of regularity in attendance. Resolved to make the eligibility to appear for 1st mid is 75% of attendance for the2nd mid it would be 75% , for 75% of attendance for semester examination and 90% for practical examinations .Also it is resolved that the student should attend at least one internal exam to appear for the Semester end examination.
7. To approve the Analytical Skills paper (Foundation Course) should be taught to all the groups of the second year , following the directions of Adikavi Nannayya University.
8. Resolved to give extra credits for MOOCS courses, N.S.S., N.C.C., winners of zonal level sports and games competitions, participation in state level/ National level competitions, blood donations camps, environmental programs like extending services in facing the natural calamities etc.

9. Resolved to Engaging of 7th hour of time table.
10. Resolved to conduct International / National , Webinar / Seminar like Data Science , Artificial Intelligence, etc.,
11. Resolved to introduce new courses of study whenever necessary.
12. Resolved to follow the admission criteria for the programmes offered by the department.
13. Resolved to conduct extension lectures by the eminent persons.

P.R. GOVT. COLLEGE (A), KAKINADA							
BOS CHANGES FROM DEPARTMENT OF MATHEMATICS - ACADEMIC YEAR 2022-23							
S.No.	Semester, Program	Paper Number & Paper Title	Titles of Topics deleted	Topics to be added during BOS meeting October 2022	Percentage of changes made in syllabus	Justification per each topic deleted	Justification per each topic added
1	I & B.Sc all Groups	I & Differential equations	Change of variables - Unit - I Equations that do not contain x or y and Equations homogeneous in x and y - Unit - II Linear differential equations with non-constant coefficients - Unit - V	Legendre's linear equations - Unit – V.	20%	Not much application oriented	It is appropriate to include this topic in view of further studies and computative exams.
2	II & B.Sc all Groups	II & Solid Geometry	Simplified form of the equations of two spheres - Unit-IV		5%	Not much application oriented	
3	III & B.Sc all Groups	III & Abstract Algebra		Homomorphism & Isomorphism-Unit-III	20%		These topics added for the continuation of higher studies
4	IV & B.Sc all Groups	IV & Real Analysis	Bolzano –Weierstras theorem Uniform continuity - Unit-III		20%	In view of average and slow learners reduced the content	
5	IV & B.Sc all Groups	V & Linear Algebra	Matrices , elementary properties , Inverse matrix, Rank of a matrix		10%	In view of syllabus coverage more over these topics were covered in the lower classes.	
6	V & B.Sc all Groups	VII-A & Special Functions	Laguerre Polynomials	Power series and Power series solutions of ordinary differential equations - Unit-II	20%		These topics added for the continuation of higher studies.
7	V & All Degree Groups	Certificate Course-I		Cerificate Course on Basic Mathematics (for non-maths students)			Helps an individual to showcase his competency, commitment

	V & B.Sc all Groups	Certificate Course-II	Certificate course on Competative Mathematics			for the profession, build expertise in his professional subject area, and helps with job advanement.
Practical exams will be included for the students joining the academic year 2022-23						
First year paper-I & II model can be changed.						
Second year , III semester Skill Development paper-Analytical Skills paper model can be changed.						

M. Surekha

Signature of the I/C of department

P. R. GOVERNMENT COLLEGE (A), KAKINADA

ACTION PLAN FOR THE ACADEMIC YEAR 2022-2023

Department of Mathematics

S.No	Month	Week	Item as approved in BOS and to be incorporated in AC meeting agenda as Institution Plan	Outcome of the activity
1	June	IV	Community outreach Programmes	To developed and enhanced the subject's academic skills , leadership qualities and responsibilities toward the rural community.
2	July	II	Mathematics Guest lecture.	To Update the students knowledge.
3	August	III	Independence day	To understand the importance of commemoration of Independence day and how they can make changes in the world.
4	September	I II	Teacher's Day Quiz Computation	To knowledge the challenges, hardship and special roles that teachers play in our lives. The competitive spirit will be improved.
5	November	II	Field Trip	It's provides real-world experience, increases the quality of education and improves the social relations.
6	December	II III	Town level Quiz and elocution computations. Celebration of Mathematics Day on 22nd Dec-2022.	The competitive spirit will be improved among the students. The students will be motivated to pursue higher education in Mathematics.
7	January	III	Mathematics Guest lecture.	Students are able to understand the role of Mathematics in real world.
8	February	IV	Science Day Celebrations.	Students will get more interest to do projects and there is a scope to know the applicability of all subjects.
9	March	II	π Day celebrations Carrier Guidance	To import the knowledge on the significance of π

P. R. GOVERNMENT COLLEGE (A), KAKINADA

Department of Mathematics

Board of Studies Meeting 2022 -23

LIST OF EXAMINERS & PAPER SETTERS IN MATHEMATICS

S.No.	Name of the Lecturer	Address
1	Smt. Gayatri	Lecturer in Mathematics, Government College (A), Rajamahendravaram.
2	Dr. D. Sai Baba	Lecturer in Mathematics, Sri.ASNM Govt college (A),Palkolu
3	Dr. Ch. Srinivas	Lecturer in Mathematics, Government College (A), Rajamahendravaram
4	Sri. K. Chitti Babu	Lecturer in Mathematics, Government Degree College, Ramachandrapuram. 9493654033
5	Dr. V.S. Patnayak	Lecturer in Mathematics, M.R College, Vizaianagaram
6	Sri. K. Kameswara Rao	Lecturer in Mathematics, Government Degree College,Pithapuram
7	Ms. Y. Padmaja	Lecturer in Mathematics, Government Degree College, Ramachandrapuram. 9951773314
8	Sri. T. Srinivas Reddy	Lecturer in Mathematics, Government Degree College, Ramachandrapuram. 7981598769
9	Sri N. Kiran Kumar	Lecturer in Mathematics, Government Degree College, Mandapeta 9866522999
10	Dr. SK. Sajana	Lecturer in Mathematics, S.R.R. Government Degree College, Vijayawada. 7893918849
11	M. Madhavi	Lecturer in Mathematics, Government Degree College, Tuni. 9247380632

P. R. GOVERNMENT COLLEGE (A), KAKINADA

Department of Mathematics

Certificate of Submission

These following documents are submitted to the Academic Coordinator and Controller of Examinations:

1. Hard copy of the approved curriculum which includes minutes of Board of studies, Approved syllabus, blue print for the question papers and model question papers for all semesters, and list of approved examiners .
2. Soft Copy containing the approved curriculum which includes minutes of Board of Studies, approved syllabus, blue print for the question papers and model question papers for all semesters and list of approved examiners.

Chairman
M. Surekha
(M.Surekha)

Academic Coordinator

Controller of Examinations 2|XI|2022

Blue Print of C.B.C.S. Model Curriculum in B.Sc. Mathematics

Yr.	Course & Theory / Lab	Paper	Title	Workload Hrs / Week	Credits	Max. Marks			Practical
						Intrnl	Extrnl	Total	
I	Sem I	I	Differential Equations	6 Hrs (4T+2P)	4+1	50	50	100	50
	Sem II	II	Three Dimensional Analytical Solid Geometry	6 Hrs (4T+2P)	4+1	50	50	100	50
II	Sem III	III	Abstract Algebra	6 Hrs	5	50	50	100	-
		Life Skill Course:	Analytical Skills	2 Hrs	2	-	50	50	-
	Sem IV	IV	Real Analysis	6 Hrs	5	50	50	100	-
		V	Linear Algebra	6 Hrs	5	50	50	100	-
III	Sem V	VI A	**(To choose One pair from the Three alternate pairs)** Numerical Methods	6 Hrs	5	40	60	100	-
		VII A	Mathematical Special Functions	6 Hrs	5	40	60	100	-
			OR						
		VI B	Multiple integrals and Applications of Vector Calculus	6 Hrs	5	40	60	100	-
		VII B	Integral transforms with Applications	6 Hrs	5	40	60	100	-
			OR						
		VI C	Partial Differential Equations and Fourier Series	6 Hrs	5	40	60	100	-
		VII C	Number theory	6 Hrs	5	40	60	100	-

Total number of hours for each paper in the academic year 2022-2023:

Paper I & II : 192 Hrs (96+96)

Paper III : 72 Hrs

Paper IV & V : 144 Hrs (72+72)

Paper VIA : 66 Hrs

Paper VII A : 66 Hrs

Analytical Skills (F.C) : 60 Hrs

Note 1: For Semester–V, for the domain subject **MATHEMATICS**, any one of the three pairs of Skill Enhancement Courses shall be chosen as courses 6 and 7, i.e., 6A & 7A or 6B & 7B or 6C & 7C. The pair shall not be broken (ABC allotment is random, not on any priority basis).

Note 2: One of the main objectives of Skill Enhancement Courses (SEC) is to inculcate field skills related to the domain subject in students. The syllabus of SEC will be partially skill oriented. Hence, teachers shall also impart practical training to students on the field skills embedded in the syllabus citing related real field situations.

Note 3: To insert assessment methodology for Internship/ on the Job Training/Apprenticeship under the revised CBCS as per APSCHE Guidelines.

- **First internship (After 1st Year Examinations):** Community Service Project. To inculcate social responsibility and compassionate commitment among the students, the summervacationintheintervening1stand2ndyearsofstudy shall be for Community Service Project (the detailed guidelines are enclosed).
- **CreditForCourse:04 for 100 marks**
- **Second Internship (After 2nd Year Examinations):** Apprenticeship / Internship / on the job training / In-house Project / Off-site Project. To make the students employable, this shallbeundertakenbythestudentsintheinterveningsummervacationbetweenthe2nd and 3rd years (the detailed guidelines are enclosed).
- **CreditForCourse:04for 100 marks**

- **Third internship/Project work(6th Semester Period):**

  During the entire 6th Semester, the student shall undergo Apprenticeship / Internship / On the Job Training. This is to ensure that the students develop hands on technical skills which will be of great help in facing the world of work (the detailed guidelines are enclosed).

- **CreditFor Course:12for 200 marks**

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

Objectives of the Department

- To impart knowledge on various Mathematical concepts like Differential Equations, Solid Geometry, Group Theory, Real Analysis, Ring Theory and Vector Calculus, Linear Algebra, Numerical Analysis and Special Functions.
- To equip our students with good quality to appear for competitive examinations.
- To make the students to understand the needs of Mathematics in Science and Technology.
- To inculcate research atmosphere among students by assigning projects.

The Department of Mathematics is offering B.Sc. courses involving mathematics (10 courses), B.Sc. Professional (B. Voc.) for undergraduate courses.

PROGRAMME OUTCOMES

For every degree program expectations are listed out by the institution under the Program Outcomes. For all Degree Streams the following are set as Programme Outcomes.

Knowledge and Understanding:-

On completion of this programme the successful student will have knowledge and understanding of

- Core areas of pure mathematics including geometry, algebra, mathematical analysis and discrete mathematics.
- Core areas of applied mathematics including statistics, operational research and differential equations.
- Several specialized areas of advanced mathematics and its applications.
- The correct use of mathematical language to express both theoretical concepts and logical arguments.
- The use of computers both as an aid and as a tool to study problems in mathematics.

Cognitive (thinking) Skills:-

On completion of this programme the successful student will be able to

- To formulate problems in appropriate theoretical frameworks to facilitate their solution.
- To develop strategies to solve mathematical problems in a range of relevant areas.
- To construct logical arguments solving abstract or applied mathematical problems.
- To criticise mathematical arguments developed by themselves and others.

Practical Skills:-

On completion of the programme the successful student will be able

- To solve practical problems in a range of areas of mathematics.
- To determine the appropriateness of different methods of solving mathematical problems.
- To communicate mathematics effectively to a wide range of audiences.
- To use computer packages where appropriate to develop a deeper understanding of mathematical problems.

Graduate Skills:-

On completion of this programme the successful student will be able

- To work effectively and constructively as part of a team.
- To motivate and communicate complex ideas accurately using a range of formats.
- To identify and benefit from opportunities for personal and career development.
- To work confidently and accurately with formulae and numerical information.

Programme Specific Outcomes of Mathematics Stream Courses

PROGRAMME	Program Specific Outcomes
MPC	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Physics and Chemistry
	PSO 2: To identify and understand the theoretical concepts of physical and chemical properties of materials and the role of mathematics in dealing with them in a quantitative way.
	PSO 3: To learn problem solving techniques related to Mathematics, Physics and Chemistry
	PSO 4: To gain insights procedures of safe handling of Chemicals and Equipments.
	PSO 5: To carry out hands on experiments and to analyze results.
MPE	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Physics and Electronics.
	PSO 2: To identify and understand the mechanism behind various electronic and physical systems and quantify them with mathematical tools.
	PSO 3: To learn problem solving techniques related to Mathematics, Physics and Electronics.
	PSO 4: To gain skills needed to handle the instruments and design circuits with analysis of results.
MPCs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Physics and Computer Science.
	PSO 2: To identify and understand the concepts of Mathematics, Physics and Computer Science and then relate them in numerical programming of physical system models.
	PSO 3: To learn problem solving techniques related to Mathematics, Physics and Computer Science.
	PSO 4: To gain skills required to develop programming techniques and implementation of numerical algorithms by using various programming languages.
MCPC	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Chemistry and Petro Chemicals.
	PSO 2: To identify and understand the theoretical concepts of Mathematics and Chemistry and utilize them in Petro Chemicals.
	PSO 3: To examine the Mathematical Modelling and Chemical procedures in the field of Petro chemicals.
	PSO 4: To get the employability skills in chemical industries as well as petro chemical industries.
MECs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Electronics and Computer Science.
	PSO 2: To identify and understand the concepts of Mathematics and Computer Science and utilize them in numerical programming of electronical system models.
	PSO 3: To gain insights to design circuits and provide mathematical modelling.
	PSO 4: To design circuits and understand the variations by simulation.
MCCs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Chemistry and Computer Science.

	PSO 2: To analyse the concepts of Mathematics, Chemistry and Computer Science and identify the relation among them like deriving the equations in chemistry, mathematical modelling of chemistry.
	PSO 3: To carry out problem solving and to demonstrate the real life applications of Mathematics and Chemistry in Computer Science.
	PSO 4: To gain insights procedures of safe handling of Chemicals and Equipments.
MSCs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Statistics and Computer Science.
	PSO 2: To identify and analyse the concepts of mathematics, statistics and computers science and then to find their applications in different areas like physical sciences, life sciences, various industries, etc.
	PSO 3: To solve various real life problems by developing mathematical model and applying various statistical tools with the help of computer programming knowledge.
	PSO 4: To develop thinking about research to solve critical problems.
MSAs	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Statistics and Actuarial Science.
	PSO 2: To identify and analyse the concepts of mathematics, statistics and Actuarial science and then to find their applications in different areas like physical sciences, life sciences, various industries, Insurance, etc.
	PSO 3: To solve various real life problems by developing mathematical model and applying various statistical tools with the help of suitable economic, finance and risk policies.
	PSO 4: To acquire the skill of collection of data, analyzing it and to give conclusions
	PSO 5: To develop thinking about research to solve critical problems.
MCAc	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Chemistry and Analytical Chemistry.
	PSO 2: To identify and understand the concepts of Mathematics, Chemistry and Analytical chemistry and then to understand the relation among them like mathematical modelling of chemistry and derivation of chemical equations.
	PSO 3: To gain insights procedures of safe handling of Chemicals and Equipments
	PSO 4: To get the employability skills especially chemical industries.
ME.IOT	PSO 1: To understand nature, scope, basic concepts and terminology of Mathematics, Electronics and Internet of thinking.
	PSO 2: To identify and understand the concepts of Mathematics and Electronics and utilize them in Internet programming system models.
	PSO 3: To gain insights to design networking.

Courses (Papers) offered under B.Sc. Mathematics Stream

S. No.	Sem. No.	Domain Specific course/Clusters	Title
1	I	General Core	**Differential Equations**
2	II	General Core	**Three Dimensional Analytical Solid Geometry**
3	III	General Core	**Abstract Algebra**
4	IV	General Core	**Real Analysis**
5		General Core	**Linear Algebra**
6	V	Skill Enhancement Courses(Elective)	**(To choose One pair from the Three alternate pairs)** **Numerical Methods**
			Mathematical Special Functions
			OR
			Multiple integrals and Applications of Vector Calculus
			Integral transforms with Applications
			OR
			Partial Differential Equations and Fourier Series
			Number theory

MATHEMATICS COURSE OUTCOMES

Year	Semester	Title of the Paper	Course Outcomes
I	I	Differential Equations	CO 1. Solve linear differential equations CO 2. Convert non exact homogeneous equations to exact differential equations by using integrating factors. CO 3. Know the methods of finding solutions of differential equations of the first order but not of the First degree. CO 4. Solve higher-order linear differential equations, both homogeneous and non homogeneous, with constant coefficients. CO 5. Understand the concept and apply appropriate methods for solving differential equations.
	II	Three Dimensional Analytical Solid Geometry	CO 1. Get the knowledge of planes. CO 2. Basic idea of lines, sphere and cones. CO 3. Understand the properties of planes, lines, spheres and cones. CO 4. Express the problems geometrically and then to get the solution.
II	III	Abstract Algebra	CO 1. To analyse the abstract algebraic concept Group theory. CO 2. To understand the concepts in group theory like groups, subgroups, normal subgroups, permutation groups and cyclic groups with examples. CO 3. To understand the theorems on these concepts and also to solve problems on it. CO 4. To analyse and understand the applications of group theory in various fields. CO 5. To understand the ring theoretic concepts with the help of knowledge in group theory and to prove the theorems on it. CO 6. To understand the applications of ring theory in various fields.
	IV	Real Analysis	CO 1. To get clear idea about the real numbers and real valued functions. CO 2. To obtain the skills of analyzing the concepts and applying appropriate methods for testing converges of a sequence or series. CO 3. To analyse the concepts of continuity, differentiability and Riemann integrability

			of a function and also to gain the skills about how to test these conditions of functions defined on the subsets of the real line. CO4. To know the Geometrical interpretation of mean value theorems.
		Linear Algebra	CO 1. To understand the different concepts of linear algebra. CO 2. To analyse the concepts of vector space, subspace and homomorphism between them. CO 3. To understand how to solve the system of linear equations and this concept used in balancing of chemical equations. CO 4. To analyse the concepts of eigen values, inner product spaces and orthogonality and also gain the problem solving ability on them.
III	V	Numerical Methods	CO 1. Understand the subject of various numerical methods that are used to obtain approximate solutions . CO 2. Understand various finite difference concepts and interpolation methods. CO 3. Work out numerical differentiation and integration whenever and wherever routine methods are not applicable. CO 4. Find numerical solutions of ordinary differential equations by using various numerical methods. CO 5. Analyze and evaluate the accuracy of numerical methods.
		Mathematical Special Functions	CO 1. Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations. CO 2. Find power series solutions of ordinary differential equations. CO 3. solve Hermite equation and write the Hermite Polynomial of order (degree) n, also find the generating function for Hermite Polynomials, study the orthogonal properties of Hermite Polynomials and recurrence relations. CO 4. Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials. CO 5. Solve Bessel equation and write the

			Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel unction.
		OR	
		Multiple integrals and Applications of Vector Calculus	CO 1. Learn multiple integrals as a natural extension of definite integral to a function of two variables in the case of double integral / three variables in the case of triple integral. CO 2. Learn applications in terms of finding surface area by double integral and volume by triple integral. CO 3. Determine the gradient, divergence and curl of a vector and vector identities. CO 4. Evaluate line, surface and volume integrals. CO 5. understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral (Green's theorem), relation between line and surface integral (Stokes theorem).
		Integral transforms with Applications	CO 1. Evaluate Laplace transforms of certain functions, find Laplace transforms of derivatives and of integrals. CO 2. Determine properties of Laplace transform which may be solved by application of special functions namely Dirac delta function, error function, Bessel function and periodic function. CO 3. Understand properties of inverse Laplace transforms, find inverse Laplace transforms of derivatives and of integrals. CO 4. Solve ordinary differential equations with constant/ variable coefficients by using Laplace transform method. CO 5. Comprehend the properties of Fourier transforms and solve problems related to finite Fourier transforms.
		OR	
		Partial Differential Equations and Fourier Series	CO 1.Classify partial differential equations, formation of partial differential equations and solve Cauchy's problem for first order equations. CO 2. Solve Lagrange's equations by various methods, find integral Surface passing through a given curve and Surfaces orthogonal to a given system of Surfaces.

		CO 3. Find solutions of nonlinear partial differential equations of order one by using Char pit’s method. Co 4. Find solutions of nonlinear partial differential equations of order one by using Jacobi's method. CO 5. Understand Fourier series expansion of a function f(x) and Parseval’s theorem.
	Number theory	CO 1. Find quotients and remainders from integer division, study divisibility properties of integers and the distribution of primes. CO 2. Understand Dirichlet multiplication which helps to clarify interrelationship between various arithmetical functions. CO 3. Comprehend the behaviour of some arithmetical functions for large n. CO 4. Understand the concepts of congruencies, residue classes and complete residues systems. CO 5. Comprehend the concept of quadratic residues mod p and quadratic non residues mod p.

	P.R.Government College (Autonomous): KAKINADA	Program&Semester			
Course Code MAT-101/1201	TITLEOFTHECOURSE **Differential Equations**	IB.Sc. (ISem)			
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge	4	0	-	4

Course Objectives:

To provide students with an introduction to the theory of ordinary differential equations through applications, methods of solution, and numerical approximations.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Solve linear differential equations
CO2	Convert non - exact homogeneous equations to exact differential equations by using integrating factors.
CO3	Know the methods of finding solutions of differential equations of the first order but not of the first degree.
CO4	Understand the concept and apply appropriate methods for solving differential equations.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

COURSE SYLLABUS:

UNIT – I: Differential Equations of first order and first degree (12 Hours)

Linear Differential Equations; Differential equations reducible to linear form; Exact differential equations; Integrating factors.

UNIT – II: Orthogonal Trajectory and Differential Equations of first order but not of the first degree (12 Hours)

Orthogonal Trajectories, Equations solvable for p; Equations solvable for y; Equations solvable for x; Equations of the first degree in x and y – Clairaut's Equation.

UNIT – III: Higher order linear differential equations-I (12 Hours)

Solution of homogeneous linear differential equations of order n with constant coefficients; Solution of the non-homogeneous linear differential equations with constant coefficients by means of polynomial operators. General Solution of f(D)y=0. General Solution of f(D)y=Q when Q is a function of x, $\frac{1}{f(D)}$ is expressed as partial fractions.

P.I. of f(D)y = Q when $Q = be^{ax}$

P.I. of f(D)y = Q when Q is $b \sin ax$ or $b \cos ax$.

UNIT – IV: Higher order linear differential equations-II (12 Hours)

Solution of the non-homogeneous linear differential equations with constant coefficients.

P.I. of f(D)y = Q when $Q = b\, x^k$

P.I. of f(D)y = Q when $Q = e^{ax}\, V$, where V is a function of x.

P.I. of f(D)y = Q when $Q = x\, V$, where V is a function of x.

UNIT –V: Higher order linear differential equations-III (12 Hours)

Method of variation of parameters; The Cauchy-Euler Equation, Legendre's linear equations.

Co-Curricular Activities: (15 Hours)

Seminar/ Quiz/ Assignments/ Applications of Differential Equations to Real life Problem /Problem Solving.

Prescribed Text Book:

Differential Equations and Their Applications by Zafar Ahsan, published by Prentice-Hall of India Pvt. Ltd, New Delhi-Second edition.

Reference Books :

1. A text book of Mathematics for B.A/B.Sc, Vol 1, by N. Krishna Murthy & others, published by S. Chand & Company, New Delhi.
2. Ordinary and Partial Differential Equations by Dr. M.D,Raisinghania, published by S. Chand & Company, New Delhi.
3. Differential Equations with applications and programs – S. Balachandra Rao & HR Anuradha- Universities Press.
4. Differential Equations -Srinivas Vangala & Madhu Rajesh, published by Spectrum University Press.

Additional Inputs :

Total Differential Equations

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN

COURSE-I, DIFFERENTIAL EQUATIONS

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Differential Equations of first order and first degree	2	2	20
II	Differential Equations of first order but not of the first degree	2	2	20
III	Higher order linear differential equations-I	2	2	20
IV	Higher order linear differential equations-II	1	2	16
V	Higher order linear differential equations-III	1	2	16
Total		8	10	92

S.A.Q. = Short answer questions (4 marks)

E.Q = Essay questions (6 marks)

Short answer questions : 5X4= 20M

Essay questions : 5X6=30M

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Total Marks = 50M

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P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2022-23 Admitted Batch)
Model Paper (w.e.f. 2022-2023)

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Time: 2Hrs **Max. Marks: 50M**

PART - I

Answer any FIVE questions. Each question carries FORE marks. 5 X 4=20M

1. Solve $\left(y - e^{sin^{-1}x}\right)\frac{dx}{dy} + \sqrt{1-x^2} = 0$
2. Solve $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$
3. Solve $y + px = p^2x^4$
4. Find the Orthogonal trajectories of family of curves $r = a(1 + cos\theta)$.
5. Solve $(D^2 - 3D + 2) = \cos hx$
6. Solve $(D^3 + 2D^2 + D)y = e^{2x}$
7. Solve$(D^2 - 4D + 4)y = x^3$
8. Solve $(x^2\mathrm{D}^2 - \mathrm{xD} + 1)y = 2\log x$

PART - II

Answer the following questions. Each question carries SIX marks. 5 X 6=30M

9. Solve $\left(xy^3 + y\right)dx + 2(x^2y^2 + x + y^4)dy = 0$

(OR)

10. Solve $\frac{dy}{dx}\left(x^2y^3 + xy\right) = 1$

11. Solve $p^2 + 2py\cot x = y^2$

(OR)

12. Find the orthogonal trajectories of the family of curves $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, where a is the parameter.

13. Solve $(D^2 + a^2)y = tanax$.

(OR)

14. Solve $(D^2 - 4D + 3)y = sin3xCos2x$.

15. Solve $(D^2 - 2D + 4)y = 8(\mathrm{x}^2 + \mathrm{e}^{2\mathrm{x}} + \sin 2\mathrm{x})$

(OR)

16. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}sin2x$

17. Solve $(D^2 - 2D)y = e^x \sin x$ by the method of variation of parameters.

(OR)

18. Solve $[\,(1 + x)^2 D^2 + (1 + x)D + 1\,]\,y = 4\cos\log(1 + x)$.

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA

I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2022-2023 Admitted Batch)

QUESTION BANK
Short Answer Questions

Unit-I

1. Solve $x\frac{dy}{dx}+2y-x^2\log x=0$

2. Obtain the equation of the curve satisfying the differential equation $(1+x^2)\frac{dy}{dx}+2xy-4x^2$ =0 and passing through the origin

3. Solve $\frac{dy}{dx}+2xy=e^{-x^2}$.

4. Solve $(x+2y^3)\frac{dy}{dx}=y$.

5. Solve $(1+y^2)dx=(tan^{-1}y-x)dy$

6. Solve $\frac{dy}{dx}-y\ tanx=\frac{sinxcos^2x}{y^2}$

7. Solve $x\frac{dy}{dx}+y\log y=xye^x$

8. Solve $\frac{dy}{dx}+\frac{y}{x}=y^2x$.

9. Define an Integrating factor and solve $xdy-ydx=xy^2dx$.

10. Solve $x\,dy+y\,dx+\frac{xdy-ydx}{x^2+y^2}=0$

11. Solve $ydx-xdy+(1+x^2)dx+x^2\sin y\,dy=0$

12. Solve $(\sin x\ \cos y+e^{2x})dx+\ (\cos x\sin y+\tan y)dy=0$

13. Solve $(1+xy)x\,dy+(1-xy)y\,dx=0$.

14. Solve $(x^2+y^2+2x)dx+2y\,dy=0$.

Unit-II

18. Find the Orthogonal trajectories of the family of coaxial circles $x^2+y^2+2fy+1=0, f$ being the parameter.

19. Find the Orthogonal trajectories of family of straight lines in a plane and passing through the origin.

20. Find the Orthogonal trajectories of family of coaxial circles $x^2+y^2+2gx+c=0$,where g is parameter.

21. Find the Orthogonal trajectories of family of curves $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

22. Find the orthogonal trajectories of the family of parabolas through the origin and foci on y – axis .
23. Solve $p^2x^2 = y^2$.
24. Solve $y^2 - xyp - x^2p^2 = 0$.
25. Solve $x = 2p^3 + (y/p)$.
26. Solve $y = xp^2 + p$
27. Solve $p^2x^2 - 2pxy + y^2 + 4p = 0$.
28. Define Clairaut's equation and solve $(y - xp)(p - 1) = p$

Unit-III

29. Solve $(D^3 - 2D^2 - 3D) x = 0$ where $D = d / dt$.
30. Solve $(D^4 - 4D^3 + 6D^2 - 4D + 1)y = 0$.
31. Solve $(D^4 + 8D^2 + 16) y = 0$.
32. Solve $(D^2 - 2D - 3) y = 5$.
33. Solve $(D^2 - 3D + 2)y = Coshx.$
34. Solve $(D^2 + 9)y = Cos\ \ 3x.$
35. Solve$(D^2 - 5D + 6)y = e^{4x}.$
36. Solve$(D^2 - D - 2)y = Sin2x.$
37. Solve$(D^2 + 4)y = Sin2x.$
38. Solve $(D^2 + 4)y = \tan 2x$
39. Solve $(D^2 + 1)y = cosec\ x.$
40. Solve $(D^2 + 9)y = cos^3\ x.$

Unit-IV

41. Solve$(D^2 - 4D + 4)y = x^3$
42. Solve $(D^4 - 2D^3 + D^2) y = x^3$
43. Solve$(D^2 - 2D + 1)y = x^2e^{3x}.$
44. Solve $(D^2 - 2D) y = e^x \text{ Sin } x$
45. Solve $(D^2 - 2D + 5) y = e^{2x} \text{ Sin } x$
46. Solve$(D^2 + 4)y = xSinx.$
47. Solve $(D^2 + 4) y = x \text{ Cos}2x$
48. Solve $(D^2 - 2D + 1) y = x e^x \text{ Sin } x$

Unit-V

49. Solve$(D^2 + 1)y = Secx$ by method of variation of parameters.
50. Solve$(D^2 - 3D + 2)y = Cos\ e^{-x}$ by the method of variation of parameters.
51. Solve $(D^2 + 1) y = x \text{ Cos } x$ by the method of variation of parameters.

52. Solve ($D^2 - 3D + 2$) y = Sin (e^{-x}) by the method of variation of parameters.

53. Solve $(x^2 D^2 + xD - 1)y = x3$.

54. Solve $(x^2 D^2 - xD + 1)y = 2\log x$.

55. Solve $(x^2 D^2 - 3xD + 5)y = x^2 \sin(\log x)$.

56. Solve $(x^2 D^2 + xD - 4)y = x^2$.

Essay Answer Questions

Unit - I

1. Solve $Cos^2 x \frac{dy}{dx} + y = \tan x$.

2. Solve $x\, cosx \frac{dy}{dx} + (\ x\, sinx + cosx\ )y = 1$.

3. Solve $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{(1 - x^2)}$.

4. Define Bernoulli's equation of first order and solve $x\frac{dy}{dx} + y = y^2 \log x$.

5. Solve $\frac{dy}{dx}\left(x^2 y^3 + xy\right) = 1$

6. Solve $\frac{dy}{dx} + x\sin 2y = x^3 cos^2 y$.

7. Solve $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$.

8. Solve $x^2 y\, dx - (x^3 + y^3)dy = 0$.

9. Solve $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$.

10. Solve $y(xy + 2x^2 y^2)dx + x(xy - x^2 y^2)dy = 0$.

11. Solve $\left(x^2 y^2 + xy + 1\right)ydx + (x^2 y^2 - xy + 1)xdy = 0$

12. Solve $(x^3 y^3 + x^2 y^2 + xy + 1)y\, dx + (x^3 y^3 - x^2 y^2 - xy + 1)x\, dy = 0$.

13. Solve $(1 + xy)x\, dy + (1 - xy)ydx = 0$.

14. Solve $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)dy = 0$.

15. Solve $(x^3 - 2y^2)dx + 2xy\, dy = 0$.

16. Solve $\left(xy^3 + y\right)dx + 2(x^2 y^2 + x + y^4)dy = 0$

17. Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

Unit - II

18. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$ where λ is the parameter .

19. Show that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal, where λ is the Parameter.

20. Show that the orthogonal trajectories of the parabola $y^2 = 4a(x + a)$ belongs to the same itself, a being parameter.
21. Find the Orthogonal trajectories of the family of curves in polar coordinates $r = \frac{2a}{(1+\cos\theta)}$ Where 'a' is the parameter?
22. Find the orthogonal trajectories of the family of curves in polar coordinates $r = a(1 + \cos\theta)$
23. Find the orthogonal trajectories of the family of curves in polar coordinates $r\sin 2\theta = \lambda$
24. Find the orthogonal trajectories of the family of curves $r^n \sin(n\theta) = a^n$ where 'a' is the parameter .
25. Solve $p^2 + 2py\cot x = y^2$
26. Solve $xy^2(p^2+2) = 2py^3 + x^3$.
27. Solve $x^2(\frac{dy}{dx})^2 + xy\frac{dy}{dx} - 6y^2 = 0$.
28. Solve $y^2\log y = xpy + p^2$.
29. Solve $2px = 2\tan y + p^3 cos^2 y$.
30. Solve $y = 2px + y^2p^3$.
31. Solve $y + px = p^2x^4$.
32. Solve $y = 2xp + x^2p^4$.
33. Solve $2xp^3 - 6yp^2 + x^4 = 0$.
34. Solve $(py + x)(px - y) = 2p$.
35. Solve $x^2(y - px) = p^2y$.

Unit-III

36. Solve $(D^2 + a^2)y = secax$
37. Solve $(D^2 + a^2)y = tanax$.
38. Solve $(D^3 - 12D + 16) y = (e^x + e^{-2x})^2$.
39. Solve $(D^2 - 4D + 3)y = sin3xCos2x$.
40. Solve $(D^2 - 3D + 2)y = Cos3x.Cos2x$.
41. Solve $(D^2 + 4)y = e^x + sin2x + Cos2x$.
42. Solve $(D^2 + 4)y = sin2x$
43. Solve $(D^2 - 4)y = e^x + \sin 2x + cos^2 x$

Unit-IV

44. Solve $(D^2 - 2D + 4) y = 8 (x^2 + e^{2x} + \sin2x)$
45. Solve $(D^2 - 4D + 4)y = x^2 + e^x + Cos2x$
46. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}sin2x$
47. Solve $(D^2 + 4)y = x^2e^{3x} + e^xCos2x$

48. Solve $(D^2 + 1)y = e^{-x} + \cos x + x^3 + e^x \cos x$

49. Solve $(D^2 + 4)y = x\,Sinx$

50. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}sin2x$

51. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x sinx$

52. Solve $(D^2 + 1)y = x^2 \sin 2x$

53. Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$

54. Solve $(D^2 + 2D + 1)y = x\,Cosx$

Unit-V

55. Solve $(D^2 + a^2)y = \tan ax$ by the method of variation of parameters

56. Solve $y'' + 4y = 4\tan 2x$ by the method of variation of parameters

57. Solve $(D^2 + a^2)y = \sec ax$ by the method of variation of parameters

58. Solve $(D^2 + 1)y = cosecx$ by the method of variation of parameters

59. Solve $(D^2 + a^2)y = \operatorname{cosec} ax$ by the method of variation of parameters

60. Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10(x + \frac{1}{x})$.

61. Solve $(x^2D^3 + 3xD^2 + D)y = x^2 \log x$.

62. Solve $(x^4D^3 + 2x^3D^2 - x^2D + x)y = 1$.

63. Solve $x^2D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$.

64. Solve $(x^2D^2 - xD + 2)y = x \log x$.

65. Solve $[(1 + 2x)^2D^2 - 6(1 + 2x)D + 16]y = 8(1 + 2x)^2$.

66. Solve $[(1 + x)^2 D^2 + (1 + x)D + 1]y = 4\cos\log(1 + x)$.

67. Solve $[(2x + 3)^2D^2 - 2(2x + 3)D - 12]y = 6x$.

****

	P.R.Government College (Autonomous) KAKINADA	**Program & Semester** IB.Sc. (ISem) w.e.f.2022-23 admitted Batch			
Course Code MAT-101P	**TITLE OF THE COURSE** **Differential Equations**				
Teaching	Hours Allocated:30(**Practical's**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge	-	-	2	1

UNIT – I: Differential Equations of first order and first degree

- Linear Differential Equations
- **Differential equations reducible to linear form**
- Exact differential equations
- Integrating factors

UNIT – II: Orthogonal Trajectory and Differential Equations of first order but not of the first degree

- Orthogonal Trajectories
- Equations solvable for *p*
- Equations solvable for *y*
- Equations solvable for *x*
- **Equations of the first degree in *x* and *y* – Clairaut's Equation**

UNIT – III: Higher order linear differential equations-I

- Solution of homogeneous linear differential equations of order n with constant coefficients
- . Solution of the non-homogeneous linear differential equations with constant coefficients by means of polynomial operators.
- General Solution of f(D)y=0
- General Solution of f(D)y=Q when Q is a function of x, $\frac{1}{f(D)}$ is expressed as partial fractions.
- P.I. of f(D)y = Q when $Q = be^{ax}$
- P.I. of f(D)y = Q when Q is $b \sin ax$ or $b \cos ax$.

UNIT – IV: Higher order linear differential equations-

- P.I. of f(D)y = Q when $Q = b\, x^k$
- P.I. of f(D)y = Q when $Q = e^{ax}\, V$, where V is a function of x.
- P.I. of f(D)y = Q when $Q = x\, V$, where V is a function of x.

UNIT –V: Higher order linear differential equations-III

- Method of variation of parameters
- The Cauchy-Euler Equation
- Legendre's linear equations.

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COURSE-I, DIFFERENTIAL EQUATIONS

Unit	TOPIC	E.Q	Marks allotted to the Unit
I	Differential Equations of first order and first degree	2	12
II	Differential Equations of first order but not of the first degree	2	12
III	Higher order linear differential equations-I	1	06
IV	Higher order linear differential equations-II	2	12
V	Higher order linear differential equations-III	1	06
Total		08	48

Semester – I end Practical Examinations

Scheme of Valuation for Practical's

Time : 2 Hours **Max. Marks : 50**

- **Record - 10 Marks**
- **Viva voce - 10 Marks**
- **Test - 30 Marks**

Answer any 5questions. At least 2 questions from each section. Each question carries 6 marks.

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2022-23 Admitted Batch)
Practical Model Paper (w.e.f. 2022-2023)

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Time: 2Hrs **Max. Marks: 50M**

Answer any 5questions. At least 2 questions from each section. **5 x 6 = 30 Marks**

SECTION - A

1. Solve $\frac{dy}{dx}\left(x^2y^3+xy\right)=1$
2. Solve $x^2y\,dx-(x^3+y^3)dy=0$.
3. Show that the family of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self-orthogonal, where λ is the Parameter.
4. Solve $2px=2\tan y+p^3\,cos^2\,y$

SECTION - B

5. Solve $(D^2-4D+3)y=sin3xCos2x$.
6. Solve $(D^2-4D+4)y=8x^2e^{2x}sin2x$
7. Solve $(D^4+2D^2+1)y=x^2\cos x$
8. Solve $[(1+x)^2D^2+(1+x)D+1]y=4\cos\log(1+x)$.

- **Record - 10 Marks**
- **Viva voce - 10 Marks**

Practical Question Bank

Unit - I

1. Solve $Cos^2x \frac{dy}{dx} + y = \tan x$.

2. Solve $x\, cosx \frac{dy}{dx} + (\ x\, sinx + cosx\ )y = 1$.

3. Solve $(1 - x^2)\frac{dy}{dx} + 2xy = x\sqrt{(1 - x^2)}$.

4. Define Bernoulli's equation of first order and solve $x\frac{dy}{dx} + y = y^2 \log x$.

5. Solve $\frac{dy}{dx}\left(x^2y^3 + xy\right) = 1$

6.Solve $\frac{dy}{dx} + x \sin 2y = x^3 cos^2 y$.

7.Solve $\left(1 + e^{x/y}\right) dx + e^{x/y}\left(1 - \frac{x}{y}\right) dy = 0$.

8.Solve $x^2y\, dx - (x^3 + y^3)dy = 0$.

9.Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

10. Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.

11. Solve $\left(x^2y^2 + xy + 1\right) ydx + (x^2y^2 - xy + 1)xdy = 0$

12. Solve $(x^3y^3 + x^2y^2 + xy + 1)y\, dx + (x^3y^3 - x^2y^2 - xy + 1)x\, dy = 0$.

13. Solve $(1 + xy)x\, dy + (1 - xy)ydx = 0$.

14.Solve $\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right) dx + \frac{1}{4}(x + xy^2)dy = 0$.

15.Solve $(x^3 - 2y^2)dx + 2xy\, dy = 0$.

16.Solve $\left(xy^3 + y\right)dx + 2(x^2y^2 + x + y^4)dy = 0$

17.Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$.

Unit - II

18.Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$ where λ is the parameter .

19.Show that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal, where λ is the Parameter.

20.Show that the orthogonal trajectories of the parabola $y^2 = 4a(x + a)$ belongs to the same itself, a being parameter.

21.Find the Orthogonal trajectories of the family of curves in polar coordinates $r = \frac{2a}{(1+\cos\theta)}$ Where 'a' is the parameter?

22.Find the orthogonal trajectories of the family of curves in polar coordinates $r = a(1 + \cos\theta)$

23.Find the orthogonal trajectories of the family of curves in polar coordinates $r \sin 2\theta = \lambda$

24.Find the orthogonal trajectories of the family of curves $r^n \sin(n\theta) = a^n$ where 'a' is the parameter .

25.Solve $p^2 + 2py \cot x = y^2$

26.Solve $xy^2(p^2+2) = 2py^3 + x^3$.

27.Solve $x^2(\frac{dy}{dx})^2 + xy\frac{dy}{dx} - 6y^2 = 0$.

28.Solve $y^2\log y = xpy + p^2$.

29.Solve $2px = 2\tan y + p^3 \cos^2 y$.

30.Solve $y = 2px + y^2p^3$.

31.Solve $y + px = p^2x^4$.

32.Solve $y = 2xp + x^2p^4$.

33.Solve $2xp^3 - 6yp^2 + x^4 = 0$.

Unit-III

34.Solve $(D^2 + a^2)y = secax$

35.Solve $(D^2 + a^2)y = tanax$.

36.Solve $(D^2 - 2D - 3) y = 5$.

37.Solve $(D^2 - 3D + 2)y = Coshx$.

38.Solve $(D^3 - 12D + 16) y = (e^x + e^{-2x})^2$.

39.Solve $(D^2 - 4D + 3)y = sin3xCos2x$.

40.Solve $(D^2 - 3D + 2)y = Cos3x.Cos2x$.

41.Solve $(D^2 + 4)y = e^x + sin2x + Cos2x$.

42.Solve $(D^2 + 4)y = sin2x$

43.Solve $(D^2 - 4)y = e^x + \sin 2x + cos^2 x$

Unit-IV

44.Solve $(D^2 - 2D + 4) y = 8 (x^2 + e^{2x} + \sin 2x)$

45.Solve $(D^2 - 4D + 4)y = x^2 + e^x + Cos2x$

46.Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}sin2x$

47.Solve $(D^2 + 4)y = x^2e^{3x} + e^xCos2x$

48.Solve $(D^2 + 1) y = e^{-x} + \text{Cos } x + x^3 + e^x \text{Cos} x$

49.Solve $(D^2 + 4)y = x\ Sinx$

50.Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}sin2x$

51.Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^xsinx$

52.Solve $(D^2 + 1)y = x^2 \sin 2x$

53.Solve $(D^4 + 2D^2 + 1) y = x^2 \text{ Cos } x$

54.Solve $(D^2 + 2D + 1)y = x\ Cosx$

Unit-V

55. Solve $(D^2 + a^2)y = \tan ax$ by the method of variation of parameters

56. Solve $(D^2 + a^2)y = \sec ax$ by the method of variation of parameters

57. Solve $(D^2 + a^2)y = \operatorname{cosec} ax$ by the method of variation of parameters

58. Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10(x + \frac{1}{x})$.

59. Solve $(x^2D^3 + 3xD^2 + D)y = x^2 \log x$.

60. Solve $(x^4D^3 + 2x^3D^2 - x^2D + x) y = 1$.

61. Solve $x^2D^2 + 3xD + 1)y = \frac{1}{(1-x)^2}$.

62. Solve $(x^2D^2 - xD + 2) y = x \log x$.

63. Solve $[(1 + 2x)^2D^2 - 6(1 + 2x) D + 16] y = 8(1 +2x)^2$.

64. Solve $[(1 + x)^2 D^2 + (1 + x)D + 1] y = 4 \cos \log (1 + x)$.

65. Solve $[(2x + 3)^2D^2 - 2(2x + 3)D - 12] y = 6x$.

	P.R.Government College (Autonomous): KAKINADA	**Program&Semester** IB.Sc. (IISem)
CourseCode MAT-201/2201	**TITLE OF THE COURSE** **Solid Geometry**	

		L	T	P	C
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	BasicMathematicsKnowledge on 2-D Geometry	4	-	-	4

Course Objectives:

The student will demonstrate knowledge of geometry and its applications in the real world.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Get the knowledge of planes.
CO2	Basic idea of lines, sphere and cones.
CO3	Understand the properties of planes, lines, spheres and cones.
CO4	Express the problems geometrically and then to get the solution.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

COURSE SYLLABUS:

UNIT – I: The Plane **(12 Hours)**

Equation of plane in terms of its intercepts on the axis, Equations of of angles between two planes, Combined equation of two planes, Orthogonal projection on a plane. te plane through the given points, Length of the perpendicular from a given point to a given plane, Bisectors

UNIT – II: The Line **(12 Hours)**

Equation of a line; Angle between a line and a plane; The condition that a given line may lie in a given plane; The condition that two given lines are coplanar; Number of arbitrary constants in the equations of straight line; Sets of conditions which determine a line; The shortest distance between two lines; The length and equations of the line of shortest distance between two straight lines; Length of the perpendicular from a given point to a given line.

UNIT – III: The Sphere **(12 Hours)**

Definition and equation of the sphere; Equation of the sphere through four given points; Plane sections of a sphere; Intersection of two spheres; Equation of a circle; Sphere through a given circle; Intersection of a sphere and a line; Power of a point; Tangent plane; Plane of contact; Polar plane; Pole of a Plane; Conjugate points; Conjugate planes.

UNIT – IV: The Sphere and Cones **(12 Hours)**

Angle of intersection of two spheres; Conditions for two spheres to be orthogonal; Radical plane; Coaxial system of spheres.

Definitions of a cone; vertex; guiding curve; generators; Equation of the cone with a given vertex and guiding curve; equations of cones with vertex at origin are homogenous; Condition that the general equation of the second degree should represent a cone.

UNIT –V: Cones **(12 Hours)**

Enveloping cone of a sphere; right circular cone: equation of the right circular cone with a given vertex, axis and semi vertical angle: Condition that a cone may have three mutually perpendicular generators; intersection of a line and a quadric cone; Tangent lines and tangent plane at a point; Condition that a plane may touch a cone; Reciprocal cones; Intersection of two cones with a common vertex.

Co-Curricular Activities: **(15 Hours)**

Seminar/ Quiz/ Assignments/ Applications of Differential Equations to Real life Problem /Problem Solving.

Prescribed Text Book:

Analytical Solid Geometry by Shanti Narayan and P.K. Mittal, published by S. Chand &Company Ltd. 7th Edition.

Reference Books :

1. A text book of Mathematics for BA/B.Sc Vol 1, by V Krishna Murthy & Others, published by S. Chand & Company, New Delhi.
2. A text Book of Analytical Geometry of Three Dimensions, by P.K. Jain and Khaleel Ahmed, published by Wiley Eastern Ltd., 1999.
3. Co-ordinate Geometry of two and three dimensions by P. Balasubrahmanyam, K.Y. Subrahmanyam,

4. G.R. Venkataraman published by Tata-MC Gran-Hill Publishers Company Ltd., New Delhi.

Additional Inputs :

Definition of Cylinder and Right Circular Cylinder .

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

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COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**The Plane**	2	2	20
II	**The Line**	2	2	20
III	**The Sphere**	1	2	16
IV	**The Sphere and Cones**	2	2	20
V	**Cones**	1	2	16
	Total	8	10	92

S.A.Q. = Short answer questions (4 marks)

E.Q = Essay questions (6 marks)

Short answer questions : 5X4 = 20M

Essay questions : 5X6 =30M

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Total Marks = 50M

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P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - II Semester
Mathematics Course-II: Three Dimensional Analytical Solid Geometry
(w.e.f. 2022-23 Admitted Batch)
Model Paper (w.e.f. 2022-2023)

Time: 2Hrs **Max. Marks: 50M**

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PART - I

Answer any FIVE questions. Each question carries FOUR marks. 5 X 4=20M

1. Find the equation of the plane through the point (-1,3,2) and perpendicular to the planes x+2y+2z=5 and 3x+3y+2z=8.
2. Find the equation to the plane through the points (1, 1, 1), (1, -1, 1) and (-7, -3, -5). Show that it is parallel to y- axis.
3. Find the image of the point (2,-1,3) in the plane 3x-2y+z =9.
4. Find the equations of the line through the point (1, 1, 1) and intersecting the lines $2x - y - z - 2 = 0 = x + y + z - 1$; $x - y - z - 3 = 0 = 2x + 4y - z - 4$.
5. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.
6. Find the limiting points of the coaxal system of spheres determined by $x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0$, $x^2 + y^2 + z^2 + 2x - 4y + 2z + 6 = 0$.
7. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$
8. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).

PART - II

Answer the following questions. Each question carries SIX marks. 5 X 6 = 30M

9. A plane meets the coordinate axes in A, B, C. If the centroid of ΔABC is (a, b, c). Show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$.

(OR)

10. Show that the equation $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6 = 0$ represents a pair of parallel planes and find the distance between them.

11. Find the shortest distance between the lines $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

(OR)

12. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar . Also find their point of intersection and the plane containing the lines.

13. Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z = 2$; $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere.

(OR)

14. Find the equation of the sphere through the circle $x^2+y^2+z^2 = 9$, $2x+3y+4z-5 = 0$ and the point (-1,- 2,3).

15. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts orthogonally to the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.

(OR)

16. Prove that if the angle between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\pi/2$, then $a + b + c = 0$ and is $\pi/3$, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

17. Prove that the equation $\sqrt{fx} \mp \sqrt{gy} \mp \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone.

(OR)

18. Find the equation to the right circular cone whose vertex is P(2,-3,5) , axis PQ which makes equal angles with the axis and semi-vertical angle 30^0 .

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - II Semester
Mathematics Course-II: Three Dimensional Analytical Solid Geometry
(w.e.f. 2022-23 Admitted Batch)

QUESTION BANK
Short Answer Questions
Unit-I

1. Find the equation of the plane through (4, 4, 0)and perpendicular to the planes x + 2y + 2z = 5 and 3x + 3y + 2z – 8 = 0 .
2. Find the equation of the plane through (1, 0, -2)and perpendicular to the planes 2x + y -2 = z and x – y - z= 3 .
3. Find the equation of the plane through (-1, 3, 2)and perpendicular to the planes and x +2y +2z = 5 and 3x + 3y + 2z = 8 .
4. Find the equation to the plane through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane 2x + 6y + 6z = 9 .
5. Show that the equation of the plane passing through the points (2, 2, -1), (3, 4, 2), (7, 0, 6) is 5x + 2y – 3z – 17 = 0 .
6. Find the angles between the planes 2x – y + z = 0 , x + y + 2z = 7 .
7. Find the equation of the plane through the point (-1, 3, 2) and perpendicular to the planes x+2y+2z =5 and 3x+3y+2z =8.
8. Find the equation of the plane through the line of intersection of x – y +3z = 5 = 0 and 2x + y – 2z + 6 = 0and passing through (-3, 1, 1).
9. Find the equation of the plane through the line of intersection of the planes $x + y + z - 1 = 0$, $2x + 3y$+$4z - 5 = 0$ and perpendicular to the plane $x - y + z = 0$.

UNIT-II

10. Find the equation of the line through the point $(1, 2, 3)$ and parallel to X- axis.
11. Find the point of intersection of the line through $(2, -3,1), (3, -4,5)$ with the plane $2x + y + z = 7$.
12. Find the image of the point (2, -1, 3) in the plane 3x - 2y + z = 9.
13. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$
14. Find the symmetric form of the equation of the line $x + y + z + 1 = 0 = 4x + y - 2z + 2$.
15. Find the equations of the line through the point (1, 1, 1) and intersecting the lines $2x - y - z - 2 = 0 = x + y + z - 1$; $x - y - z - 3 = 0 = 2x + 4y - z - 4$.
16. Show that the lines $2x + y - 4 = 0 = y + 2z$ and $x + 3z - 4 = 0 = 2x + 5z - 8 = 0$ are coplanar.

17. Find the length and equations to the line of the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$.

UNIT-III

18. Find the equation of the sphere through O = (0, 0, 0) and making intercepts a, b, c on the axes .
19. A plane passes through a fixed point (a, b, c) and intersect the axes in A, B, C . Show that the centre of the sphere OABC lies on $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$
20. Find the centre and radius of the circle $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$, $x + 2y + 2z - 15 = 0$.
21. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$, $x - 2y + 4z = 9$ and the centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0$.
22. Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle .Also find its centre and radius .
23. Find the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which are parallel to $2x + 2y - z = 0$
24. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$, and find the point of contact.

UNIT-IV

25. Find the limiting points of the coaxal system of spheres determined by $x^2 + y^2 + z^2 + 4x - 2y + 2z + 6 = 0$, $x^2 + y^2 + z^2 + 2x - 4y + 2z + 6 = 0$.
26. Find the limiting points of the coaxal system of spheres determined by $x^2 + y^2 + z^2 + 3x - 3y + 6 = 0, x^2 + y^2 + z^2 - 6x - 6y - 6z + 6 = 0$.
27. Show that the general equation of the cone of the second degree which pass through Coordinate axes is $fyz + gzx + hxy = 0$.
28. Find the equation to the cone which passes through the three coordinate axes and the lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ *and* $\frac{x}{2} = \frac{y}{1} = \frac{z}{1}$.
29. Find the equation of the cone with vertex at (1, 2, 3) guiding curve $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$.
30. Find the equation of the cone whose vertex is (1, 2, 3) and base curve $y^2 = 4ax$, $z = 0$.

UNIT-V

31. Find the enveloping cone at the (1, 1, 1) and generators touching the sphere $x^2 + y^2 + z^2 - 2x + 4z - 1 = 0$
32. Find the equations of the cone touches the 3 co-ordinate planes and the plane $x + 2y + 3z = 0, 2x + 3y + 4z = 0$.

33. Find the equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 + 2x - 2y = 2$ with its vertex at (1, 1, 1).

34. Show that the two lines of intersection of the plane ax + by + cz = 0 with the cone yz + zx + xy = 0 will be perpendicular if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

35. Show that if a right circular cone has sets of three mutually perpendicular generators , its semi-vertical angle must be $\tan^{-1}\sqrt{2}$.

36. If $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone then prove that $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$.

37. Find the equation of the right circular cone whose vertex is (1, -2, -1), axis is the line $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z+1}{5}$ and the semi- vertical angle 60° .

38. Show that the reciprocal cone of $ax^2 + by^2 + cz^2 = 0$ is the cone $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$

Essay Questions

UNIT-I

1. If a plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r) then show that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$.

2. A variable plane is at a constant distance 3p from the origin and meets the axes in A, B, C. Show that the locus of the centroid of Δ ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

3. A variable plane makes intercepts on the axes, the sum of whose squares is k^2 (a constant). Show that the locus of the foot of the perpendicular from origin to the plane is $(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2)^2 = k^2$.

4. Find the planes bisecting the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$. Point out which of the planes bisects the acute angle and which bisects the obtuse angle in which the origin lies.

5. Find the bisecting plane of the acute angle between the planes 3x – 2y – 6z + 2 = 0 , -2x + y – 2z + 2 = 0 .

6. Prove that the equation represents a pair of planes , and find the angle between them .
   (i) $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$
   (ii) $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$

7. Show that the equation $x^2 + 4y^2 + 9z^2 - 12yz - 6zx + 4xy + 5x + 10y - 15z + 6 = 0$ represents a pair of parallel planes and find the distance between them.

8. Show that the equation $x^2 + 4y^2 + 4z^2 + 8yz + 4zx + 4xy - 9x - 18y - 18z + 18 = 0$ represents a pair of parallel planes and find the distance between them.

UNIT-II

9. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

10. Show that the lines $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ and $x + 2y + 3z - 8$=2x+3y+4z-11 are intersecting, find the point their intersection and the equation to the plane containing them.

11. Find the length and equations of shortest distance between the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

12. Find the S.D between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Find also the equations and the points in which the S.D meets the given lines .

13. Find the length and equation of the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$.

14. Find the S.D and the equations of the line of S.D between the lines 3x – 9y + 5z = 0 = x = y – z and 6x + 8y + 3z – 10 = 0 = x + 2y + z – 3 .

15. Find the equation of the plane containing line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z - 12$ =0.

UNIT-III

16. A sphere of radius k passes through the origin and meet the axes in A, B, C. Show that the centroid of the triangle ABC lies on the sphere 9 $(x^2 + y^2 + z^2) = 4k^2$

17. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

18. Show that the two circles $x^2 + y^2 + z^2 - y + 2z = 0$, $x - y + z = 2$; $x^2 + y^2 + z^2 + x - 3y + z - 5 = 0$, $2x - y + 4z - 1 = 0$ lie on the same sphere , and find its equation .

19. Find the equation of the sphere passing through the circle $x^2+y^2=4$, z=0 and is intersected by the plane x+2y+2z=0 in circle of radius 3 .

20. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

21. Find the polar plane of the point (0, -1, 1) with respect to the sphere
$$x^2 + y^2 + z^2 - 2x + 4y + 6z - 11 = 0.$$

22. Find the pole of the plane x-y+5z-3=0 with respect to the sphere $x^2+y^2+z^2=9$.

UNIT-IV

23. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ and cuts orthogonally to the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$.
24. Show that the radical line of the spheres $x^2 + y^2 + z^2 - 4x + 3 = 0$, $x^2 + y^2 + z^2 - 6y + 3 = 0$, $x^2 + y^2 + z^2 + 4x + 2y - 4z + 3 = 0$, $is\ \frac{x}{3} = \frac{y}{2} = \frac{z}{7}$.
25. Find the radical centre of the spheres $x^2 + y^2 + z^2 + 4y = 0$, $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$, $x^2 + y^2 + z^2 + 3x - 2y + 8z + 6 = 0$, $x^2 + y^2 + z^2 - x + 4y - 6z - 2 = 0$
26. If r_1 , r_2 are the radii of two orthogonal spheres then the radius of the circle of their intersection is $\frac{r_1 r_2}{\sqrt{{r_1}^2 + {r_2}^2}}$.
27. Find the equation to the sphere of the coaxal system $x^2 + y^2 + z^2 - 5 + \lambda(2x + y + 3z - 3) = 0$ which touch the plane 3x+4y-15=0 .
28. Prove that if the angle between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\pi/2$, then $a + b + c = 0$ and is $\pi/3$, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.
29. Show that two lines of intersection of the plane $ax + by + cz = 0$ with the cone $yz + zx + xy = 0$ will be perpendicular if 1/a + 1/b +1/c =0.
30. Find the angle between the lines of section of the plane $x - 3y + z = 0$ and the cone $x^2 - 5y^2 + z^2 = 0$.

UNIT-V

31. The equation of a right circular cone with vertex at (α, β, γ) semi vertical angle θ and axis having direction ratios (l, m, n) is $[l(x-\alpha) + m(y-\beta) + n(z-\gamma)]^2 = (l^2 + m^2 + n^2)((x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2)\cos^2\theta$.
32. Find the equation to the right circular cone whose vertex is P(2,-3,5) and axis PQ which makes equal angles with the axis and which passes through A(1,-2,3)
33. Find the vertex of the cone $7x^2 + 2y^2 + 2z^2 + 10zx + 10xy + 26x - 2y + +2z - 17 = 0$.
34. Prove that the equation $2y^2 + 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $(\frac{-7}{6}, \frac{1}{2}, \frac{5}{6})$.
35. Find the vertex of the following cones .
    i) $4x^2-y^2+2z^2+2xy-3yz+12x-11y+6z+4=0$
    ii) $x^2-2y^2+3z^2-4xy+5yz-6zx+8x-19y-2z-20=0$.

36. Show that the general equation to a cone which touches the three co ordinate planes is $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$.

37. Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone .

38. The semi-vertical angle of a right circular cone having three mutually perpendicular

(i) generators is $tan^{-1}(\sqrt{2})$.

(ii) tangent planes is $tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

<table>
<tr><td></td><td>P.R.Government College (Autonomous): KAKINADA</td><td colspan="4" rowspan="2">Program&Semester
IB.Sc. (IISem)
w.e.f 2022-23 admitted batch</td></tr>
<tr><td>CourseCode
MAT-201P</td><td>TITLE OF THE COURSE
Solid Geometry</td></tr>
<tr><td>Teaching</td><td>HoursAllocated:30(Practical)</td><td>L</td><td>T</td><td>P</td><td>C</td></tr>
<tr><td>Pre-requisites:</td><td>Basic Mathematics Knowledge on 2-D Geometry</td><td>-</td><td>-</td><td>2</td><td>1</td></tr>
</table>

UNIT – I: The Plane

- Equation of plane in terms of its intercepts on the axis,
- Equations of of angles between two planes,
- Combined equation of two planes,
- Orthogonal projection on a plane.
- the plane through the given points,
- Length of the perpendicular from a given point to a given plane,
- Bisectors

UNIT – II: The Line

- Equation of a line;
- Angle between a line and a plane;
- The condition that a given line may lie in a given plane;
- The condition that two given lines are coplanar;
- Number of arbitrary constants in the equations of straight line;
- Sets of conditions which determine a line;
- The shortest distance between two lines;
- The length and equations of the line of shortest distance between two straight lines;
- Length of the perpendicular from a given point to a given line.

UNIT – III: The Sphere

- Definition and equation of the sphere;
- Equation of the sphere through four given points;
- Plane sections of a sphere;
- Intersection of two spheres;
- Equation of a circle;
- Sphere through a given circle;
- Intersection of a sphere and a line;

- Power of a point;
- Tangent plane;
- Plane of contact;
- Polar plane;
- Pole of a Plane;
- Conjugate points;
- Conjugate planes.

UNIT – IV: The Sphere and Cones

- Angle of intersection of two spheres;
- Conditions for two spheres to be orthogonal;
- Radical plane;
- Coaxial system of spheres.
- Definitions of a cone;
- vertex; guiding curve; generators;
- Equation of the cone with a given vertex and guiding curve;
- equations of cones with vertex at origin are homogenous;
- Condition that the general equation of the second degree should represent a cone.

UNIT –V: Cones

- Enveloping cone of a sphere;
- right circular cone:
- equation of the right circular cone with a given vertex, axis and semi vertical angle:
- Condition that a cone may have three mutually perpendicular generators;
- intersection of a line and a quadric cone;
- Tangent lines and tangent plane at a point;
- Condition that a plane may touch a cone;
- Reciprocal cones;
- Intersection of two cones with a common vertex.

Semester – II end Practical Examinations

Scheme of Valuation for Practical's

Time : 2 Hours **Max.Marks : 50**

- **Record - 10 Marks**
- **Viva voce - 10 Marks**
- **Test - 30 Marks**
- **Answer any 5questions. At least 2 questions from each section. Each question carries 6 marks.**

BLUE PRINT FOR PRACTICAL PAPER PATTERN

COURSE-II, THREE DIMENSIONAL ANALYTICAL SOLID GEOMETRY

Unit	TOPIC	E.Q	Marks allotted to the Unit
I	**The Plane**	2	12
II	**The Line**	2	12
III	**The Sphere**	1	6
IV	**The Sphere and Cones**	2	12
V	**Cones**	1	6
	Total	08	48

P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
I year B.Sc., Degree Examinations - I Semester
Mathematics Course-I: Differential Equations
(w.e.f. 2022-23 Admitted Batch)
Practical Model Paper (w.e.f. 2022-2023)

……

Time: 2Hrs **Max. Marks: 50M**

Answer any 5questions. At least 2 questions from each section. **5 x 6 = 30 Marks**

SECTION - A

1. Find the planes bisecting the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$. Point out which of the planes bisects the acute angle and which bisects the obtuse angle in which the origin lies.

2. Show that the equation $x^2 + 4y^2 + 4z^2 + 8yz + 4zx + 4xy - 9x - 18y - 18z + 18 = 0$ represents a pair of parallel planes and find the distance between them.

3. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

4.Find the length and equation of the shortest distance between the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ and $x + y + 2z - 3 = 0 = 2x + 3y + 3z - 4$.

SECTION -B

5. Find the equation of the sphere passing through the circle $x^2+y^2=4$, z=0 and is intersected by the plane x+2y+2z=0 in circle of radius 3 .

6. Find the radical centre of the spheres $x^2 + y^2 + z^2 + 4y = 0$, $x^2 + y^2 + z^2 + 2x + 2y + 2z + 2 = 0$, $x^2 + y^2 + z^2 + 3x - 2y + 8z + 6 = 0$, $x^2 + y^2 + z^2 - x + 4y - 6z - 2 = 0$.

7. Prove that if the angle between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\pi/2$, then $a + b + c = 0$ and is $\pi/3$, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

8. Find the vertex of the cone $7x^2 + 2y^2 + 2z^2 + 10zx + 10xy + 26x - 2y + +2z - 17 = 0$.

- **Record - 10 Marks**
- **Viva voce - 10 Marks**

PRACTICAL QUESTION BANK

UNIT-I

1. Find the equation of the plane through (4, 4, 0)and perpendicular to the planes x + 2y + 2z = 5 and 3x + 3y + 2z – 8 = 0 .

2. If a plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r) then show that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$.

3. A variable plane is at a constant distance 3p from the origin and meets the axes in A, B, C. Show that the locus of the centroid of Δ ABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$.

4. A variable plane makes intercepts on the axes, the sum of whose squares is k^2 (a constant). Show that the locus of the foot of the perpendicular from origin to the plane is $(x^{-2}+y^{-2}+z^{-2})(x^2+y^2+z^2)^2=k^2$.

5. Find the planes bisecting the angles between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$. Point out which of the planes bisects the acute angle and which bisects the obtuse angle in which the origin lies.

6. Find the bisecting plane of the acute angle between the planes 3x – 2y – 6z + 2 = 0 , -2x + y – 2z + 2 = 0 .

7. Prove that the equation represents a pair of planes , and find the angle between them .

(i) $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$

(ii) $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$

8. Show that the equation $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$ represents a pair of parallel planes and find the distance between them.

9. Show that the equation $x^2+4y^2+4z^2+8yz+4zx+4xy-9x-18y-18z+18=0$ represents a pair of parallel planes and find the distance between them.

UNIT-II

10. Find the image of the point (2, -1, 3) in the plane 3x - 2y + z = 9.

11. Find the equations of the line through the point (1, 1, 1) and intersecting the lines $2x-y-z-2=0=x+y+z-1$; $x-y-z-3=0=2x+4y-z-4$.

12. Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$; $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Also find their point of intersection and the plane containing the lines.

13. Show that the lines $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ and $x+2y+3z-8=2x+3y+4z-11$ are intersecting, find the point their intersection and the equation to the plane containing them.

14. Find the length and equations of shortest distance between the line $\frac{x-2}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$.

15. Find the S.D between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Find also the equations and the points in which the S.D meets the given lines .

16. Find the length and equation of the shortest distance between the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{1}$ and $x+y+2z-3=0=2x+3y+3z-4$.

17. Find the S.D and the equations of the line of S.D between the lines 3x – 9y + 5z = 0 = x = y –z and 6x + 8y + 3z – 10 = 0 = x + 2y + z – 3 .

18. Find the equation of the plane containing line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and perpendicular to the plane $x+2y+z-12=0$.

UNIT-III

19. A sphere of radius k passes through the origin and meet the axes in A, B, C. Show that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$

20. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

21. Show that the two circles $x^2+y^2+z^2-y+2z=0$, $x-y+z=2$; $x^2+y^2+z^2+x-3y+z-5=0$, $2x-y+4z-1=0$ lie on the same sphere , and find its equation .

22. Find the equation of the sphere passing through the circle $x^2+y^2=4$, $z=0$ and is intersected by the plane $x+2y+2z=0$ in circle of radius 3 .

23. Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

24. Find the polar plane of the point (0, -1, 1) with respect to the sphere $x^2+y^2+z^2-2x+4y+6z-11=0$.

25. Find the pole of the plane $x-y+5z-3=0$ with respect to the sphere $x^2+y^2+z^2=9$.

UNIT-IV

26. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at $(1,-2,1)$ and cuts orthogonally to the sphere $x^2+y^2+z^2-4x+6y+4=0$.

27. Show that the radical line of the spheres $x^2+y^2+z^2-4x+3=0$, $x^2+y^2+z^2-6y+3=0$, $x^2+y^2+z^2+4x+2y-4z+3=0$, is $\frac{x}{3}=\frac{y}{2}=\frac{z}{7}$.

28. Find the radical centre of the spheres $x^2+y^2+z^2+4y=0$, $x^2+y^2+z^2+2x+2y+2z+2=0$, $x^2+y^2+z^2+3x-2y+8z+6=0$, $x^2+y^2+z^2-x+4y-6z-2=0$

29. If r_1 , r_2 are the radii of two orthogonal spheres then the radius of the circle of their intersection is $\frac{r_1r_2}{\sqrt{r_1{}^2+r_2{}^2}}$.

30. Find the equation to the sphere of the coaxal system $x^2+y^2+z^2-5+\lambda(2x+y+3z-3)=0$ which touch the plane $3x+4y-15=0$.

31. Prove that if the angle between the lines of intersection of the plane $x+y+z=0$ and the cone $ayz+bzx+cxy=0$ is $\pi/2$, then $a+b+c=0$ and is $\pi/3$, if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$.

32. Show that two lines of intersection of the plane $ax + by + cz = 0$ with the cone $yz + zx + xy = 0$ will be perpendicular if 1/a + 1/b +1/c =0.

33. Find the angle between the lines of section of the plane $x - 3y + z = 0$ and the cone $x^2 - 5y^2 + z^2 = 0$.

UNIT-V

34. The equation of a right circular cone with vertex at (α, β, γ) semi vertical angle θ and axis having direction ratios (l, m, n) is $[l(x-\alpha) + m(y-\beta) + n(z-\gamma)]^2 = (l^2 + m^2 + n^2)((x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2)\cos^2\theta$.

35. Find the equation to the right circular cone whose vertex is P(2,-3,5) and axis PQ which makes equal angles with the axis and which passes through A(1,-2,3)

36. Find the vertex of the cone $7x^2 + 2y^2 + 2z^2 + 10zx + 10xy + 26x - 2y + +2z - 17 = 0$.

37. Prove that the equation $2y^2 + 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone whose vertex is $\left(\frac{-7}{6}, \frac{1}{2}, \frac{5}{6}\right)$.

38. Find the vertex of the following cones .

i) $4x^2-y^2+2z^2+2xy-3yz+12x-11y+6z+4=0$

ii) $x^2-2y^2+3z^2-4xy+5yz-6zx+8x-19y-2z-20=0$.

39. Show that the general equation to a cone which touches the three co ordinate planes is $\sqrt{ax} + \sqrt{by} + \sqrt{cz} = 0$.

40. Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the coordinate planes and find its reciprocal cone .

41. The semi-vertical angle of a right circular cone having three mutually perpendicular

(i) generators is $tan^{-1}(\sqrt{2})$.

(ii) tangent planes is $tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

42. Show that the reciprocal cone of $ax^2 + by^2 + cz^2 = 0$ is the cone $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$.

	P.R.Government College (Autonomous) **KAKINADA**	**Program&Semester** IIB.Sc. (IIISem)
Course Code MAT-301/3201	**TITLEOFTHECOURSE** **Abstract Algebra**	

		L	T	P	C
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on sets and number system.	5	1	-	5

Course Objectives:

To provide the learner with the skills, knowledge and competencies to carry out their duties and responsibilities in pure Mathematic environment.

CourseOutcomes:

On Completion of the course, the students will be able to-	
CO1	Acquire the basic knowledge and structure of groups, subgroups and cyclic groups.
CO2	Get the significance of the notation of a normal subgroups.
CO3	Understand the ring theory concepts with the help of knowledge in group theory and to prove thetheorems.
CO4	Study the homomorphisms and isomorphisms with applications.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	

UNIT I : **(12 Hours)**

GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II: **(12 Hours)**

SUBGROUPS:Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition- examples-criterion for a complex to be a subgroups. Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. **Co-sets and Lagrange's Theorem:** Cosets Definition-properties of Cosets–Index of a subgroups of a finite groups–Lagrange's Theorem.

UNIT III: **(12 Hours)**

NORMAL SUBGROUPS: Definition of normal subgroup – proper and improper normal subgroup– Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group –quotient group – criteria for the existence of a quotient group.

HOMOMORPHISM : Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties– kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

UNIT IV: **(12 Hours)**

PERMUTATIONS: Definition of permutation – permutation multiplication – Inverse of a permutation –cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

UNIT V: (12 Hours)

RINGS

Definition of Ring and basic properties, Boolean Rings, divisors of zero and cancellation laws Rings,Integral Domains, Division Ring and Fields, The characteristic of a ring - The characteristic of an Integral Domain, The characteristic of a Field. Sub Rings.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

TEXT BOOK :

1. A text book of Mathematics for B.A. / B.Sc. by B.V.S.S. SARMA and others, published by S.Chand & Company, New Delhi.

REFERENCE BOOKS :

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna.
3. Rings and Linear Algebra by Pundir & Pundir, published by Pragathi Prakashan.

Additional Inputs :

Cyclic Groups, Maximal Ideals and Prime Ideals .

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	2	1	20
II	Subgroups , Co-sets and Lagrange's Theorem	2	1	20
III	Normal subgroups, Homomorphism	1	2	25
IV	Permutations.	1	1	15
V	Rings	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

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Total Marks = 50 M

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P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Mathematics Course: Abstract Algebra
Paper III (Model Paper w.e.f. 2021-22)

Time: 2Hrs **Max. Marks: 50**

PART - I

Answer any FOUR questions. Each question carries FIVE marks. 4 X 5 M=20 M

1. Prove that in a group $G(\neq \emptyset)$, for $a, b, x, y \in G$,the equations $ax = b, ya = b, \forall\, a, b \in G$ have unique solutions.
2. If G is a group,for $a, b \in G$ prove that $(ab)^{-1} = b^{-1}a^{-1}$
3. If a non empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
4. If G is a finite group and $a \in G$ then show that O(a) divides O(G) .
5. Define Normal subgroup. Prove that a subgroup H of a Group (G,.) is a normal subgroup of G if and only if $xHx^{-1} = \mathrm{H}\ \forall\ \mathrm{x} \in \mathrm{G}$.
6. Express the product (2 5 4)(1 4 3)(2 1) as a product of disjoint cycles and find its inverse.
7. Prove that the characteristic of an integral domain is either a prime or zero.

PART - II

Answer Any THREE questions. Each question carries Ten marks. 3 X 10 M = 30 M

9. A finite semi –Group (G, ·) satisfying the cancellation laws is a group.

10. Prove that a non empty complex H of a group G is a subgroup of G if and only if

$a, b \in H \Rightarrow ab^{-1} \in H$.

11. If H is a normal subgroup of a group (G,.) then prove that the product of two right (or) left cosets of H is also a right (or) left coset of H in G.

12. State and prove fundamental theorem on homomorphisms of groups .

13 . State and prove Cayley's theorem .

14. Prove that the ring of integers Z is a principal ideal ring.

P.R.GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank

PAPER–III: ABSTRACT ALGEBRA
Short Answer Questions
UNIT-1

1. Prove that in a group the identity element is unique and the inverse of every element is unique.
2. If G is a group,for $a, b \in G$ prove that $(ab)^{-1} = b^{-1}a^{-1}$
3. Prove that the cancellation laws hold in a group.
4. In a group G , if $a^2 = e$ for every $a \in G$, prove that G is an abelian .
5. If every element of a group G is its own inverse , prove that G is abelian .
6. Prove that in a group $G(\neq \emptyset)$, for $a, b, x, y \in G$,the equations $ax = b, ya = b, \forall\, a, b \in G$ have unique solutions.
7. If $(G, .)$ is a group such that $(ab)^n = a^n b^n, \forall\, a, b \in G$ for three consecutive positive integers n, then show that $(G, .)$ is an abelian group.
8. Show that the fourth roots of unity form an abelian group under multiplication.

UNIT-II

9. Show that the identity of a subgroup H of a group G is same as the identity of G .
10. If a non empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
11. If H is a subgroup of a group G then show that HH = H .
12. If H and K are two subgroups of a group G then show that $H \cap K$ is also a subgroup of G.
13. $(Z_6 = \{0,1,2,3,4,5\}, +_6)$ is a group. Prove that $S = \{0,2,4\}, T = \{0,3\}$ are subgroups of Z_6 and $S \cup T$ is not a subgroup of Z_6.
14. Let G be a group .If a ∈ G then show that $O(a) = O(a^{-1})$.
15. If G is a finite group and $a \in G$ then show that O(a) divides O(G) .
16. If G is a group and $a \in G$, then prove that the normalizer $N(a) = \{x \in G / ax = xa\}$ is a subgroup of G.

UNIT-III

17. Define Normal subgroup. Prove that a subgroup H of a Group (G,.) is a normal subgroup of G if and only if $xHx^{-1} = H\ \forall\ x \in G$.
18. Prove that every subgroup of an abelian group is normal.
19. Prove that intersection of two normal sub-groups of a group is a normal sub-group .
20. If G is a group then the centre Z of G is a normal subgroup of G.
21. Show that every sub-group of prime order is simple .
22. Prove that the homomorphic image of an abelian group is abelian.
23. Prove that the necessary and sufficient condition for a homomorphism f of a group G onto a group G' with kernel K to be an isomorphism of G into G' is that $K = \{e\}$.
24. Show that if $f: G \to G$ defined by $f(a) = a^{-1}\ \forall a \in G$ .Prove that f is one-one onto . Also prove that f is an automorphism if and only if G is Commutative.

UNIT-IV

25. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ as a product of disjoint cycles .

26. Write down the following permutation as product of disjoint cycles $f = (1\ 3\ 2\ 5)(1\ \ 4\ \ 3)(2\ \ 5\ \ 1)$.
27. Show that the permutation $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 6\,1\,4\,3\,2\,5\,7\,9\,8 \end{pmatrix}$ is odd permutation .
28. Verify whether the permutation $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 2\,5\,4\,3\,6\,1\,7\,9\,8 \end{pmatrix}$ is even or odd .
29. Find the order of the cycle (1 4 5 7).
30. Define a even and odd permutations and give one example.
31. Find the regular permutation group isomorphic to the multiplicative group { 1, -1, i, -i } .

UNIT-V

32. Prove that, a ring R has no zero divisors iff the cancellation laws hold in R.
33. Prove that every field is an integral domain.
34. Prove that, the characteristic of a Boolean ring is 2.
35. Give an example of a division ring which is not a field .
36. Prove that, the intersection of two subrings of a ring R is a subring of R.
37. Prove that the characteristic of an integral domain is either a prime or zero.
38. A field has no proper ideals .
39. Show that the set $I = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in Z \right\}$ is a right ideal but not a left ideal of the ring of 2 x 2 matrices over integers .

Essay Questions

UNIT-1

1. Show that the set of rational numbers other than 1 with operation * such that $a * b = a + b - ab\ \forall a, b\ \in Z$ is an abelian group .
2. Show that the set Q_+ of all positive rational numbers forms an abelian group under the composition defined by 'o' such that $a\, ob = (ab)/3\ \ for\ a, b\ \in\ Q_+$.
3. Prove that the set G of real numbers other than -1 with operation * such that $a * b = a + b + ab\ \forall\ a, b \in Z$ is an abelian group .
4. Prove that the set $G = \{A_\alpha / \alpha \in R\}\ where\ A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ forms a group with respect to '.'
5. Prove that the set of n^{th} roots of unity forms an abelian group w.r.t. '.'
6. Prove that a finite semi-group (G , •) satisfying the cancelation laws is a group.
7. Prove that the group (G , •) is abelian iff $(ab)^2 = a^2 b^2$, $\forall\ a, b \in G$.

UNIT-II

8. Prove that a non empty complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G .
9. Prove that a non empty finite complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab \in H.$
10. If H and K be two sub-groups of a group g then $H \cup K$ is a sub-group iff either $H \subseteq K\ or\ K \subseteq H$.
11. If H and K are two sub-groups of a group G , then show that HK is a sub-group of G if and only if HK = KH.
12. If a and b are any two distinct elements of a group G and H is a sub-group of G then prove that $i)\ a \in bH \Leftrightarrow aH = bH\ \ ii)\ a\ \in Hb \Leftrightarrow Ha = Hb$.

13. Let H be a sub-group of a group G and a , b ∈ G . Then prove that

$$i)\ Ha = Hb \ \Leftrightarrow ab^{-1} \in H \ \ ii)\ aH = bH \ \Leftrightarrow a^{-1}b \in H$$

14. Prove that any two left(right) cosets of a sub-group are either disjoint are identical .
15. State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true.

UNIT-III

16. Prove that H of a group G is normal sub-group of G if and only if each left coset of H in G is a right coset of H in G .
17. Prove that H is a normal sub-group of G if and only if product of two right right (left) cosets of H in G is again a right (left) coset of H on G .
18. Prove that a sub-group of index 2 in a group is a normal sub-group .
19. If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N.
20. If H is a normal subgroup of G. The set $\frac{G}{H}$ of all cosets of H in G with respect to coset multiplication is a group.
21. State and prove Fundamental theorem of homomorphism of groups .
22. If f is a homomorphisms of a group G into a group G^1 then show that the kernel of f is a normal subgroup og G .
23. Let G be a group. Prove that the set A(G) of all Automorphisms on G is a group with respect to the composition of mappings.

UNIT-IV

27. $f = (1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8)$, $g = (4\ \ 1\ \ 5\ \ 6\ \ 7\ \ 3\ \ 2\ \ 8)$ are cyclic permutations. Show that $(fg)^{-1} = g^{-1}f^{-1}$.
28. Let S_n be a symmetric group of n symbols and let A_n be the group of even permutations , then show that A_n is a normal in S_n and $O(A_n) = \frac{n!}{2}$.
29. State and prove Cayley's theorem.
30. The set A_n of all even permutations on n symbols is a normal subgroup of the permutation group S_n on then symbols.

UNIT-V

34. Prove that every finite integral domain is a field.

35. Prove that the set $Z(i) = \{a + ib : a, b \in Z, i^2 = -1\}\ of$ Gaussion integers is an Integral domain with respect to addition & multiplication of numbers is a field.

36. Prove that $Q(\sqrt{2}) = \{a + b\sqrt{2} :\ a, b \in Q\}$ is a field w.r.to ordinary addition and multiplication of numbers.

37. Prove that the characteristic of an integral domain is either a prime or zero.

38. Let S be a non empty subset of a ring R. Then prove that S is a subring of R iff $a - b \in S$ and $ab \in S$, for all $a, b \in S$.

39. If U_1 and U_2 are two ideals of a ring R, then prove that $U_1 \cup U_2$ is an ideal of R iff $U_1 \subseteq U_2$ or $U_2 \subseteq U_1$.

40. Prove that the ring of integers Z is a principal ideal ring.

	P.R.Government College (Autonomous) KAKINADA	Program&Semester			
CourseCode **SDC-AS/3023**	**TITLE OF THE COURSE** **Analytical Skills**	IIB.Sc (IIISem)			
Teaching	HoursAllocated:30(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge	2	0	-	2

Course Objectives:

Intended to inculcate quantitative analytical skills and reasoning as an inherent ability in students.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the basic concepts of arithmetic ability, quantitative ability, logical reasoning, business computations and data interpretation and obtain the associated skills.
CO2	Acquire competency in the use of verbal reasoning.
CO3	Apply the skills and competencies acquired in the related areas.
CO4	Solve problems pertaining to quantitative ability, logical reasoning and verbal ability inside and outside the campus.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

UNIT – 1 (10 Hrs)

Arithmetic ability:

Algebraic operations BODMAS, Fractions, Divisibility rules, LCM& GCD(HCF).

Verbal Reasoning:

Number Series, Coding & Decoding, Blood relationship, Clocks, Calendars.

UNIT – 2: (10 Hrs)

Quantitative aptitude:

Averages, Ratio and proportion, Problems on ages, Time-distance–speed.

Business computations:

Percentages, Profit & loss, Partnership, simple compound interest.

UNIT – 3: (07 Hrs)

Data Interpretation:

Tabulation, Bar Graphs, Pie Charts, line Graphs. Venn diagrams.

Recommended Co-Curricular Activities (03 Hrs)

Surprise tests / Viva-Voice / Problem solving/Group discussion.

Text Book:

Quantitative Aptitude for Competitive Examination by R.S. Agrawal, S.Chand Publications.

Reference Books:

1. Analytical skills by Showick Thorpe, published by S Chand And Company Limited, Ramnagar,New Delhi-110055.
2. Quantitative Aptitude and Reasoning by R V Praveen, PHI publishers.
3. Quantitative Aptitude for Competitive Examination by Abhijit Guha, Tata Mc Graw HillPublication

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':No Correlation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-III

Unit	**TOPIC**	**Multiple choice questions (1 mark)**	**S.A.Q**	**Marks allotted to the Unit**
I	Arithmetic ability &Verbal reasoning	10	2	20
II	Quantitative aptitude & Business computations	10	2	20
III	Data interpretation	10	2	20
	TOTAL	30	6	60

M.C.Q. = multiple choise questions (1 marks)

S.A.Q = Short answer questions (5 Marks)

Very Short answer questions : 30 x 1 = 30M

Short answer questions : 4 x 5 = 20 M

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Total Marks = 50M

P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Foundation course:Analytical Skills
(Model Paper w.e.f. 2021-22)

Time: 2 Hrs **Total Marks: 50M**

Section-A

Answer the following Questions. Each question carries one mark. **30 x 1 = 30 M**

1. $25 - [18 - \{25 + 3(10 - \overline{7 - 5})\}] = ?$

a) **86** b) 68 c) 52 d) None of these

2. Which of the following fraction is arranged in ascending order of their value ?
a) 1/4 . 2/7 , 3/4 , 4/7 , 5/7 , 6/5 b) **1/4 , 2/7 , 4/7 , 5/7 , 3/4 , 6/5**
c) 2/7 , 1/4 , 4/7 , 3/4 , 5/7 , 6/5 d) 2/7 , 1/4 , 4/7 , 5/7 , 3/4 , 6/5

3. If HCF , LCM of two numbers are 16 , 240 . If one number is 48 , find the other ?

a) 58 b) **80** c) 85 d) 72

4. ($x^n - a^n$) is completely divisible by (x-a), when

a) **n is any natural number** b) n is an even natural number

c) n is an odd natural number d) n is prime

5. 11,13,17,19,23,25,?

a) 26 b) 27 **c) 29** d) 37

6. A and B are young ones of C . If C is the father of a but B is not the son of C . How are B and C related

a) niece and Uncle b) **Daughter and Father** c) Niece and Father d) Daughter and Mother

7. In certain code 'FROZEN' is written as 'OFAPSG'.How is 'MOLTEN' is written in that code?

a)OFPOMN b)OFSMPN c)**OFUMPN** d)OFUNPM

8. How many times do the hands of a clock coincide in a day ?

a) 24 b) 21 c) 20 d) **22**

9. What was the day India attained Independence ?

a) Wednesday b) **Friday** c) Monday d) Thursday

10. Today is Tuesday . After 62 days it will be

a) **Monday** b) Wednesday c) Tuesday d) Friday

11. What will be the average of numbers from 1 to 51 .

a) 25 b) **26** c) 27 d) 28

12. The average age of 14 girls and their teacher's age is 15 yr . If teacher's age is excluded , then the average reduced by 1 . What is the teacher's age ?

a) **29 yr** b) 35 yr c) 32 yr d) 30 yr

13. The sum of three consecutive odd numbers is 285 . What is the ratio of the smallest and largest numbers respectively ?

a) 97 : 95 b) 93 : 95 c) 95 : 93 d) **93 : 97**

14. The ages of Surekha and Arunima are in the ratio of 9 : 8 respectively . After 5 yr , the ratio of their ages will be 10 : 9 . what is the difference in years between their ages ?

a) 4 yr b) **5 yr** c) 6 yr d) 7 yr

15. If a student walks from his house to school at 5 km / h , he is late by 30 min . However , if he walks at 6 km / h , he is late by 5 min only . The distance of his school from his house is

a) 2.5 km b) **12.5 km** c) 3.6 km d) 5.5 km

16. Raman scored 456 marks in an exam and Sita got 54% marks in the same exam , which is 24 marks less than Raman . If the minimum passing marks in the exam is 34% , then how much more marks did Raman score than the minimum passing marks ?

a) **184** b) 196 c) 190 d) 180

17. By selling 18 chocolates , a vendor loses the selling price of 2 chocolates . Find his loss per cent .
a) 95 b) **10%** c) 11% d) 12%

18. A , B and C invested their capitals in the ratio of 5 : 6 : 8 . At the end of the business , they received the profits in the ratio of 5 : 3 : 1 . Find the ratio of time for which they contributed their capitals .

a) 12 : 9 : 7 b) 25 : 18 : 8 c) 5 : 6 : 8 d) **8 : 4 : 1**

19. The compound interest accrued on an amount at the end of 2 yr at the rate of 12% p.a. is Rs.2862 . What is the amount ?

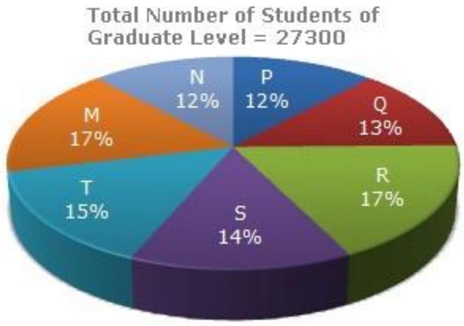
a) **Rs.11250** b) Rs.12200 c) Rs.13500 d) Rs. 10000

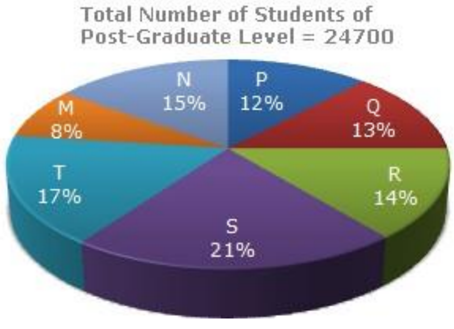
20. A sum of Rs.3200 becomes Rs.3456 in 2 yr at a certain rate of simple interest . What is the rate of interest per annum ?

a) 5.5% b) 6% c) **4%** d) 4.5%

21.The following pie-charts show the distribution of students of graduate and post-graduate levels

in seven different institutes in a town. **Distribution of students at graduate and post-graduate levels in seven institutes:**





1. What is the total number of graduate and post-graduate level students is institute R?
   A. 8320 B. 7916 C. 9116 **D. 8099**
2. What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?
   A. 14:19 B. 19:21 C. 17:21 **D. 19:14**

3. How many students of institutes of M and S are studying at graduate level?

   A. 7516 **B. 8463** C. 9127 D. 9404

4. What is the ratio between the number of students studying at post-graduate level from institutes S and the number of students studying at graduate level from institute Q?

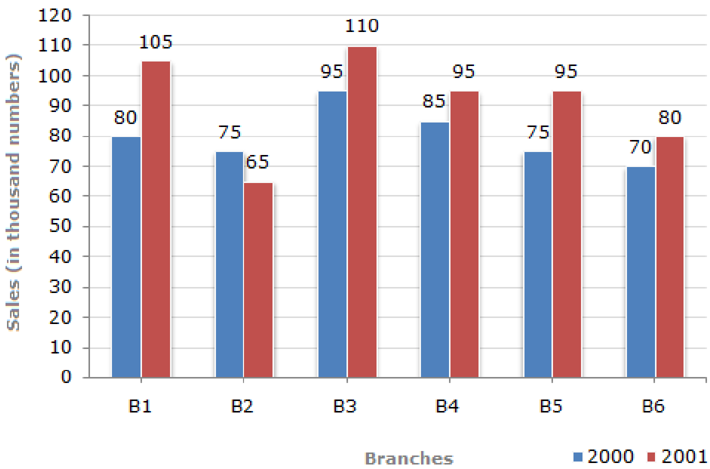
   A. 13:19 B. 21:13 C. 13:8 **D. 19:13**

5. Total number of students studying at post-graduate level from institutes N and P is

   A. 5601 B. 5944 **C. 6669** D. 8372

22. The bar graph given below shows the sales of books (in thousand number) from six branches of a publishing company during two consecutive years 2000 and 2001.

Sales of Books (in thousand numbers) from Six Branches - B1, B2, B3, B4, B5 and B6 of a publishing Company in 2000 and 2001.



1. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?
   A. 2:3 B. 3:5 C. 4:5 **D. 7:9**
2. Total sales of branch B6 for both the years is what percent of the total sales of branches B3 for both the years?
   A. 68.54% B. 71.11% **C. 73.17%** D. 75.55%
3. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000?
   A. 75% B.77.5% C. 82.5% **D.87.5%**
4. What is the average sales of all the branches (in thousand numbers) for the year 2000?
   A. 73 **B. 80** C. 83 D. 88

5. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is ?

   A. 250 B. 310 C. 435 **D. 560**

SECTION – B

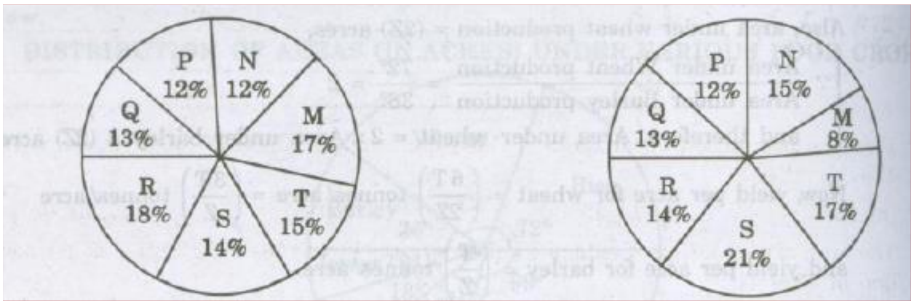
Answer any four questions . Each question carries 5 Marks . **4 x 5 = 20 M**

1. Explain the divisibility rules .
2. Explain the BODMAS rule.
3. Explain the types of Ratio and Proportion .
4. Explain simple and compound interest .
5. Explain the Venn diagrams .

6. ***The following pie-charts show the distribution of students of graduate and post graduate levels in seven different institute-M,N,P,Q,R,S and T in a town.***

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES-M,N,P,Q,R,S AND T.

Total Number of students of graduate level **Total Number of students of post graduate level**



1. How many students of institutes M and S are studying at graduate level?
2. Total number of students studying at post -graduate level from institutes N and P is:
3. What is the total number of graduate and post-graduate level students in institute R?
4. What is the ratio between the number of students studying at post graduate and graduate levels respectively from institute S?
5. What is the ratio between the number of students studying post graduate level from institute S and the number of students studying at graduate level from institute Q?

P. R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUESTION BANK FOR ANALYTICAL SKILLS

UNIT-1 ARITHMETIC ABILITY

BODMASRULE AND SIMPLIFICATION

1. $4003 \times 77 - 21015 = ? \times 116$

a) 2477 b) 2478 c) 2467 d) **2476**

2. $(5696 \div 4 - ?) \times 5 = 1020$

a) 1180 b) 1200 c) 1240 d) **1220**

3. $23 \times 15 - 60 \div ? \div 31 = 292$

a) 218 b) 186 c) **217** d) 201

4. $16 \times 12 - 672 \div 21 = ? - 211$

a) 381 b) **371** c) 372 d) 311

5. $36 \times 15 - 56 \times 784 \div 112 = ?$

a) 138 b) 238 c) 158 d) **None of these**

6. $(4444 \div 40) + (645 \div 25) + (3991 \div 26) = ?$

a) 280.4 b) **290.4** c) 295.4 d) 285.4

7. $25 - \left[18 - \left\{25 + 3\left(10 - \overline{7 - 5}\right)\right\}\right] = ?$

a) **86** b) 68 c) 52 d) None of these

8. $2 - [3 - \left\{6 - \left(5 - \overline{4 - 3)}\right\}\right\} = ?$

a) 2 b) **1** c) 5 d) None of these

9. $3\frac{3}{4} + 4\frac{2}{5} - 3\frac{1}{8} = ?$

a) $4\frac{1}{40}$ b) $\mathbf{5\frac{1}{40}}$ c) $6\frac{1}{40}$ d) $5\frac{3}{40}$

10. $\frac{3}{5} \times \frac{-1}{2} + \frac{1}{2} of \frac{3}{4} - \frac{1}{8} \div \frac{5}{8} = ?$

a) $\mathbf{\frac{-1}{8}}$ b) $\frac{-1}{8}$ c) $\frac{3}{10}$ d) $\frac{1}{5}$

DECIMAL FRACTIONS

1. Which of the following fraction is the smallest ?

a) 13/16 b) **15/19** c) 17/21 d) 7/8

2. The value of 0.57575757.... is

a) 57/10 b) 26/45 c) **57/99** d) 52/99

3. Which of the following series is in ascending order .

a) 3/5 , 7/9 , 6/7 b) 7/9 , 6/7 , 3/5 c) 7/9 , 3/5 , 6/7 d) **6/7 , 7/9 , 3/5**

4. Which of the following fraction is less than 7/8 and greater than 1/3 ?

a) 1/4 b) 11/12 c) **17/24** d) 23/24

5. On simplification 1/0.04 is equal to

a) 2.5 b) 2/5 c) **25** d) 1/40

6. 3 x 0.3 x 0.03 x 0.003 x 30 = ?

a) 0.0000243 b) **0.00243** c) 0.000243 d) 0.0243

7. Which of the following fraction is the largest ?

a) **8/9** b) 18/23 c) 16/21 d) 14/17

8. $(25.732)^2 - (15.732)^2 = ?$

a) **414.64** b) 4146.4 c) 41.464 d) 4.1464

9. $8^{1.1} \times 4^{2.7} \times 2^{3.3} = 2^?$

a) 7.1 b) 12 c) 0.5 d) 14

10. Which of the following fraction is arranged in ascending order of their value?

a) 1/4 . 2/7 , 3/4 , 4/7 , 5/7 , 6/5 b) **1/4 , 2/7 , 4/7 , 5/7 , 3/4 , 6/5**

c) 2/7 , 1/4 , 4/7 , 3/4 , 5/7 , 6/5 d) 2/7 , 1/4 , 4/7 , 5/7 , 3/4 , 6/5

DIVISIBILITY RULE

1. The largest 4-digit number exactly divisible by 88 is

a) **9944** b)9768 c) 9988 d) 8888

2. Which of the following number will completely divide (49^{15}-1) ?

a) **8** b) 14 c) 48 d) 50

3. Which one of the following number will completely divide ($4^{61}+4^{62}+4^{63}+4^{64}$)

a) 3 b) **10** c) 11 d) 13

4. How many numbers between 11 and 90 are divisible by 7 ?

a) **11** b) 13 c) 15 d) 17

5. Find the least value of * for which 7*5462 is divisible by 9 ?

a) 5 b) 7 c) **3** d) 9

6.If the product 4864 x 9P2 is divisible by 12 , the value of P is

a)2 b) **1** c) 6 d) 8

7.How many 3 digits numbers are divisible by 6 in all ?

a) 149 b) **150** c) 151 d) 166

8. What will be the remainder when 17^{200} is divisible by 18 ?

a) 17 b) 16 c) **1** d) 2

9. ($x^n - a^n$) is completely divisible by (x-a), when

a) **n is any natural number** b) n is an even natural number c) n is an odd natural number d) n is prime

10. When a number is divided by 13 , the remainder is 11 . when the same number is divided by 17 , the remainder is 9. What is the number ?

a) 339 b) **349** c) 369 d) data inadequate

LCM and HCF:

1. The LCM of $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{9}$, $\frac{4}{27}$ is

a) $\frac{1}{54}$ b) $\frac{10}{27}$ c) $\boldsymbol{\frac{20}{3}}$ d) $\frac{27}{3}$

2. The sum of two numbers is 45 and their product is 500 . Their HCF is

a) **5** b) 9 c) 10 d) 15

3. If HCF , LCM of two numbers are 16 , 240 . If one number is 48 , find the other ?

a) 58 b) **80** c) 85 d) 72

4. Find the least number which when divided by 27 , 35 , 45 , 49 leaves the remainder 6 in each case ?

a) 6615 b) **6621** c) 6156 d) 6261

5. HCF of two numbers each of 4 digits is 103 and their LCM is 19261 . Sum of the numbers is

a) **2884** b) 2488 c) 4288 d) 4882

6. LCM of two numbers 7700 , HCF of two numbers is 11 and one of the number is 275 then the second number is

a) 279 b) 283 c) **308** d) 318

7. The ratio of three numbers is 3 : 4 : 5 and their LCM is 2400 then HCF is

a) **40** b) 80 c) 120 d) 200

8. Sum of the two numbers is 216 and their HCF is 27 then the numbers are

a) **27 , 189** b) 81 , 189 c) 108 , 108 d) 154 , 162

9. The LCM of two numbers is 495 and their HCF is 5 . Also sum of the two numbers is 100 then find the difference ?

a) **10** b) 46 c) 70 d) 90

10. When a number is divided by 2, 3, 4, 5 and 6 , remainder in each case is 1 . But the number is exactly divisible by 7 . If the number lies between 250 and 350 , the sum of digits of the number will be

a) **4** b) 5 c) 7 d) 10

VERBAL REASONING:

In each of the following questions, a number series is given with one term missing. Choose the correct alternative that will continue the same pattern and replace the question mark in the given series.

1. 1,9,25,49,?,121

a)64 **b)81** c)91 d)100

2. 11,13,17,19,23,25,?

a)26 b)27 **c)29** d)37

3. 6,11,21,36,56,?

a)42 b)51 **c)81** d)91

4. 10,18,28,40,54.70,?

a)85 b)86 c)87 **d)88**

5. 22,24,28,?,52,84

a)36 b)38 c)42 d)46

6. 28,33,31,36,?,39

a)32 **b)34** c)38 d)40

7. 6,17,39,72,?

a)83 b)94 **c)116** d)127

8. 325,259,204.160,127,105,?

a)94 b)96 c)98 d)100

In each of the following questions, one term in the number series is wrong. Find out the wrong term

1.3,10,27,4,16,64,5,25,105

a)3 b)4 **c)10** d)27

2. 8,13,21,32,47,63,83

a)13 b)21 c)32 **d)47**

3. 105,85,60,30,0,-45,-90

a)105 b)60 **c)0** d)-45

a)94 b)127 **c)202** d)259

5. 1,2,4,8,16,32,64,96

a)4 b)32 c)64 **d)96**

6. 10,26,74,218,654,1946,5834

a)26 b)74 c)218 **d)654**

7. 1,3,10,21,64,129,356,777

a)21 b)129 c)10 **d)356**

8. 3,4,10,32,136,685,4116

a) 10 b) **32** c) 136 d) 4116

Blood Relation

1. A and B are young ones of C. If C is the father of a but B is not the son of C . How are

B and C related

a) niece and Uncle b) **Daughter and Father** c) Niece and Father d) Daughter and Mother

2. F is the brother of A, C is the daughter of A , K is the sister of F and G is the brother of C then who is the uncle of G ?

a) C b) **A** c) C d) None of the above

3. A women introduces a man as "the son of the brother of her mother" . How is the man related to the women?

a) Uncle b) Grandson c) **Cousin** d) Son

4. A is the mother of B and C . If D is the husband of C . What is A to D .

a) Mother b) Sister c) Aunt d) **Mother-in – law**

5. If S is the brother of N , the sister of N is M , the brother of P is J and the daughter of S is P then who is the uncle of J ?

a) N b) S c) **P** d) M

6. Pointing towards a person , a man said to a women , " His mother is the only daughter of your father '' . How is the women related to that person ?

a) Sister b) Daughter c) **Mother** d) Wife

7. A is the brother of B and K , D is the brother of B and E is the father of A . Which of the following statement is not definitely true ?

a) A is the son of D b) **A is the father of K** c) B is the brother of K d) A is the son of E

8. A girl introduced a boy as the son of the daughter of the father of her uncle . The boy is girl's

a) Uncle b) Nephew c) **Brother** d) Son

CODING AND DECODING:

1. In a certain code 'MISSIONS' is written as 'MSIISNOS'.How is 'ONLINE' written in that code?
   a)**OLNNIE**
   b)ONILEN
   c)NOILEN
   d)ONNLIE
2. In certain code 'TIGER' is written as 'QDFHS'.How is 'FISH' written in that code?
   a)GERH
   b)**GRHE**
   c)GREH
   d)GHRE
3. In certain code 'FROZEN' is written as 'OFAPSG'.How is 'MOLTEN' is written in that code?
   a)OFPOMN
   b)OFSMPN
   c)**OFUMPN**
   d)OFUNPM
4. In certain code 'ROAR' is written as 'URDU'.How is 'URDU' written in that code?
   a)VXDQ
   b)**XUGX**
   c)ROAR
   d)VSOV
5. In certain code 'LIMCA' is written as 'HJLDZ'.Which of the following word is written as 'IFWJBP'?
   a)MERCURY
   b)**MEXICO**
   c)JAPAN
   d)HONDUS
6. In certain code 'HILTON' is written as 'HJLDZ'.How is 'BILLION' written in that code?
   a)IBLLION
   b)IBOILLN
   c)**IBLLOIN**
   d)IBLOILN
7. If in the English alphabet,every alternate letter from B onwards is written i small letters while others are written in capitals then how will the third day from Tuesday will be coded?
   a)WeDNeSdAY
   b)WEdnESdAY
   c)**FridAY**
   d)THURSdAY

8. In a certain code 'CERTAIN' is coded as 'BFQUZJM'. How is 'MUNDANE' coded in that code?
   a)**LVMEZOD**
   b)NTCOMBF
   c)NTOCNBF
   d)LTMCZO

CLOCKS

1. How many times in a day the hands of a clock are straight ?

a) 22 b) 24 c) **44** d) 48

2. How many times in a day the hands of a clock at right angles in a day ?

a) 22 b) **44** c) 24 d) 48

3. How many times do the hands of a clock coincide in a day ?

a) 24 b) 21 c) 20 d) **22**

4. How many times do the hands of a clock point towards each other in a day ?

a) 12 b) **22** c) 20 d) 24

5. The angle between the minut hand and the hour hand of a clock when the time is 4.20 is

a) 0^0 b) 20^0 c) 5^0 d) $\mathbf{10^0}$

6. The reflex angle between the hands of a clock at 10.25 is

a) 180^0 b) $\mathbf{197\frac{1^0}{2}}$ c) 195^0 d) $19\frac{1^0}{2}$

7. A clock rings at 1 O'clock , 2 o'clock , 3 o'clock and so on . The number of rings in a week would be

a) 256 b) 176 c) 156 d) **168**

8. An accurate clock shows 8 o'clock in the morning . through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon ?

a) 144 degree b) 150 degree c) **180 degree** d) 168 degree

9. By what time (in minutes) between 3 o'clock and 4 o'clock , both the needles coincide each other ?

a) $12\frac{4}{11}min$ b) $5\frac{1}{11}min$ c) $13\frac{4}{11}min$ d) $\mathbf{16\frac{4}{11}min}$

CALENDAR

1. Mahatma Gandhi was born on 2^{nd} October , 1869 . What was the week day on that day ?

a) **Saturday** b) Wednesday c) Thursday d) Friday

2. The last day of century cannot be …….

a) Monday b) Wednesday c) **Tuesday** d) Friday

3. What was the day India attained Independence ?

a) Wednesday b) **Friday** c) Monday d) Thursday

4. What was the day of week on 1st March 2016 ?

a) Wednesday b) Friday c) Monday d) **Tuesday**

5. The first day after a previously concluded century cannot be ….

a) Saturday b) Thursday c) **Sunday** d) Tuesday

6. The first republic day of India was celebrated on 26th January 1950 . It was ….

a) a) Saturday b) **Thursday** c) Sunday d) Tuesday

7. Today is Tuesday . After 62 days it will be

a) **Monday** b) Wednesday c) Tuesday d) Friday

8. What will be the day of the week on 1st January 2001 ?

a) a) Saturday b) Thursday c) **Sunday** d) Tuesday

9. On what dates of April 1994 did Sunday fall ?

a) **3, 10 , 17, 24** b) 2, 9, 16, 23 c) 4, 11, 18, 25 d) 5, 12, 19, 26

10. How many days are there from 2nd January 1995 to 15th March 1995 ?

a) 82 b) **73** c) 91 d) 74

UNIT-2 QUANTITATIVE APTITUDE

AVERAGE:

1. What will be the average of numbers from 1 to 51 .

a) 25 b) **26** c) 27 d) 28

2. What will be the average of 1, 2, 3, … 51, 52, 52 .

a) 26 b) **27** c) 25 d) 29

3. Find the average of all odd numbers and average of all even numbers from 1 to 45 .

a) 23 and 25 b) 23 and 24 c) **23 and 23** d) 20 and 21

4. The average of six observations is 12 . The average decreases by 1 , when a new observation is included . What is the seventh observation ?

a) 1 b) 3 c) **5** d) 6

5. A average height of 16 students is 142 cm . If the height of the teacher is included , the average height increases by 1 cm . The height of the teacher is

a) 156 cm b) 159 cm c) 159.5 cm d) 157 cm

6. The average age of 14 girls and their teacher's age is 15 yr . If teacher's age is excluded , then the average reduced by 1 . What is the teacher's age ?

a) **29 yr** b) 35 yr c) 32 yr d) 30 yr

7. The average marks of 65 students in a class was calculated as 150 . It was later realized that the marks of one of the student was calculated as 142 , where as his actual marks were 152 . What is the actual average marks of the group of 65 students ? (rounded off to two digits after comal).

a) 151.25 b) **150.15** c) 151.10 d) 150.19

8. Average score of Rahul Manish and suresh is 63 . Rahul's score is 15 less than Ajay and 10 more than Manish . It Ajay scored 30 marks more than the average score of Rahul , Manish and Suresh . Then what is the sum of Manish's and Suresh's score ?

a) 120 b) **111** c) 117 d) Cannot be determined

9. The average of four consecutive even numbers P , Q , R and S respectively (in increasing order) is 51 . What is the product of P and R ?

a) 2592 b) 2400 c) **2496** d) 2808

10. The average of 5 numbers is 306.4 . The average of the first two numbers is 431 and the average of the last two numbers is 214.5 . What is the third number ?

a) 108 b) **241** c) 52 d) 321

11. Average of 10 numbers is zero . At most hoe many numbers may be greater than zero .

a) 0 b) 1 c) 5 d) 9

RATIO & PROPORTION:

1. The sum of three consecutive odd numbers is 285 . What is the ratio of the smallest and largest numbers respectively ?

a) 97 : 95 b) 93 : 95 c) 95 : 93 d) **93 : 97**

2. The fourth proportional to 5 , 8 , 15 is

a) 18 b) **24** c) 19 d) 20

3. The mean proportional between 234 and 104 is

a) 128 b) 139 c) 145 d) **156**

4. The third proportional to 0.36 and 0.48 is

a) **0.64** b) 0.1728 c) 0.42 d) 0.94

5. A particular sum was divided among A , B , and C in theratio 2 : 6 : 7 respectively . If the amount received by A was Rs.4908 , what was the difference between the amounts received by B and C ?

a) **Rs.2454** b) Rs.3494 c) Rs.2135 d) Rs.2481

6. The sides of a triangle are in the ratio 1/2 , 1/3 , 1/4 and its perimeter is 104 cm . The length of the longest side (in cm) is

a) 26 b) 32 c) **48** d) 52

7. 20 boys and 25 girls form a group of special workers . During their membership drive , the same number of boys and girls joined the group . How many members does the group have if ratio of boys to girls is 7 : 8 ?

a) **75** b) 65 c) 70 d) None of these

8. Seats of Maths , physics and Biology are in the ratio of 5 : 7 : 8 respectively . There is a proposal to increase these seats by 40% , 50% and 75% respectively . What will be the respectively ratio of increased seats ?

a) 6 : 7 : 8 b) **2 : 3 : 4** c) 6 : 8 : 9 d) cannot be determind

9. If 50% of a certain number is equal to $3/4^{th}$ of another number , what is the ratio between the numbers ?

a) **3 : 2** b) 3 ; 2 c) 2 ; 5 d) 3 ; 4

10. The ratio of third propotional to 12 and 30 and the mean proportional between 9 and 25 is

a) 2 : 1 b) 7 : 15 c) **5 : 1** d) 9 : 14

PROBLEMS ON AGES:

1. The total present ages of p and Q is 25 yr more than the present age of R . If at present Q is 5 yr older than R . what is P's present age in years ?

a) **20** b) Data provided are not adequate to answer the question c) 40 d) 35

2. The ages of Surekha and Arunima are in the ratio of 9 : 8 respectively . After 5 yr , the ratio of their ages will be 10 : 9 . what is the difference in years between their ages ?

a) 4 yr b) **5 yr** c) 6 yr d) 7 yr

3. The respective ratio of the ages of Anubha and her mother is 1 : 2 . After 6 yr , the ratio of their ages will be 11 : 20 . 9 yr before what was the respective ratio of their ages ?

a) 3 : 5 b) 2 : 7 c) 1 : 4 d) **2 : 5**

4. The present ages of Vishal and Shekhar are in the ratio of 14 : 17 respectively . Six year from now , their ages will be in the ratio of 17 : 20 respectively . What is Shekhar's present age ?

a) 17 yr b) 51 yr c) **34 yr** d) 28 yr

5. The respective ratio between the present ages of Ram , Rohan and Raj is 3 : 4 : 5 . If the average of their present ages is 28 yr , then what would be the sum of the ages of Ram and Rohan togrther after 5 yr ?

a) 45 yr b) 55 yr c) 46 yr d) **59 yr**

TIME - DISTANCE-SPEED

1. A bike covers a certain distance at the speed of 64 km/h in 8 h . If the bike was to cover the same distance in approximately 6 h , at what approximate speed should the bike travel ?

a) 80 km/h b) **85 km/h** c) 90 km/h d) 75 km/h

2. A man walked at a speed of 4 km/h from point A to B and came back from point B to A at the speed of 6 km/h . What would be the be ratio between the time taken by man in walking from point A to B to point b to A respectively ?

a) 5 : 3 b) 2 : 3 c) 2 : 1 d) **3 : 2**

3. A person travels from P to Q at speed of 40 km/h and returns to Q by increasing his speed by 150% . What is his average speed for both the trips ?

a) 36 km/h b) 45 km/h c) **48 km/h** d) 50 km/ h

4. The average speed of a train is $1\frac{3}{7}$ times the average speed of a car . The car covers a distance of 588 km in 6 h . How much distance will the train cover in 13 h ?

a) 1750 km b) 1760 km c) 1720 km d) **None of these**

5. If a student walks from his house to school at 5 km / h , he is late by 30 min . However , if he walks at 6 km / h , he is late by 5 min only . The distance of his school from his house is

a) 2.5 km b) **12.5 km** c) 3.6 km d) 5.5 km

6. A bus started its journey from Ramgarh and reached Devgarh in 44 min with its average speed of 50 km/h . If the average speed of the bus is increased by 5 km/ h . How much time will it take to cover the same distance ?

a) **40 min** b) 38 min c) 36 min d) 31 min

7. A 280 m long train moving with an average speed of 108 km/h crosses a platform in 12s . a man crosses the same platform in 10s . What is the speed of the man in m/s ?

a) 5 m/s b) **8 m/s** c) 12 m/s d) cannot be determined

8. A car covers a distance of 258 km in a certain time at a speed of 66 km/h . How much distance would a truck cover at an average speed which is 24 km/h less than that of the speed of the car in time which is 7 h more than that taken by the car ?

a) 336 km b) 682 km c) 598 km d) **630 km**

PERCENTAGES

1. One-fourth of two-fifth of 30% of a number is 15 . What is 20% of that number ?
a) **100** b) 50 c) 200 d) 75
2. One-fifth of a number is 81 . What will be 68% of that number ?
a) 195.2 b) **275.4** c) 225.6 d) 165.8
3. Sum of three consecutive numbers is 2262 . What is 41% of the highest number ?
a) 301.51 b) 309.14 c) 308.73 d) **309.55**
4. What is 35% of 45% of 7/9 th of 36000 ?
a) **441** b) 414 c) 444 d) 411
5. Raman scored 456 marks in an exam and Sita got 54% marks in the same exam , which is 24 marks less than Raman . If the minimum passing marks in the exam is 34% , then how much more marks did Raman score than the minimum passing marks ?
a) **184** b) 196 c) 190 d) 180
6. If the numerator of a fraction is increased by 20% and the denominator is increased by 25% , the fraction obtained is 3/5 . What was the original fraction ?
a) 5/7 b) 4/7 c) **cannot be determined** d) 3/8
7. The price of rice is reduced by 5% . How many kilograms of rice can now be bought for the money , which was sufficient to buy 50 kg of rice earlier ?
a) 50.40 kg b) **52.63 kg** c) 42.30 kg d) None of these
8. A papaya tree was planted 2 yr ago . It increaser at the rate of 20% every year . If at present , the height of the tree is 540 cm , what was it when the tree was planted ?
a) 324 cm b) 400 cm c) **375 cm** d) 432 cm

PROFIT AND LOSS

1. Meenal purchased a car for Rs.25000 and sold it for Rs.34800 , What is the percent profit she made on the car ?
a) 40% b) **39.2%** c) 38.4% d) 38%
2. Lokesh bought an article for Rs.2500 . He spent Rs.320 on its shipping . He then sold it for Rs.4089 . What was the percent profit he gained in this transaction ?
a) 38% b) **45%** c) 46% d) 35%
3. Prasad sold his work tools for Rs.1850 and earned a profit of 25% . At what price did Prasad buy the work tools ?
a) Rs.1360 b) Rs.1300 c) Rs.1240 d) **Rs.1480**
4. Seema purchased an item for Rs.9600 and sold it for a loss of 5% . from that money , she purchased another item and sold it for a gain of 5% . What is her overall gain/loss ?
a) Loss of Rs.36 b) **Loss of Rs.24** c) Profit of Rs.24 d) Profit of Rs.36
5. The profit earned after selling an article for Rs.1754 is the same as loss incurred after selling the article for rs.1492 . what is the cost price of the article ?
a) **Rs.1623** b) Rs.1523 c) Rs.1689 d) Rs.1589

6. By selling 18 chocolates , a vendor loses the selling price of 2 chocolates . Find his loss per cent .

a) 95 b) **10%** c) 11% d) 12%

7. If on selling 12 note books any seller makes a profit equal to the selling price of 4 notebooks , what is his per cent profit ?

a) **50%** b) 25% c) Data inadequate d) None of these

8. An article was purchased for Rs.78350 . Its price was marked up by 30% it was sold at a discount of 20% on the marked up price . What was the profit per cent of the cost price ?

a) 3% b) 7% c) 5% d) **4%**

9. Vendor bought toffees at 6 for a rupee . How many for a rupee must be sell to gain 20%

a) 3 b) 4 c) 5 d) 6

10. A man buys a cycle for Rs.1400 and sells it at a loss of 15% . what is selling price of the cycle ?

a) 1090 b) 1160 c) 1190 d) 1202

11. If selling price doubled , the profit triples . Find profit percent

a) $66\frac{2}{3}$ b) 100 c) $105\frac{1}{3}$ d) 120

12. The cost price of 20 articles is the same as the selling price of X articles . If the profit is 25% . What is X ?

a) 15 b) 19 c) 18 d) 25

PARTNERSHIP

1. X , Y and Z were sharing profits in the ratio 4 : 3 : 2 : Y , retired from the firm and X and Z decide to share profits in the ratio 3 : 2 . Calculate the gaining ratio .

a) **7 : 8** b) 5 : 9 c) 4 : 7 d) 5 : 8

2. A , B and C invested their capitals in the ratio of 5 : 6 : 8 . At the end of the business , they received the profits in the ratio of 5 : 3 : 1 . Find the ratio of time for which they contributed their capitals .

a) 12 : 9 : 7 b) 25 : 18 : 8 c) 5 : 6 : 8 d) **8 : 4 : 1**

3.Tanvi started a business investing Rs.45000 . After 8 months , Anisha joined her with a capital of Rs.52000 . at the end of the year , the total profit was Rs.56165 , what is the share of profit of Anisha ?

a) Rs.21450 b) Rs.24440 c) **Rs.15618** d) Rs.31765

4. A , B and C started a shop by investing Rs.27000 , Rs.81000 and Rs.72000 respectively . At the end of 1 year , B's share in total profit was Rs.36000. What was the total profit ?

a) Rs.108000 b) Rs. 116000 c) Rs.80000 d) Rs.92000

SIMPLE , COMPOUND INTEREST

1. A sum of Rs.3200 becomes Rs.3456 in 2 yr at a certain rate of simple interest . What is the rate of interest per annum ?

a) 5.5% b) 6% c) **4%** d) 4.5%

2. The simple interest accrued on an amount at the end of 5 yr at 12.5% is Rs.1575 . What is the amount ?

a) Rs.2050 b) Rs.2550 c) Rs.2250 d) **Rs.2520**

3. What amount a man would have received on a principal of Rs.4000 after 2 yr at simple interest at the rate of 5% p.a ?

a) Rs.4161 b) Rs.5200 c) **Rs.4400** d) Rs.4100

4. A sum of Rs.5000 amounts to Rs.6050 in 2 yr . What is the rate of interest ?

a) 15% b) 13% c) 11% d) **10.5%**

5. The compound interest accrued on an amount at the end of 2 yr at the rate of 12% p.a. is Rs.2862 . What is the amount ?

a) **Rs.11250** b) Rs.12200 c) Rs.13500 d) Rs. 10000

UNIT-3 DATA ANALYSIS

1. DIRECTIONS: Study the table carefully to answer the questions that follow: Maximum and minimum Temperature (in degree Celsius) recorded on first day of each month for five different cities.

Month	Temperature									
	Bhuj		Sydney		Ontario		Kabul		Beijing	
	Max	Min	Max	Min	Max	Min	Max	Min	Max	Min
1st sep	24	14	12	2	5	1	34	23	12	9
1st oct	35	21	5	-1	15	6	37	30	9	3
1st nov	19	8	11	3	4	0	45	36	15	1
1st dec	9	2	-5	-9	-11	-7	31	23	2	-3
1st jan	-4	-7	-11	-13	-14	-19	20	11	5	-13

1. What is the difference between the max temperature of Ontario on 1st nov and the min temperature of Bhuj on 1st jan?

(1) 3 ℃ (2) 18 ˚C (3) 15 ℃ (4) 9 ℃ **(5) 11℃**

ANS: (5) Required difference = 4 - (-7) = 4 + 7 = 11

2: In which month respectively the max temperature of Kabul is 2nd highest and min temperature of Sydney is highest?

(1) 1st oct& 1st jan (2) **1st oct& 1st nov** (3) 1st dec& 1st jan (4) 1st sept& 1st jan (5) 1st dec& 1st Sept

ANS: (1)

3: In which month on 1st day is the difference between the max temperature & min temperature of Bhuj second highest?

(1) 1st sept (2) 1st oct **(3) 1st nov** (4) 1st dec (5) 1st jan

ANS: (3) Temperature difference of Bhuj: 1st Sept: 24-14=10℃, **1st Nov: 19-8=11℃**,
1st Oct: 35-21=14℃, 1st Dec: 9-2=7℃, 1st Jan -4+7=3℃

4: What is the average maximum temperature of Beijing over all the months together.
(1) 8.4℃ (2) 9.6℃ (3) 7.6℃ (4) 9.2℃ **(5) 8.6℃**

ANS: (5) Max temperature =12+9+15+2+5/5 = 43/5=8.6℃

5: What is the respective ratio between the min temperature of Beijing on 1stsept & the max temperature of Ontario on 1st Oct ?
(1) 3:4 **(2) 3:5** (3) 4:5 (4) 1:5 (5) 1:4

ANS: (2) required ratio = 9:15 = 3:5

2. Study the following table carefully answer the questions percentage of marks obtained by 6 students in 6 different subjects.

Sub/student	History (out of 50)	Geography (out of 50)	Maths(out 150)	Science(out 100)	English (out of 75)	Hindi (out of 75)
Amit	76	85	69	73	64	88
Bharat	84	80	85	78	73	92
Umesh	82	67	92	87	69	76
Nikhil	73	72	78	69	58	83
Pratiksha	68	79	64	91	66	65
Ritesh	79	87	88	93	82	72

1. What is the approximately the integral % of marks obtained by umesh in all the subjects?
(1) **80%** (2) 84% (3) 86% (4)78% (5) 77%

ANS: (1) Total marks obtained by Umesh = 41+33.5+92/100*150+87+69100*75+76/100*5

= 41+33.5+138+87+51.75+57=408.25

Required % = 408/500*100=80%

2. What is the avg % of marks obtained by all the students in hindi (approximated to two places of decimal)
(1) 77.45% **(2) 79.33%** (3) 75.52% (4) 73.52% (5) None of these

ANS: (2) Required avg of % in hindi =88+92+76+83+65+72/6=476/6=79.33%

3. What is the average marks of all the students in Mathematics?
(1) 128 (2) 112 **(3) 118** (4) 138 (5) 144

ANS: (3) avg. mark in mathematics = 15.
(69+85+92+78+64+88)/100*6 =150*476/100*6=119

4. What is the average marks obtained by all the students in geography?
(1)38.26 (2) 37.26 (3) 37.16 **(4) 39.16** (5) None of these

ANS: (4) Average marks in geography = 50(85+80+67+72+79+87)/6*1/100

= 50*470/6*1/100 = 39.16

5. What are the total marks obtained by pratiksha in all the subjects taken together?
   (1)401.75 (2) 410.75 (3) 402.75 (4) 420.75 **(5) None of these**

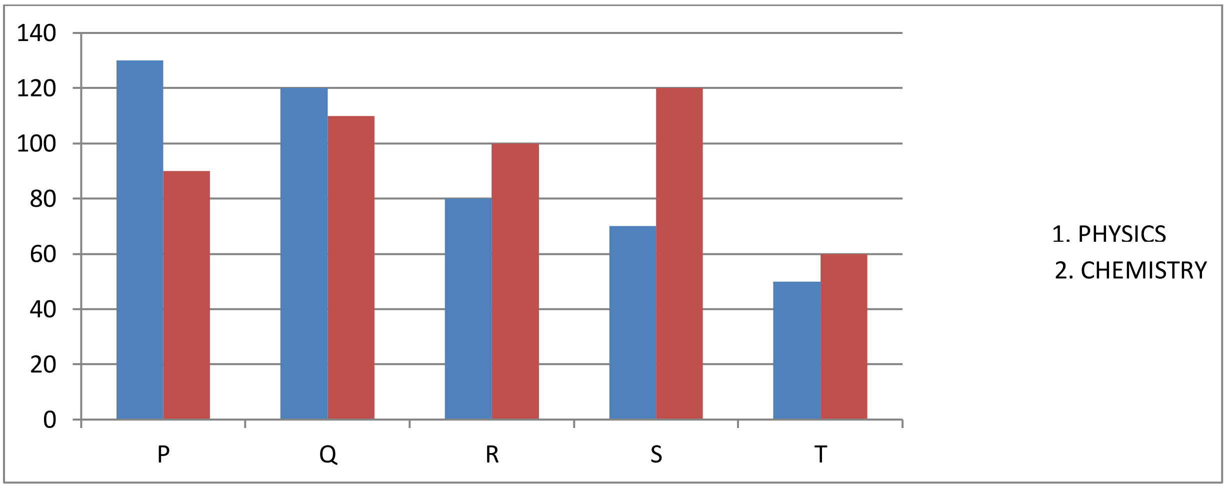
ANS: (5) marks obtained by

Ritesh = 50*68/100+50*79/100+150*64/100+91+75*66/100+50*86/100+75*65/100

=34+39.5+96+91+49.5+48.75 = 358.75

<u>BAR GRAPHS</u>

1. Study the following bar graphs carefully to answer these questions. Marks obtained by 5 students in physics & chemistry.



1. Marks obtained by S in chemistry is what percent of the total marks obtained by all the students in chemistry?
   (1) 25 (2) 28.5 (3) 35 (4) 31.5 (5) 22

ANS: (1) required %=120/90+110+100+120+60*100=120/480*100=25%

2. If the marks obtained by T in physics were increased by 14% of the original marks, what would be his new approximate % in physics if the max marks in physics were 140?
   (1)57 (2) 32 (3) 38 (4) 48 **(5) 41**

ANS: (5) increase in marks in physics of T=50*1.14=57. Required %= 57/140*100=40.7=41

3. What is the respective ratio between the total obtained by P in physics & chemistry together to the total marks obtained by T in physics & chemistry together?
   (1) 3:2 (2) 4:3 (3) 5:3 **(4) 2:1** (5)None of these

ANS: (4) required ratio = 130+90/60+50 = 220/110 = 2:1

4. What is the respective ratio between the total marks obtained by Q & S together in chemistry to the total marks obtained by P & R together in physics?

   (1) 23:25 **(2) 23:21** (3) 17:19 (4) 17:2 (5) None of these

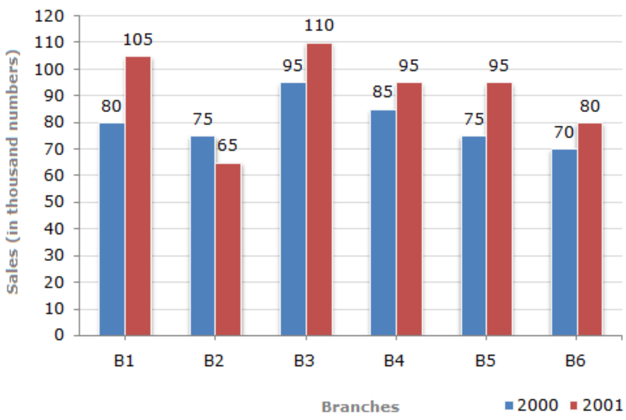
ANS: (2) Marks obtained by Q & S in chemistry=110+120=230.

Marks obtained by P & R in physics=130+80=210.

Required ratio=230/210=23:21.

2. The bar graph given below shows the sales of books (in thousand number) from six branches of a publishing company during two consecutive years 2000 and 2001.

Sales of Books (in thousand numbers) from Six Branches - B1, B2, B3, B4, B5 and B6 of a publishing Company in 2000 and 2001.



6. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?

B. 2:3 B. 3:5 C. 4:5 **D. 7:9**

Ans: D

Explanation: Required ratio $= \frac{(75+65)}{(85+95)} = \frac{140}{180} = \frac{7}{9} = 7:9$

7. Total sales of branch B6 for both the years is what percent of the total sales of branches B3 for both the years?

A. 68.54% B. 71.11% **C. 73.17%** D. 75.55%

Ans: 73.17%

Explanation: Required percentage $= \frac{70+80}{95+110} \times 100 = \frac{150}{205} \times 100 = 73.17$

8. What percent of the average sales of branches B1, B2 and B3 in 2001 is the average sales of branches B1, B3 and B6 in 2000?

B. 75% B.77.5% C. 82.5% **D.87.5%**

Ans: 87.5%

Explanation:

Average sales (in thousand number) of branches B1, B3 and B6 in 2000 $= \frac{1}{3} \times (80+95+70) = \frac{245}{3}$

Average sales (in thousand number) of branches B1, B2 and B3 in 2001$= \frac{1}{3} \times (105+65+110) = \frac{280}{3}$

Therefore, Required percentage = $= \frac{\frac{245}{3}}{\frac{280}{3}} \times 100 = \frac{245}{280} \times 100 = 87.5$

9. What is the average sales of all the branches (in thousand numbers) for the year 2000?

A. 73 **B. 80** C. 83 D. 88

Ans. 80

Explanation: Average sales of all the six branches (in thousand numbers) for the year 2000=

$\frac{1}{6} \times (80+75+95+85+7+70) = 80$

10. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is ?

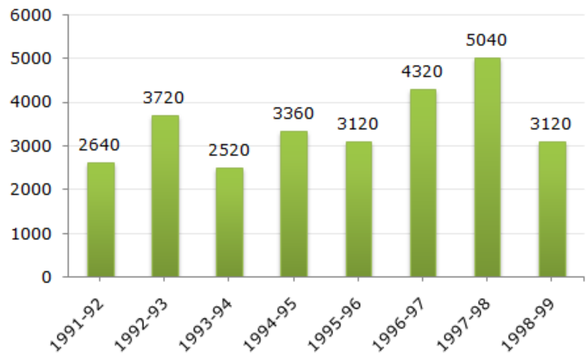
A. 250 B. 310 C. 435 **D. 560**

Explanation: Total sales of branches B1, B3 and B5 for both the years (in thousand numbers)

= (80 + 105) + (95 + 110) + (75 + 95) = 560.

3. The bar graph given below shows the foreign exchange reserves of a country (in million US $) from 1991 - 1992 to 1998 - 1999.

Foreign Exchange Reserves of a Country. (in million US $)



1. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves is?

A. 2:6 B. 3:4 **C. 3:5** D. 4:4

Ans: 3:5

Explanation: Average foreign exchange reserves over the given period = 3480 million US $.

The country had reserves above 3480 million US $ during the years 1992-93, 1996-97 and 1997-98, i.e., for 3 years and below 3480 million US $ during the years 1991-92, 1993-94, 1994-95, 1995-56 and 1998-99 i.e., for 5 years.

Hence, required ratio = 3:5.

2. The foreign exchange reserves in 1997 - 98 was how many times that in 1994-95?

A. 0.7 B. 1.2 C. 1.4 **D. 1.5**

Ans:1.5

Explanation: Required ratio=$\frac{5040}{3360} = 1.5$

3. For which year, the percent increase of foreign exchange reserves over the previous year, is the highest?

A. 1992-93 B. 1993-94 C. 1994-95 D. 1996-97

Ans:1992-93

Explanation: There is an increase in foreign exchange reserves during the years 1992 - 1993, 1994 - 1995, 1996 - 1997, 1997 - 1998 as compared to previous year (as shown by bar-graph).

The percentage increase in reserves during these years compared to previous year are:

For 1992 – 1993 = $\frac{3720 - 2640}{2640} \times 100 = 40.91$

For 1996 – 1997 = $\frac{4320 - 3120}{3120} \times 100 = 38.46$

For 1997 – 1998 = $\frac{5040 - 4320}{4320} \times 100 = 16.67$

Clearly, the percentage increase over previous year is highest for 1992 - 1993.

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review?

A. 95% B. 110% C. 115% **D. 125%**

Ans: 125%

Explanation: Average foreign exchange reserves over the given period

$$= \frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)$$

$= 3480$ million US $

Foreign exchange reserves in 1996 - 1997 = 4320 million US $.

Required percentage= $\frac{4320}{3480} \times 100 = 124.14 \approx 125.$

5. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94?

A. 100 B. 150 C. 200 D. 620

Ans:100%

Explanation: Foreign exchange reserves in 1997 - 1998 = 5040 million US $.

Foreign exchange reserves in 1993 - 1994 = 2520 million US $.

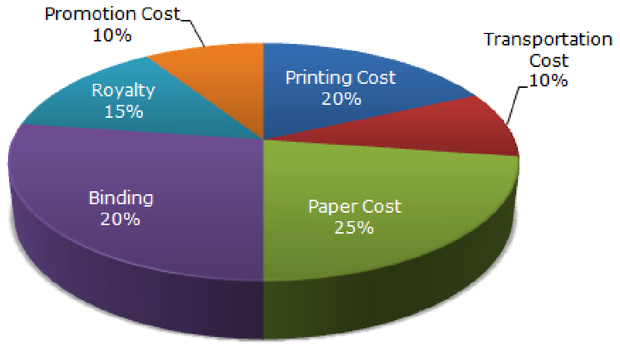
Increase = 5040 – 2520 = 2520 US $.

Percentage increase = $\frac{2520}{2520} \times 100 = 100$

PIE CHARTS

1. The following pie-chart shows the percentage distribution of the expenditure incurred in publishing a book. Study the pie-chart and the answer the questions based on it.

Expenditures (in percentage) Incurred in Publishing a Book



1. If for a certain quantity of books, the publisher has to pay Rs. 30, 600 as printing cost, then what will be amount of royalty to be paid for these books?

   A. Rs.19,450 B. Rs.21,200 **C. Rs.22,950** D. Rs.26,150

   Ans: Rs.22,950

   Explanation: Let the amount of Royalty to be paid for these books be Rs. *r*.

   Then, 20 : 15 = 30600 : *r* $\Rightarrow \frac{30600 \text{ x } 15}{20} = \text{Rs. } 22{,}950$

2. What is the central angle of the sector corresponding to the expenditure incurred on Royalty?

   A. 15º B. 24º **C. 54º** D. 48º

   Ans: 54º

   Explanation: Central angle corresponding to Royalty = (15% of 360)º = $\left(\frac{15}{100} \times 360\right)^{\circ} = 54^{\circ}$

3. The price of the book is marked 20% above the C.P. If the marked price of the book is Rs. 180, then what is the cost of the paper used in a single copy of the book?

   A. Rs.36 **B. Rs.37.50** C. Rs.42 D. Rs.44.25

   Explanation:

   Clearly, marked price of the book = 120% of C.P.

   Also, cost of paper = 25% of C.P

   Let the cost of paper for a single book be Rs. *n*.

   Then, 120 : 25 = 180 : *n* $\Rightarrow$ $n = \frac{25 \times}{120}$ = Rs. 37

4. If 5500 copies are published and the transportation cost on them amounts to Rs. 82500, then what should be the selling price of the book so that the publisher can earn a profit of 25%?

   A. Rs.187.50 B. Rs.191.50 C. Rs.175 D. Rs.180

   Ans: Rs.187.50

   Explanation:

For the publisher to earn a profit of 25%, S.P. = 125% of C.P.

Also Transportation Cost = 10% of C.P.

Let the S.P. of 5500 books be Rs. x.

Then, $10:125 = 82500:x \Rightarrow x = \frac{125 \text{ x } 82500}{10} = 1031250$, Rs

S.P. of one book= Rs. $\frac{1031250}{5500} = 187.50$

5. Royalty on the book is less than the printing cost by:

A. 5% B. 33 1/5 % C. 20% **D. 25%**

Ans: 25%

Explanation:

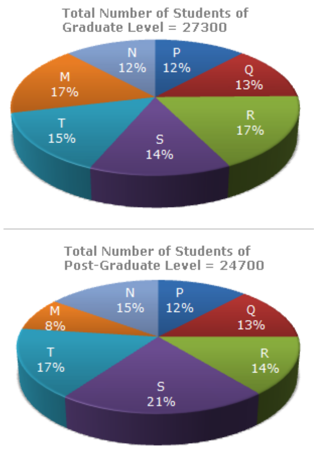
Printing Cost of book = 20% of C.P.

Royalty on book = 15% of C.P.

Difference = (20% of C.P.) - (15% of C.P) = 5% of C.P.

Percentage difference=$\frac{\text{difference}}{\text{printing cost}} \times 100 = \frac{5\% \text{ of C.P.}}{\text{Printing Cost}} \times 100 = 25\%$

2. The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes in a town.



Distribution of students at graduate and post-graduate levels in seven institutes

1.What is the total number of graduate and post-graduate level students is institute R?

B. 8320 B. 7916 C. 9116 **D. 8099**

Ans: 8099

Explanation: Required number = (17% of 27300) + (14% of 24700) = 4641+3458=8099.

2.What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?

B. 14:19   B. 19:21   C. 17:21   **D. 19:14**

Ans: 19:14

Explanation: Required ratio $=\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{(21 \text{ x } 24700)}{14 \text{ x } 27300} = \frac{19}{14}$

3. How many students of institutes of M and S are studying at graduate level?

A. 7516   **B. 8463**   C. 9127   D. 9404

Explanation: Students of institute M at graduate level= 17% of 27300 = 4641.
Students of institute S at graduate level = 14% of 27300 = 3822.

Total number of students at graduate in institutes M and S = (4641 + 3822) = 8463

4. What is the ratio between the number of students studying at post-graduate level from institutes S and the number of students studying at graduate level from institute Q?

A. 13:19   B. 21:13   C. 13:8   **D. 19:13**

Explanation: Required ratio = $\frac{21\% \text{ of } 24700}{13\% \text{ of } 27300} = \frac{21 \times 24700}{13 \times 27300} = \frac{19}{13}$

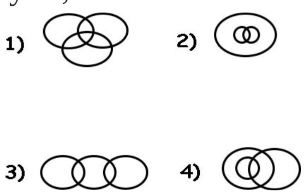
5. Total number of students studying at post-graduate level from institutes N and P is

A. 5601   B. 5944   **C. 6669**   D. 8372

Explanation: Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669

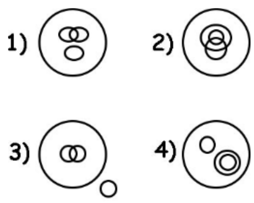
VENN DIAGRAM

1. Which of the following Venn- diagram correctly illustrates the relationship among the classes : Tennis fans, Cricket players, Students



Ans. 1

2. In a dinner party both fish and meat were served. Some took only fish and Some only meat. There were some vegetarians who did not accept either. The rest accepted both fish and meat. Which of the following Venn-diagrams correctly reflects this situation?



Ans. 1

Short answer questions

1. Explain the BODMAS rule .

2. Explain the divisibility rules .

3. Find the HCF of 12 , 36 , 48 .

4.LCM of two numbers is 28 times their HCF . The sum of HCF and LCM is 1740 , if one of these numbers is 240 . Then the sum of digits of the other number ?

5. Explain the types of Ratio and Proportion .

6. Write the relation between Speed , Time and Distance .

7. Explain the calculation of Profit and Loss percentage .

8. Explain the types of partnerships .

9. Explain simple and compound interest .

10. Explain profit and loss .

11.Write about Pie-charts.

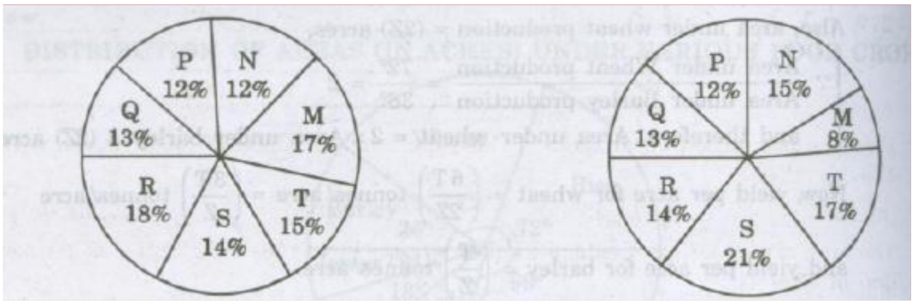
12. Explain types of Venn diagrams .

13. Wright about Bar graphs.

14. ***The following pie-charts show the distribution of students of graduate and post graduate levels in seven different institute-M,N,P,Q,R,S and T in a town.***

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES-M,N,P,Q,R,S AND T.

Total Number of students of graduate level | **Total Number of students of post graduate level**



1.How many students of institutes M and S are studying at graduate level?

2.Total number of students studying at post -graduate level from institutes N and P is:

3.What is the total number of graduate and post-graduate level students in institute R?

4 .What is the ratio between the number of students studying at post graduate and graduate levels respectively from institute S?

5.What is the ratio between the number of students studying post graduate level from institute S and the number of students studying at graduate level from institute Q?

Solution :

1.(b):Students of institute M at graduate level = 17% of 27300 = 4641.

Students of institute S at graduate level = 14% of 27300 = 3822

Total number students at graduate level in institutes M and S = 4641+3822=8463

2.(c):Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.

3.(d):Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.

4.(d):Required ratio = (21% of 24700) / (14% of 27300)
= 21 * 24700 / 14 * 27300
= 19 /14

5.(d):Required ratio = (21% of 24700)/ (13% of 27300)
= 21 * 24700 /13 * 27300
= 19 / 13

<table>
<tr><td rowspan="1"></td><td colspan="1">P.R.Government College (Autonomous)
KAKINADA</td><td colspan="4" rowspan="2">Program&Semester

IIB.Sc. (IVSem)</td></tr>
<tr><td>CourseCode

MAT-401/4201</td><td>TITLEOFTHECOURSE

Real Analysis</td></tr>
<tr><td>Teaching</td><td>HoursAllocated:60(Theory)</td><td>L</td><td>T</td><td>P</td><td>C</td></tr>
<tr><td>Pre-requisites:</td><td>Basic Mathematics Knowledge on number system.</td><td>5</td><td>1</td><td>-</td><td>5</td></tr>
</table>

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Get clear idea about the real numbers and real valued functions.
CO2	Obtain the skills of analyzing the concepts and applying appropriate methods for testing convergence of a sequence/ series.
CO3	Test the continuity and differentiability and Riemann integration of a function.
CO4	Know the geometrical interpretation of mean value theorems.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

UNIT I **(12 Hours)**

Introduction of Real Numbers (No question is to be set from this portion)

Real Sequences: Sequences and their limits, Range and Boundedness of Sequences, Limit of a sequence and Convergent sequence. The Cauchy's criterion, properly divergent sequences, Monotone sequences, Necessary and Sufficient condition for Convergence of Monotone Sequence, Limit Point of Sequence, Subsequences , Cauchy Sequences – Cauchy's general principle of convergence theorem.

UNIT II: (12 Hours)

INFINITIE SERIES :

Series : Introduction to series, convergence of series. Cauchy's general principle of convergence forseries tests for convergence of series, Series of Non-Negative Terms.

1. P-test
2. Cauchy's n^{th} root test or Root Test.
3. D'-Alemberts' Test or Ratio Test.
4. Alternating Series – Leibnitz Test.
5. Absolute Convergence and Conditional Convergence.

UNIT III: (12 Hours)

CONTINUITY:

Limits: Real valued Functions, Boundedness of a function, Limits of functions. Some extensions of the limitconcept, Infinite Limits. Limits at infinity. (No question is to be set from this portion).

Continuous functions: Continuous functions, Combinations of continuous functions, Continuous Functionson interval.

UNIT IV: **(12 Hours)**

DIFFERENTIATION AND MEAN VALUE THEOREMS: The derivability of a function, on an interval, at a point, Derivability and continuity of a function, Graphical meaning of the Derivative, Mean valueTheorems; Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem.

UNIT V: **(12 Hours)**

RIEMANN INTEGRATION : Riemann Integral, Riemann integral functions, Darboux theorem. Necessary and sufficient condition for R – integrability, Properties of integrable functions, Fundamental theorem of integral calculus, First mean value Theorem.

Co-Curricular Activities **(15 Hours)**

Seminar/ Quiz/ Assignments/ Real Analysis and its applications / Problem Solving.

TEXT BOOK:

1. Introduction to Real Analysis by Robert G.Bartle and Donlad R. Sherbert, published by JohnWiley.

REFERENCE BOOKS:

1. A Text Book of B.Sc Mathematics by B.V.S.S. Sarma and others, published by S. Chand &Company Pvt. Ltd., New Delhi.
2. Elements of Real Analysis as per UGC Syllabus by Shanthi Narayan and Dr. M.D. Raisinghania,published by S. Chand & Company Pvt. Ltd., New Delhi

Additional Inputs :

Taylor's Theorem , McLaren theorem .

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**Real Sequences**	1	1	15
II	Infinite Series	2	2	30
III	Continuity	2	1	20
IV	Differentiation And Mean value theorems	1	1	15
V	Riemann Integrations	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)
E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M
Essay questions : 3 X 10 = 30 M
……………….
Total Marks = 50 M
………………..

P.R. Government College (Autonomous), Kakinada

II Year, B.Sc., Degree Examinations - IV Semester

Mathematics Course: Real Analysis

Paper–IV (Model Paper ((w.e.f. 2021-22 Admitted Batch)

Time: 2Hrs **Max. Marks: 50**

PART-I

Answer any FOUR questions. Each question carries FIVE marks. 4 X 5 =20 M

1. Prove that every convergent sequence is a Cauchy sequence
2. If $\sum u_n$ convergences absolutely then prove that $\sum u_n$ converges.
3. Test for the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n} x^{n-1} \ (x > 0)$
4. Examine for continuity the function f defined by $f(x) = |x| + |x-1|$ at 0 and 1
5. Show that $f: R \to R$ defined by f(x) = 1 if x ∈ Q ; f(x) = -1 if x ∈ $R - Q$ is discontinuous for all x ∈ R .
6. Show that $f(x) = x\, sin(1/x), x \neq 0; f(x) = 0, x = 0$ is continuous but not derivable at $x = 0$.
7. By considering the integral $\int_0^1 \frac{1}{1+x} dx$ show that $log 2 = \lim_{n\to\infty} [\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}]$

PART - II

Answer any THREE questions. Each question carries TEN marks. 3 X 10 = 30 M

8. Show that a monotonic sequence is convergent iff it is bounded .
9. State and prove Cauchy's n^{th} root test .
10. Test for absolutely convergence or conditionally convergence of

    $\sum_{n=1}^{\infty} (-1)^{n+1} (\sqrt{n^2+1} - n)$.
11. State and Prove Intermediate value theorem.
12. State and Prove Rolle's theorem .
13. State and prove fundamental theorem of Integral Calculus.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–IV: REAL ANALYSIS

Short Answer Questions

UNIT-I

1. Show that every convergent sequence is bounded . Give an example to show the converse is not true .
2. By the definition of limit of a sequence show that lim $\sqrt[n]{n} = 1$.
3. Prove that lim $\left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right] = 0$
4. Show that $\lim_{n\to\infty}\left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \cdots + \sqrt{\frac{1}{n^2+n}}\right] = 1.$
5. State and prove sandwich theorem .
6. Define a Cauchy's sequence . Prove that every convergent sequence is a Cauchy sequence
7. Prove that if $\{S_n\}$ is a Cauchy sequence, then $\{S_n\}$ is bounded.
8. Prove that every Cauchy's sequence is convergent.

UNIT-II

9. If $\sum u_n$ convergence then show that lim u_n = 0 . Is the converse true ? Justify your answer .
10. Test for convergence of $\sum \frac{1}{2^n+3^n}$
11. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$.
12. Test for convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3-1}\,\right)$
13. Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^3}\left(\frac{n+2}{n+3}\right)^n x^n,\ \forall\, x > 0$
14. Test for the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n} x^{n-1}\ (x > 0)$.
15. Test for convergence of $\sum \frac{(-1)^{n-1}}{n\sqrt{n}}$.
16. An absolutely convergent series is always convergent. Is it Converse true? Justify your answer.
17. Test for absolute convergence and conditional convergence of $\sum(-1)^{n-1}(\sqrt{n^2+1} - n)$

UNIT-III

18. Examine the continuity of the function defined by f(x) = $|x|$ +$|x-1|$ at x = 0, 1.
19. Show that $f\colon R \to R$ defined by f(x) = 1 if x ∈ Q ; f(x) = -1 if x ∈ $R - Q$ is discontinuous for all x ∈ R .

20. Discuss the continuity of the following function at the origin $f(x) = x\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ $if\ x \neq 0$ and f(0)=1 .

UNIT-IV

21. If $f: [a, b] \to R$ is derivable at $c \in [a, b]$, then prove that f is continuous on $[a, b]$. Is its converse true? Justify your answer.
22. Test the continuity and differentiability of $f(x) = x\left(\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right)$ $if\ x \neq 0$ and $f(0) = 0$ at $x = 0$.
23. Prove that $f(x) = x\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ $if\ x \neq 0$ and f(0)=0 is continuous but not derivable .
24. Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in [a, b] where $0 < a < b$.
25. Verify Cauchy's mean value theorem for f(x) = x^2 , g(x) = x^3 in [1,2] .
26. Examine the applicability of Rolle's theorem for $f(x) = 1 - (x-1)^{2/3}$ on [0, 2].
27. Discuss the applicability of Rolle's theorem for the function $f(x) = log\frac{x^2+ab}{x(a+b)}$ in [a,b] where $0 \notin [a, b]$.
28. Find 'c' of Lagrange's Mean Value theorem for f(x) = (x-1) (x-2) (x-3) on [0,4].
29. Using Lagrange's Mean Value theorem prove that $1+x < e^x < 1+xe^x$, $\forall x > 0$

UNIT-V

30. If $f(x) = x^2\ \forall\ x \in [0,1]$ and P= {0,1/4 , 2/4, 3/4, 1}, then find $U(P,f)$ and $L(P,f)$.
31. Find the lower and upper Riemann sums of f(x) = 2x -1 on [0,1] when P= { 0, 1/3 , 2/3 , 1 } .
32. Show that the function f defined by $f(x) = \begin{cases} 1\ if & x\ \ is\ \ rational \\ -1 if & x\ \ is\ \ irrational \end{cases}$ for all x in [0,1] is not R-integrable on [0,1].
33. If $f \in R[a, b]$ and m , M are the infimum and supremum of f on [a,b] then prove that $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$.
34. If f is R-integrable on [a,b] then show that | f | is R-integrable on [a,b] .
35. Evaluate $\int_0^{\pi/4}(sec^4 x - tan^4 x)dx$
36. By considering the integral $\int_0^1 \frac{1}{1+x}dx$ show that $log2 = \lim_{n\to\infty}[\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}]$
37. Show that $\lim_{n\to\infty}[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{3n}] = log3$

Essay Questions

UNIT-I

1. State and prove Cauchy's first theorem on limits .
2. Show that a monotonic sequence is convergent iff it is bounded .
3. Prove that the sequence $\{s_n\}$ defined by $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ is convergent.
4. Show that the sequence $\{s_n\}$ where $s_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$ is convergent .
5. Show that the sequence $\{a_n\}$ $defined\ by\ a_n = \left(1 + \frac{1}{n}\right)^n$ is convergent.
6. If $S_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)}$ the show that $\{s_n\}$ is convergent .
7. State and prove Cauchy's general principle of convergence .

UNIT –II

8. State and prove Limit comparison test.
9. State and prove Cauchy's n^{th} root test .
10. State and prove D'Alemberts ratio test.
11. State and prove Leibnitz test.
12. Test for convergence of i) $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$ ii) $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$
13. Test for convergence of $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n})$.

UNIT-III

14. If a function f is continuous on [a,b] show that f is bounded on [a,b] . Is this result is true for functions on open intervals? Justify your answer .
15. State and Prove Intermediate value theorem.
16. If $f: [a, b] \to R$ is continuous on $[a, b]$, then f is bounded on $[a, b]$ and attains its bounds or infimum and supremum.
17. Test the continuity of of $f(x) = \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\ if\ x \neq 0\ and\ f(0) = 0$ at $x = 0$.
18. Let f: R→ R be such that f(x) = $\frac{\sin(a+1)x + \sin x}{x}$ for $x < 0, f(x) = c\ for\ x\ = 0$ and

    $f(x)$ =$\frac{(x+bx^2)-x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}$ for x >0. Determine the values of a, b, c which the function is continuous at x = 0.
19. Determine the constants a,b so that the function defined by f(x) = 2x + 1 if x ≤ 1 ;

    f(x) = ax^2 + b if 1< x < 3 ; f(x) = 5x + 2a if x ≥ 3 is continuous every where .

UNIT-IV

20. State and prove Roll's theorem .

21. State and Prove Lagrange's mean value theorem .

22. State and Prove Cauchy's mean value theorem.

23. Show that f(x) = | x – 1 | + | x – 2 | is continuous but not derivable at x= 1 , 2 .

24. Using Lagrange's mean value theorem, show that

$$x > log(1 + x) > \frac{x}{1 + x}\ if\ f(x) = log(1 + x)\ \forall x > 0$$

25. Show that $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$, $\forall x > 0$

26. Show that $\frac{v-u}{1+v^2} < tan^{-1}v - \ tan^{-1}u < \frac{v-u}{1+u^2}\ for\ 0 < u < v.$ Hence deduce that

$\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$

UNIT-V

27. Show that $f(x) = 3x + 1$ is integrable on[1,2] and $\int_1^2 (3x + 1)dx = \frac{11}{2}$.

28. Prove that $f(x) = x^2$ is integrable on $[0, a]$ and $\int_0^a x^2 dx = \frac{a^3}{3}$.

29. State and prove Necessary and Sufficient condition for integrability.

30. If f is continuous on the $[a, b]$ then prove that f is Riemann Integrable on the $[a, b]$.

31. If f is monotonic on the $[a, b]$ then prove that f is Riemann Integrable on the $[a, b]$.

32. State and prove fundamental theorem of Integral Calculus.

33. Prove that $\frac{\pi^3}{24} \le \int_0^\pi \frac{x^2}{5+3\ cosx}\ dx \le \frac{\pi^3}{6}$.

34. Show that $\frac{1}{\pi} \le \int_0^1 \frac{\sin \pi x}{1+x^2} dx \le \frac{2}{\pi}$

	P.R.Government College (Autonomous) KAKINADA	Program&Semester IIB.Sc. (IVSem)			
CourseCode **MAT-402/4225**	**TITLEOFTHECOURSE** **Linear Algebra**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Abstract Algebra.	5	1	-	5

Course Objectives:

This course will cover the analysis and implementation of algorithms used to solve linear algebra problems in practice. This course will enable students to acquire further skills in the techniques of linear algebra, as well as understanding of the principles underlying the subject.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the concepts of vector spaces, subspaces, basises, dimension and their properties.
CO2	Understand the concepts of linear transformations and their properties.
CO3	Apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods.
CO4	Learn the properties of inner product spaces and determine orthogonality in inner product spaces.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

Unit - I: Vector Spaces – I **(12 Hrs)**

Vector spaces, General properties of vector spaces, n-dimensional vectors, Addition and scalar multiplication of vectors, Internal and external composition, Null Space, Vector Subspaces, Algebra of subspaces, Linear sum of two subspaces, Linear combination of vectors, Linear span, Linear dependence and linear independence of Vectors.

Unit - II: Vector spaces – II **(12 Hrs)**

Basis of vector space, Finite dimensional vector space, Basis extension, Co-ordinates, Dimension of vector space, Dimension of subspace, Quotient space and Dimension of Quotient space.

Unit - III: Linear transformations **(12 Hrs)**

Linear transformations, Linear operators, Properties of linear transformation, Sum and product of linear transformations, Algebra of Linear Operators, Range space and Null Space of LT, Rank and Nullity of a LT, Rank & Nullity theorem.

Unit - IV: Matrix **(12 Hrs)**

Linear Equations, Characteristic Values and Characteristic Vectors of square matrix – Cayley - Hamilton Theorem.

Unit - V: Inner Product Space (**12 Hrs**)

Inner Product spaces, Euclidean and Unitary spaces, Norm or length of a vector, Schwartz's inequality, Triangle Inequality, Parallelogram law, Orthogonality and orthonormal set, Complete orthonormal set, Gram-Schmidt Orthogonalisation Process, Bessel's inequlity and Parsvel's identity.

Co-Curricular: Assignment, Seminar, Quiz, etc. **(15 Hrs)**

Additional Inputs: Diagonalization of a matrix.

Prescribed Text Books:

J.N. Sharma & A.R.Vasista, Linear Agebra, Krishna Prakasham Mandir, Meerut.

Books for Reference:

1. III year Mathematics Linear Algebra and Vector Calculus, Telugu Academy.
2. A Text Book of B.Sc. Mathematics, Vol-III, S. Chand & Co.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV PAPER-V

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**Vector Spaces – I**	2	1	20
II	**Vector Spaces – II**	2	1	20
III	**Linear transformations**	1	1	15
IV	**Matrix**	1	2	25
V	**Inner Product Space**	1	1	15
Total		7	6	95

S.A.Q. **=** Short answer questions (5 marks)

E.Q **=** Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 3 X 10 = 30

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Total Marks = 50

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P.R. Government College (Autonomous), Kakinada
II Year, B.Sc., Degree Examinations - IV Semester
Mathematics Course: Linear Algebra
Paper–V (Model Paper ((w.e.f. 2021-22 Admitted Batch)

Time: 2Hrs **Max. Marks: 50**

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PART-I

Answer any FOUR questions. Each question carries FIVE marks. 4 X 5 =20 M

1. Determine whether the set of vector {(1, -2, 1), (2, 1, -1), (7, -4, 1)} is linearly dependent or Linearly independent.

2. Let p, q, r be the fixed elements of a field F.Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a vector space of $V_3(F)$.

3. Show that the set $\{(1,0,0), (1,10), (1,1,1)\}$ is a basis of $C^3(C)$. Hence find the coordinates of the vector (3+4i, 6i, 3+7i) in $C^3(C)$.

4. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is also finite dimensional and $\dim W \leq \dim V$.

5. Find T(x, y, z) where T: $R^3 \rightarrow R$ is defined by T(1,1,1)=3, T(0,1,-2)=1, T(0,0,1)= -2.

6. Solve the following system of linear equations

$2x - 3y + z = 0,\ x + 2y - 3z = 0,\ 4x - y - 2z = 0.$

7. State and prove Parallelogram Law

PART - II

Answer any THREE questions. Each question carries TEN marks. 3 X 10 = 30 M

9. Prove that a non empty subset W of a vector space $V(F)$ is a subspace of V if and only if $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.

10. Let W be a sub space of a finite dimensional vector space V(F), then prove that

$\dim(\frac{V}{W})$ = dimV – dimW.

11. State and prove rank and nullity theorem.

12. State and prove Cayley- Hamilton theorem.

13. Find the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$

14. Apply the Gram-Schmidt process to the vectors β_1 = (2, 1 , 3) , β_2 = (1, 2, 3),

β_3 = (1, 1, 1) to obtain an orthonormal basis for $V_3(R)$ with the standard product .

P.R. GOVERNMENT COLLEGE (A), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–V: LINEAR ALGEBRA

Short answers

UNIT-I

1. Prove that the intersection of any two subspaces W_1 and W_2 of vector space V(F) is subspace of V(F).

2. Prove that the union of two subspaces of V(F) need not be a subspace of V(F) .

3. The linear span L(S) of any subset S of a vector space V(F) is a subspace of V(F).

4. Express the vector α = (1, -2, 5) as alinear combination of the vectors e_1=(1, 1, 1) , e_2=(1, 2 , 3) and e_3=(2, -1, 1) .

5. Show that the vector α=(2, -5, 3) in R^3 cannot be expressed as a linear combination of the vectors e_1=(1, -3, 2) , e_2=(2, -4 , -1) and e_3=(1, -5, 7) .

6. Every superset of a linearly dependent set of vectors is linearly dependent .

7. Every non-empty subset of a linearly independent set of vectors is linearly independent .

8. If two vectors are linearly dependent , prove that one of them is a scalar multiple of the other .

9. If α, β, γ are linearly independent vectors of $V(R)$, then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also linearly independent.

UNIT-II

10. Show that the set of vectors {(1, 2, 1), (2, 1, 0), (1, -1, 2)} form a basis for R^3.

11. Show that the set S={(1, 0, 0), (0, 1, 0), (1, 1, 0), (1, 1, 1)} is a spanning set of $R^3(R)$, but not a basis.

12. Show that the vectors α_1 =(1, 1, 1), α_2 = (-1, 1, 0), α_3 = (1, 0, -1) form a basis of R^3 and express (4, 5, 6) in terms of α_1, α_2 , α_3 .

13. Find a basis for the subspace spanned by the vectors (1, 2, 0), (-1, 0, 1), (0, 2, 1) in V_3(R) .

14. W= { (x, y, 0, 0) : x, y $\in$ R } .Write a basis for the quotient space R^4 / W .

15. If W is a subspace of a finite dimensional vector space $V(F)$ then prove that W is also finite dimensional and $\dim W \le \dim V$.

UNIT-III

16. Define linear transformation and show that the function $T: R^3 \to R^3$ defined by $T(x, y, z) = (x - y, 0, y + z)$ is a linear transformation.
17. Find a linear transformation $T: R^3 \to R$ such that $T(1,1,1) = 3, T(0,1,-2) = 1$, $T(0,0,1) = -2$.
18. Find a linear transformation $T: U \to V$ be such that whose basis and range are $\{(1,2,1), (2,1,0), (1,-1,-2)\}\ and\ \{(1,0,0), (0,1,0), (1,1,1)\}$.
19. Let $T: R^3 \to R^3$ be defined by $T(x, y, z) = (x - y + 2z,\ 2x + y - z,\ -x - 2y)$. Then verify Rank-nullity theorem.
20. Describe explicitly the linear transformation $T: R^3 \to R^3$ whose range space is spanned by {(1, 0, -1), (1, 2, 2)}.
21. Let $U(F)$ and $V(F)$ be two vector spaces such that T: U(F) → V(F) be a linear transformation. Then define range set of T and prove that the range set $R(T)$ is a subspace of $V(F)$.

UNIT-IV

22. Solve the system of linear equations x + y + z = 6 , x +2y + 3z = 14 , x + 4y + 7z = 30
23. Is it the system of equations $x - 4y + 7z = 14, 3x + 8y - 2z = 13,\ 7x - 8y + 26z = 5$ are consistent.
24. Show that the system of equations x – 4y + 7z = 14 , 3x + 8y – 2z = 13 , 7x – 8y + 26z = 5 are inconsistent .
25. Prove that the matrices A and A^T have the same eigen values .
26. If λ_1 , λ_2 , … λ_n are the characteristic roots of A then prove that λ_1^2 , λ_2^2 , … λ_n^2 are characteristic roots of A^2 .
27. Prove that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent .
28. Find the eigen values and eigen vectors of the square matrix A= $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 5 \\ 0 & 0 & 4 \end{pmatrix}$.
29. Verify Cayley-Hamilton theorem for the matrix A= $\begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

30. Let V(F) be an inner product space and $\alpha \in V$, a $\in F$ then prove that

    i) ‖ α ‖ ≥ 0 and ‖ α ‖ = 0 iff α = Ō   ii) ‖ aα ‖ = |a| ‖ α ‖ .

31. Find the unit vector corresponding to (2 – I , 3 + 2i , 2 + √3 i) of $V_3(C)$ with respect to the standard inner product .

32. State and Prove Triangle-Inequality.

33. State and prove Parallelogram law in an inner product space V(F).

34. If α , β are two vectors in an inner product space V(F) and $a , b \in F$ then prove that Re (α , β) = (1/4) $\| \alpha + \beta \|^2$ – (1/4) $\| \alpha - \beta \|^2$.

35. State and prove Parseval's identity in an inner product space V(F).

36. Prove that every orthogonal set of non-zero vectors in an Inner Product Space V(F) is linearly independent.

37. Prove that every orthonormal set of vectors is linearly independent.

38. Prove that the set S = $\left\{\left(\frac{1}{3},\frac{-2}{3},\frac{-2}{3}\right),\left(\frac{2}{3},\frac{-1}{3},\frac{2}{3}\right),\left(\frac{2}{3},\frac{2}{3},\frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

Essay questions
UNIT-I

1. Let V(F) be a vector space and $W \subseteq V$. Then prove that the necessary and sufficient conditions for W to be a subspace of V are

   (i) $\alpha \in W,\ \beta \in W \Longrightarrow \alpha - \beta \in W\ \ (ii) a \in F,\ \alpha \in W \Longrightarrow a\alpha \in W$

2. Prove that the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V is that $a , b \in F,\ \alpha , \beta \in W \Longrightarrow a\alpha + b\beta \in W$.

3. Prove that the union of two subspaces of a vector space is a subspace if and only if one is contained in the other.

4. If S is a subset of a vector space V(F) then prove that (i) S is a subspace of V iff L(S)=S (ii) L (L(S)) = L(S) .

5. If W_1 and W_2 are two subspaces of a vector space V(F) then prove that

   (i) $W_1 + W_2$ is a subspace of $V(F)$ and (ii) $W_1 \subseteq W_1 + W_2\ \ and\ \ W_2 \subseteq W_1 + W_2$.

6. If S and T are the subsets of a vector space $V(F)$ then prove that

   (i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ and (ii) $L(S \cup T) = L(S) + L(T)$.

7. Let $V(F)$ be a vector space and $S = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ is a finite subset of non-zero vectors of $V(F)$. Then prove that S is linearly dependent if and only if some vector $\alpha_k \in S, 2 \leq k \leq n$ can be expressed as a linear combination of its preceding vectors.

UNTI-II

8. Let W_1 and W_2 be two subspaces of a finite dimensional vector space V(F). Then prove that $dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

9. Prove that every finite dimensional vector space has a basis .

10. Prove that every linearly independent subset of a finite dimensional vector space V(F) is either a basis of V or can be extended to form a basis of V .

11. If V(F) is a finite dimensional vector space , the prove that any two basis of V have the same number of elements .

12. Show that the set { (1, 0, 0), (1, 1, 0), (1, 1, 1) } is a basis of $C^3(C)$. Hence find the coordinates of the vectors { 3+4i , 6i , 3+7i } in $C^3(C)$.

13. If W_1 and W_2 are the subspaces of $V_4(R)$ defined by W_1 = { (a, b, c, d) / b – 2c + d = 0 } , W_2 = { (a, b, c, d) / a = d , b = 2c } . Compute $\dim W_1$, $\dim W_2$, $\dim(W_1 \cap W_2)$ and

    $\dim(W_1 + W_2)$.

14. Let W be a subspace of a finite dimensional vector space V(F) then prove that

    $\dim(V/W) = \dim\ V - \dim W.$

UNIT-III

15. Let $L(U, V)$ be the vector space of all linear transformations from $U(F)$ to $V(F)$ such that $dim\ U = n$ and $dim\ V = m$, then prove that $dim\ L(U, V) = mn$.

16. Let $U(F)$ and $V(F)$ be two vector spaces and $T: U \to V$ is a linear transformation. Then prove that the range space $R(T)$ is a subspace of $V(F)$ and null space $N(T)$ is a subspace of $U(F)$.

17. State and prove Rank - Nullity theorem.

18. Show that the mapping $T : R^3 \to R^3$ defined by T (x , y , z) = (x + 2y – z , y + z , x + y – 2z) is a linear transformation . Find rank , nullity and verify rank T + nullity T = dim R^3 .

19. Show that the mapping $T : V_2(R) \to V_3(R)$ defined as T (a , b) = (a + b , a – b , b) is a linear transformation from $V_2(R)$ into $V_3(R)$. Find the range , rank , null space and nullity of T .

20. Let $T: R^3 \to R^3$ be defined by $T(x, y, z) = (x - y + 2z,\ 2x + y - z,\ -x - 2y)$. Then verify Rank-nullity theorem.

UNIT-IV

21. For what values of λ , the equation x + y + z = 1 , x + 2y + 4z = λ , x + 4y + 10z = λ^2 have solution ? Solve them completely in each case .

22. Investigate for what values of λ , μ the simultaneous equations x + y + z = 6 , x = 2y + 3z = 10 , x + 2y + λz = μ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions .

23. Find the eigen roots and the corresponding vectors of the square matrix

$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

24. Find the characteristic roots and characteristic vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}.$$

25. State and prove Cayley-Hamilton theorem.

26. Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and verify that it is satisfied by A and hence find A^{-1} .

27. Verify Cayley -Hamilton theorem for the matrix $A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{pmatrix}$.

UNIT-V

28. State and prove Cauchy- Schwarz's inequality.

29. If α , β are vectors in an inner product space V(F) then prove that $|<\alpha,\beta>| = \|\alpha\|\,\|\beta\|$ iff α , β are linearly dependent .

30. Let $\alpha = (2, 1+i, i)$, $\beta = (2-i, 2, 1+2i)$ be two vectors in $V_3(C)$. Compute $<\alpha, \beta>$, $\|\alpha\|$,

$\|\beta\|$, $\|\alpha+\beta\|$ and verify triangle inequality .

31. State and prove Bessel's inequality .

32. If u and v are two vectors in a complex inner product space V(F), then prove that
$4\langle u, v\rangle = \|u+v\|^2 - \|u-v\|^2 + i\|u+iv\|^2 - \|u-iv\|^2$.

33. Apply the Gram-Schmidt process to the vectors { (2, 1 , 3) , (1, 2, 3), (1, 1, 1) } to obtain an orthonormal basis for $V_3(R)$ with the standard product .

34. Apply the Gram-Schmidt process to the vectors $\beta_1 = (2, 1, 3)$, $\beta_2 = (1, 2, 3)$, $\beta_3 = (1, 1, 1)$ to obtain an orthonormal basis for $V_3(R)$ with the standard product .

<table>
<tr><td></td><td colspan="1">P.R.Government College (Autonomous)
KAKINADA</td><td colspan="4" rowspan="2">Program&Semester
IIIB.Sc. (VSem)</td></tr>
<tr><td>CourseCode
MAT-601A / 5231</td><td>TITLEOFTHECOURSE
6A- Numerical Methods</td></tr>
<tr><td>Teaching</td><td>HoursAllocated:60(Theory)</td><td>L</td><td>T</td><td>P</td><td>C</td></tr>
<tr><td>Pre-requisites:</td><td>Basic Mathematics Knowledge on theory of equations</td><td>5</td><td>1</td><td>-</td><td>5</td></tr>
</table>

Course Objectives:

This course will cover the classical fundamental topics in numerical methods such as, approximation, numerical integration, numerical linear algebra, solution of nonlinear algebraic systems and solution of ordinary differential equations.

Course Outcomes:

On Completion of the course, the students will be able o-	
CO1	Understand various finite difference concepts and interpolation methods.
CO2	Work out numerical differentiation and integration whenever and wherever routine methods are not applicable.
CO3	Find numerical solutions of ordinary differential equations by using various numerical methods.
CO4	Analyze and evaluate the accuracy of numerical methods.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

Unit – 1: Finite Differences and Interpolation with Equal intervals (15h)

1. Introduction, Forward differences, Backward differences, Central Differences, Symbolic relations, nth Differences of Some functions,

2. Advancing Difference formula, Differences of a Polynomial.

3. Newton's formulae for interpolation. Central Difference Interpolation Formulae.

Unit – 2: Interpolation with Equal and Unequal intervals (15h)

1. Gauss's Forward interpolation formulae, Gauss's backward interpolation formulae, Stirling's formula, Bessel's formula.

2. Interpolation with unevenly spaced points, divided differences and properties, Newton's divided differences formula.

3. Lagrange's interpolation formula, Lagrange's Inverse interpolation formula.

Unit – 3: Numerical Differentiation **(15h)**

1. Derivatives using Newton's forward difference formula, Newton's back ward difference formula,

2. Derivatives using central difference formula, Stirling's interpolation formula,

3. Newton's divided difference formula, Maximum and minimum values of a tabulated function.

Unit – 4: Numerical Integration **(15h)**

1. General quadrature formula one errors, Trapezoidal rule,

2. Simpson's1/3– rule, Simpson's 3/8 – rule and Weddle's rules,

3. Euler – McLaurin Formula of summation and quadrature, The Euler transformation.

Unit – 5: Numerical solution of ordinary differential equations **(15h)**

1. Introduction, Solution by Taylor's Series,

2. Picard's method of successive approximations,

3. Euler's method, Modified Euler's method, Runge – Kutta methods.

III. References:

1. S.S.Sastry, Introductory Methods of Numerical Analysis, Prentice Hall of India Pvt. Ltd., New Delhi-110001, 2006.

2. P.Kandasamy, K.Thilagavathy, Calculus of Finite Differences and Numerical Analysis. S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.

3. R.Gupta, Numerical Analysis, Laxmi Publications (P) Ltd., New Delhi.

4. H.C Saxena, Finite Differences and Numerical Analysis, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

5. S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr.V.Ramesh Babu, Numerical Analysis, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities: A) Mandatory:

1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Applications of Newton's forward and back ward difference formulae.

2. Applications of Gauss forward and Gauss back ward, Stirling's and Bessel's formulae.

3. Applications of Newton's divided differences formula and Lagrange's interpolation formula.

4. Various methods to find the approximation of a definite integral.

5. Different methods to find solutions of Ordinary Differential Equations.

2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work done in the areas like the following, by choosing any one of the aspects.

1. Collecting the data from the identified sources like Census department or Electricity department, by applying the Newton's, Gauss and Lagrange's interpolation formula, making observations and drawing conclusions. (Or)

2. Selection of some region to find the area by applying Trapezoidal rule, Simpson's1/3– rule, Simpson's 3/8 – rule, and Weddle's rules. Comparing the solutions with analytical solution and concluding which one is the best method. (Or)

3.Findingsolutionof the ODE by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge–Kutta methods. Comparing the solutions with analytical solution, selecting the best method.

3. Max. Marks for Fieldwork/Project work Report: 05.

4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area .

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-V PAPER-VI A

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**Finite Differences and Interpolation with Equal intervals**	2	2	26
II	**Interpolation with Equal and Unequal intervals**	2	2	26
III	**Numerical Differentiation**	2	2	26
IV	**Numerical Integration**	1	2	21
V	**Numerical solution of ordinary differential equations**	1	2	21
Total		8	10	120

S.A.Q. **=** Short answer questions (5 marks)
E.Q **=** Essay questions (8 marks)

Short answer questions : 4 X 5 = 20
Essay questions : 5 X 8 = 40
……………….
Total Marks = 60
………………..

P.R. Government College (Autonomous), Kakinada
III year B.Sc. Degree Examinations : V Semester - Mathematics
Skill Enhancement Course (Elective) : Numerical Methods
Paper VI A: MODEL PAPER (w.e.f. 2020-21)

Time: 2 Hrs 30 Min **Max. Marks : 60 M**

PART – I

Answer any FOUR of the following questions. Each question carries 5 marks.

4 x 5 = 20 M

1. Given $y_0 = 3$, $y_1 = 12$, $y_3 = 200$, $y_4 = 100$. Find $\Delta^4 y_0$ without forming difference table .

2. Find the missing term in the following data.

x	0	1	2	3	4
y	1	3	9	__	81

3. Given that $\sqrt{12500} = 111.803399,\ \sqrt{12510} = 111.848111,\ \sqrt{12520} = 111.892806, \sqrt{12530} = 1.937483.$ *Show by gauss backward formula that* $\sqrt{12516} = 111.874930$

4. Show that $f(x_0, x_1, x_2, \ldots, x_n) = \frac{\Delta^n f(x_0)}{n! h^n}$.

5. Find $f^1(1)$ for $f(x) = \frac{1}{1+x^2}$ using the following table .

X	1.0	1.1	1.2	1.3	1.4
Y	0.5000	0.4524	0.4098	0.3717	0.3378

6. Find $f^1(2.5)$ from the following table .

X	1.5	1.9	2.5	3.2	4.3	5.9
Y	3.375	6.059	13.625	29.368	73.907	196.57

7. Evaluate $\int_0^1 (4x - 3x^2)dx$ taking 10 intervals byn trapezoidal rule .

8. Using Taylor's series method, find $y(0.1)$ correct to four decimal places if $y' = x - y^2$ and $y(0) = 1$.

SECTION – B

Answer ALL questions. Each question carries Eight marks. **5 X 8 = 40 M**

9. a) State and prove Newton – Gregory formula for forward interpolation with equal intervals .

(OR)

b) From the following table , find the number of students who obtain less than 45 marks .

Marks	30-40	40-50	50-60	60-70	70-80
No.of students	31	42	51	35	31

10. a) Apply Gauss forward formula to find the value of u_9 if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{16} = 40$.

(OR)

b) State and prove Newton's divided difference formula .

11. a) Find $f^1(0.6)$ and $f^{11}(0.6)$ from the following table .

x	0.4	0.5	0.6	0.7	0.8
f(x)	1.5836	1.7974	2.0442	2.3275	2.6510

(OR)

b) Find the maximum and the minimum values of the function y = f(x) from the following data .

x	0	1	2	3	4	5
f(x)	0	0.25	0	2.25	16	56.25

12.a) Find the value of the integral Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}$ th - rule taking $h = \frac{1}{6}$. Hence obtain an approximate value of π.

(OR)

b) Evaluate the integral $\int_4^{5.2} \log x \ dx$ using Weddle's rule.

13. a) Find the value of y at $x = 0.1$ by Picard's method, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$.

(OR)

b) Given $\frac{dy}{dx} = y - x$ with y(0) = 2 , find y(0.1) and y(0.2) correct to four decimal places by using Runge – Kutta method .

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
Question Bank for
PAPER–VI A: NUMERICAL METHODS

Short Answer Questions

Unit-I

1. Prove that (i) $\Delta f(x)g(x)$, (ii) $\Delta\frac{f(x)}{g(x)}$ and (iii) $E = e^{hD}$.
2. Prove that i) $\Delta = E - 1$ ii) $\nabla = 1 - E^{-1}$
3. Prove that i) $(1 + \Delta)(1 - \nabla) = 1$ ii) $E\nabla = \Delta$ iii) $\Delta - \nabla = \Delta\nabla$
4. Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$, (ii) $\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$
5. Prove that $\Delta log f(x) = \log(1 + \frac{\Delta f(x)}{f(x)})$
6. Evaluate $(\frac{\Delta^2}{E})x^3$, the interval of differencing being unity .
7. Prove that $e^x = \left(\frac{\Delta^2}{E}\right)e^x.\frac{Ee^x}{\Delta^2 e^x}$, the interval of differencing being unity .
8. Given $y_0 = 3$, $y_1 = 12$, $y_3 = 81$, $y_4 = 100$. Find $\Delta^4 y_0$ without forming difference table .
9. Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$, $u_5 = 8$; find $\Delta^5 u_0$.
10. Prove that i) $u_3 = u_2 + \Delta u_1 + \Delta^2 u_0 + \Delta^3 u_0$ and ii) $u_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$.
11. Evaluate $\Delta^3(1 - x)(1 - 2x)(1 - 3x)$, the interval differencing being unity .
12. Find f(2) if f(-1) = 2 , f(0) = 1 , f(1) = 0 and f(3) = -1 .
13. Find the missing term in the following data.

x	1	2	3	4	5	6	7
y	2	4	8	---	32	64	128

14. Find the missing term in the following data.

x	0	1	2	3	4
y	1	3	9	__	81

15. Find the cubic polynomial which takes the following values .

x	0	1	2	3
f(x)	0	2	1	10

16. Compute f(1.1) from the following table .

x	1	2	3	4	5
f(x)	7	12	29	64	123

Unit- II

17. Use Gauss forward forward formula to find a polynomial of degree four or less which takes the following values of the formula f(x).

x	1	2	3	4	5
f(x)	1	-1	1	-1	1

18. Given that

x	50	51	52	53	54
Tan x	1.1918	1.2349	1.2799	1.3270	1.3764

Using Gauss's backward formula , find the value of tan $51^0 42^1$.

19. Show that $f(x_0, x_1, x_2, \ldots, x_n) = \frac{\Delta^n f(x_0)}{n! h^n}$.

20. Construct a divided difference table for the following .

x	1	2	4	7	12
f(x)	22	30	82	106	216

21. Find the third divided difference with arguments 2 , 4 , 9 , 10 of the function $f(x) = x^3 - 2x$.

22. Derive Lagrange's interpolation formula.

23. By Lagrange's interpolation formula , find the value of y at x = 5 , given that

x	1	3	4	8	10
f(x)	8	15	19	32	40

24. By Lagrange's interpolation formula , find the form of the function given by

x	0	1	2	3	4
f(x)	3	6	11	18	27

UNIT - III

25. Find the first order derivative of $\sqrt{x}$ at x = 15 from the following .

x	15	17	19	21	23	25
f(x)	3.8773	4.123	4.359	4.583	4.796	5.000

26. Find $f^1(1)$ for $f(x) = \frac{1}{1+x^2}$ using the following table .

x	1.0	1.1	1.2	1.3	1.4

f(x)	0.5000	0.4524	0.4098	0.3717	0.3378

27. Find $f^1(1.5)$ from the following table .

x	0.0	0.5	1.0	1.5	2.0
f(x)	0.3989	0.3521	0.2420	0.1245	0.0540

28. Find $f^1(5)$ from the following table .

x	1	2	4	8	10
f(x)	0	1	5	21	27

29. Assuming Stirling's formula , show that

$$\frac{d}{dx}[f(x)] = \frac{2}{3}[\,f(x+1) - f(x-1)] - \frac{1}{12}[\,f(x+2) - f(x-2)]$$

upto third difference .

UNIT - IV

30. Evaluate $\int_1^1 (4x - 3x^2)\,dx$ taking 10 intervals by Trapezoidal rule .

31. Evaluate $\int_0^{\Pi} t \sin t \;\; dt$ by using Trapezoidal rule .

32. Calculate the approximate value of $\int_{-3}^{3} x^3 dx$ by using Trapezoidal Rule .

33. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ correct to three decimal places by Trapezoidal rule with h = 0.25 .

34. Evaluate the integral $\int_1^2 \sqrt{(1 - \frac{1}{x}}) \, dx$ by Simpson's 1/3 rule with five ordinates .

35. Using Simpson's 1/3 rule to prove that log 7 is approximately 1.9587 using $\int_1^7 \frac{dx}{x}$.

36. The velocities of a car at intervals of 2 minutes are given below . Apply Simpson's rule to find distance covered by the car .

Time(in minutes)	0	2	4	6	8	10	12
Velocity(Km/hr)	0	22	30	27	18	7	0

37. valuate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's Rule .

UNIT - V

38. Solve the differential equations $\frac{dy}{dx} = x + y$, with y(0) = 1 , $x \in [0,1]$ by Taylor series expansion to obtain y for x = 0.1 .

39. Given the differential equation $y'' - xy' - y = 0$ with the conditions y(0)=1 , $y^1(0)=0$, use Taylor's series method to determine the value of y(0.1) .

40. Solve $\frac{dy}{dx} = 1 + y^2 \, , y(0) = 0$ by Picard's method .

41. Given $\frac{dy}{dx} = y + x^3$, $y(0) = 1$, compute y(0.02) by Euler's method taking h = 0.01 .

42. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y = 1 when x = 0 , find y for x = 0.1 in 4 steps by Euler's method .

43. Solve $\frac{dy}{dx} = -2xy^2$ with y(0) =1 . find y(0.1) using Runge-Kutta method .

Essay Questions

Unit-I

1. Show that $\Delta^n \cos(ax + b) = (2sin\frac{ah}{2})^n \cos[a + bx + n\left(\frac{ah+\pi}{2}\right)]$
2. State and prove fundamental theorem of difference calculus .
3. State and prove Newton's – Gregory formula for forward interpolation with equal intervals .
4. The population of a country in the decennial cences were as under . Estimate the population for the year 1895 .

Year(x)	1891	1901	1911	1921	1931
Population(y) (in thousands)	46	66	81	93	101

5. The area of a circle of diameter d is given for the following values , find the approximate value for the area of a circle of diameter 82 .

d(Diameter)	80	85	90	95	100
A(Area)	5026	5674	6362	7088	7854

6. Find the value of $\sin 52^0$ from the given table .

θ	45^0	50^0	55^0	60^0
$\sin\theta$	0.7071	0.7660	0.8192	0.8660

7. From the following table , find the number of students who obtain less than 56 marks .

Marks	30-40	40-50	50-60	60-70	70-80
No.of students	31	42	51	35	31

8. State and prove Newton's – Gregory backward interpolation formula with equal intervals .
9. The population of a country in the decennial census were as under . Estimate the population for the year 1925 .

Year(x)	1891	1901	1911	1921	1931
Population(y) (in thousands)	46	66	81	93	101

10. From the following table find y value at x = 0.26

x	0.10	0.15	0.20	0.25	0.30
y = Tanx	0.1003	0.1511	0.2027	0.2553	0.3093

11. Given

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

Find f(7.5)

UNIT - II

12. State and prove Gauss forward interpolation formula .
13. Using Gauss forward formula find u_{32} from the given data u_{20} = 14.035 , u_{25} = 13.674 , u_{30} = 13.257 , u_{35} = 12.734 , u_{40} = 12.089 , u_{45} = 11.309 .
14. Apply Gauss forward formula to find the value of u_9 if u_0 = 14 , u_4 = 24 , u_8 = 32 , u_{16} = 40 .
15. State and prove Gauss backward interpolation formula .
16. Interpolate by means of Gauss backward interpolation formula the sales for the concern for the year 1936, given that

$year$	1901	1911	1921	1931	1941	1951
$sales(in\ thousands)$	12	15	20	27	39	52

17. Given that $\sqrt{12500} = 111.803399$, $\sqrt{12510} = 111.848111$, $\sqrt{12520} = 111.892806$, $\sqrt{12530} = 111.937483$, show $\sqrt{12516} = 111.8749301$ by using Gauss backward interpolation formula.
18. Given that

x	50	51	52	53	54
Tanx	1.1918	10.2349	1.2799	1.3270	1.376

19. State and prove Stirling's formula .
20. Apply Stirling's formula to find y_{28} given that y_{20}=49225, $y_{25} = 48316$, $y_{30} = 47236$, $y_{35} = 45926$, $y_{40} = 44300$.
21. Given y_{20} = 24 , y_{24} = 32 , y_{28} = 35 , y_{32} = 40 , find y_{25} by Bessel's formula .
22. State and prove Netown's divided difference formula .
23. By means of Newton's divided difference formula, find the values of $f(8), f(15)$ from the following table.

x	4	5	7	10	11	13

$f(x)$	48	100	294	900	1210	2028

24. Using the Newton's divided difference formula , find a polynomial function satisfying the following data .

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

25. Using Lagrange's interpolation formula find y at x = 301 .

X	300	304	305	307
Y	2.4771	2.4829	2.4843	2.4871

26. Using Lagrange's interpolation formula , prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}\,(y_{-1} - y_{-3})]$$

27. Apply Lagrange's formula inversely to find , to one decimal place the value of x when y = f(x) = 13.6 given the following table .

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.5

UNIT - III

28. Using the following table , compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1 .

x	1	2	3	4	5	6
y	1	8	27	64	125	216

29. Using the following table , compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.2 .

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

30. Find $f^1(1.1)$ and $f^{11}(1.1)$ from the following table .

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	0	0.1280	0.5440	1.2960	2.4320	4.0000

31. Using the following table , compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 2.2 .

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
f(x)	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

32. From the following table , find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 2.03 .

x	1.96	1.9	2.00	2.02	2.04
f(x)	0.78.25	0.7739	0.7651	0.7563	0.7473

33. Find $f^1(0.6)$ and $f^{11}(0.6)$ from the following table .

x	0.4	0.5	0.6	0.7	0.8
f(x)	1.5836	1.7974	2.0442	2.3275	2.6510

34. Find $f^1(2.5)$ from the following table .

x	1.5	1.9	2.5	3.2	4.3	5.9
f(x)	3.375	6.059	13.625	29.368	73.907	196.579

35. Find the maximum and minimum values of the function y = f(x) from the following table .

x	0	1	2	3	4	5
f(x)	0	0.25	0	2.25	16	56.25

UNIT - IV

36. Calculate an approximate value of integral $\int_0^{\frac{\pi}{2}} sinx\, dx$ by Trapezoidal rule .

37. State and prove Simpson's 1/3 rule .

38. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3 rule .

39. Evaluate the integral $\int_1^2 \frac{1}{x} dx$ by Simpson's 1/3 rule with 4 strips and 8 strips respectively . Determine the error by direct integration .

40. Using Simpson's one-third rule , find $\int_0^6 \frac{dx}{(1+x)^2}$.

41. State and prove Simpson's 3/8 rule .

42. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 1/3 and 3/8 rule . Hence obtain the approximate value of π in each case .

43. Evaluate the integral $\int_0^{1.5} \frac{x^3}{e^x-1}\, dx$ by Weddle's rule .

44. Integrate numerically $\int_4^{5.2} logx\, dx$ by Weddle's rule .

45. Evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{sinx}\, dx$ by Weddle's rule .

UNIT - V

46. Using the Taylor's series for y(x) , find y(0.1) correct to four decimal places if y(x) satisfies $y^1 = x - y^2$, $y_0 = 1$ where $x_0 = 0$.
47. Solve $y^1 = y - x^2$, y(0) = 1 by Picard's method upto the fourth approximation . Hence find the value of y(0.1) and y(0.2) .
48. Solve $\frac{dy}{dx} = y \,, y(0) = 1$ by Picard's method and compare the solution with the exact solution .
49. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y = 1 when x = 0 . Find approximately the value of y for x = 0.1 by Picard's method .
50. Determine the value of y when x = 0.1 given that y(0) = 1 and $y^1 = x^2 + y$ by Euler's modified method .
51. Find y(0.2) by using Euler's modified method for $\frac{dy}{dx} = log_{10}(x + y)$ with initial condition y = 1 for x = 0 .
52. Compute y(1.2) by using Euler's modified method upto 4 decimals when $\frac{dy}{dx} = 2 + \sqrt{xy}$ and y(1) = 1 .
53. Solve $\frac{dy}{dx} = xy$ using Runge – Kutta method for x = 0.2 given that y(0) = 1 taking h = 0.2 .
54. Given $\frac{dy}{dx} = y - x$ with y(0) = 2 find y(0.1) and y(0.2) correct to four decimal place by R – K method .
55. Apply R-K fourth order find the solution of the differential equation $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y_0 = 1$ at x = 0.1 .

<table>
<tr><td></td><td colspan="1">P.R.Government College (Autonomous)
KAKINADA</td><td colspan="4" rowspan="2">Program&Semester
IIIB.Sc. (VSem)</td></tr>
<tr><td>CourseCode
MAT-701A / 5281</td><td>TITLEOFTHECOURSE
7A -Mathematical Special Functions</td></tr>
<tr><td>Teaching</td><td>HoursAllocated:60(Theory)</td><td>L</td><td>T</td><td>P</td><td>C</td></tr>
<tr><td>Pre-requisites:</td><td>BasicMathematicsKnowledge on Integration</td><td>5</td><td>1</td><td>-</td><td>5</td></tr>
</table>

Course Objectives:

This course will cover the particular mathematical functions that have more or less established names and notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations.
CO2	Find power series solutions of ordinary differential equations.
CO3	Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials.
CO4	Solve Bessel equation and write the Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel unction.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

II. Syllabus: (Hours: Teaching: 75 (incl. unit tests etc. 05), Training: 15)

Unit – 1: Beta and Gamma functions. (15h)

1. Euler's Integrals-Bet and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.

2. Another form of Beta Function, Relation between Beta and Gamma Functions.

Unit– 2: Power series and Power series solutions of ordinary differential equations (15h)

1. Introduction, summary of useful results, power series, radius of convergence, theorems on Power series

2. Introduction of power series solutions of ordinary differential equation

3. Ordinary and singular points, regular and irregular singular points, power series solution about the ordinary point $x = x_0$.

Unit – 3: Hermite polynomials (15h)

1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials.

2. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials.

3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

Unit – 4: Legendre polynomials (15h)

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials. ,

2. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required)to show that $P_n(x)$ is the coefficient of h^n , in the expansion of $(1 - 2xh + h^2)^{-1/2}$

3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

Unit – 5: Bessel's equation (15h)

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.

2. Integration of Bessel's equation in series form=0, Definitionof $J_n(x)$, recurrence formulae for $J_n(x)$.

3. Generating function for $J_n(x)$, orthogonally of Bessel functions.

Additional Inputs :

Chebyshev Polynomiaals

II. Reference Books:

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

2. J.N.Sharma and Dr.R.K.Gupta, Differential equations with special functions, Krishna Prakashan Mandir. 3. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

4. George F.Simmons, Differential Equations with Applications and Historical Notes, Tata McGRAW-Hill Edition, 1994.

5. Shepley L.Ross, Differential equations, Second Edition, John Willy & sons, New York, 1974.

6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory: 1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Beta and Gamma functions.
2. Power series, power series solutions of ordinary differential equations,
3. Procedures of finding series solutions of Hermite equation, Legendre equation and Bessel equation.
4. Procedures of finding generating functions for Hermite polynomials, Legendre Polynomials and Bessel's function.

2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work, make observations and conclusions and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources on the properties of Beta and Gamma functions, Chebyshev polynomials, power series solutions of ordinary differential equations. (or)

2. Going through the web sources like Open Educational Resources on the properties of series solutions of Hermite equation, Legendre equation and Bessel equation.

3. Max. Marks for Fieldwork/Project work Report: 05.

4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.

3. Invited lectures and presentations on related topics by experts in the specified area.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VII - A

Paper – VII –A :Mathematical Special Functions

UNIT	TOPIC	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted

I	Beta and Gamma functions	02	02	26
II	Power series and Power series solutions of ordinary differential equations	02	02	26
III	Hermite polynomials	02	02	26
IV	Legendre polynomials	01	02	21
V	Bessel's equation	01	02	21
Total		08	10	120

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA
III B.SC MATHEMATICS – Semester V (w.e.f. 2020-2021)
Skill Enhancement Course (Elective) :VII - A Mathematical Special Functions
Paper – VII – A : MODEL PAPER

Time: 2 hrs 30 Min **Max. Marks : 60 M**

PART – I

Answer any FOUR of the following questions. Each question carries 5 marks. 4 x 5 = 20M

1. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}}dy$ *and hence show that* $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
2. Prove that $\int_0^1 x^m(\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$

3. Find the radius of convergence of the series $\frac{x}{2}+\frac{1.3}{2.5}x^2+\frac{1.3.5}{2.5.8}x^3+\cdots$

4. Show that x = 0 is an ordinary point of $(x^2-1)y''+xy'-y=0$, but x = 1 is a regular singular point .

5. Prove that $H_n'(x)=2xH_n(x)-H_{n+1}(x)$

6. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)$.

7. Prove that $P_3(x)=\frac{1}{2}(5x^3-3x)$.

8. Show that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.

SECTION – B

Answer ALL questions. Each question carries Eight marks. 5 X 8 = 40 M

9. a) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

(OR)

b) Prove that $\int_0^{\pi/2} sin^{2l-1}\theta.cos^{2m-1}\theta\, d\theta=\frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

10. a) Find the radius of convergence the exact interval of convergence of the power series $\sum\frac{(n+1)}{(n+2)(n+3)}x^n$

(OR)

b) Find the power series solution of the equation $(x^2+1)y''+xy'-xy=0$ in powers of x.

11. a) State and Prove Rodrigues formula for $H_n(x)$.

(OR)

b) Prove that $H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$

12. a) Prove that $\int_{-1}^{1} P_m(x).P_n(x)dx=0\ \ if\ m\neq n$. and 2/(2n+1) if m = n .

(OR)

b) Prove that $(2n+1)xP_n=(n+1)P_{n+1}+nP_{n-1}$

13. a) Prove that $xJ_n'(x)=-nJ_n(x)+xJ_{n-1}(x)$.

(OR)

b) Prove that $\sqrt{\frac{\pi x}{2}}J_{3/2}(x)=\frac{1}{x}\sin x-\cos x$.

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA

III B.SC MATHEMATICS – Semester V (w.e.f. 2020-2021)

Skill Enhancement Course (Elective) :VII – A - Mathematical Special Functions

QUESTION BANK

Short Answer questions

Unit - I

1. Prove that $\int_0^1 x^m(\log x)^n dx=\frac{(-1)^n n!}{(m+1)^{n+1}}$
2. Evaluate $\int_0^{\infty}\frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$.
3. Evaluate $\int_0^1\frac{dx}{\sqrt{-log_e x}}$

4. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}} dy$ *and hence show that* $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

5. Prove that if n > 0 then $\Gamma(n+1) = n\Gamma(n)$

6. Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$

7. Show that $\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{1}{2}-x\right) = \frac{\pi}{\cos \pi x}$

8. Define Gamma and Beta functions. Write the relation between Gamma and Beta functions.

Unit – II

9. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for |x| < R and divergent for |x| > R .
10. Find the radius of convergence of the series $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$
11. Find the radius of the convergence of the series $\sum(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
12. Determine whether x = 0 is an ordinary point or a regular singular point of the differential equation $2x^2\left(\frac{d^2y}{dx^2}\right) + 7x(x+1)\frac{dy}{dx} - 3y = 0.$
13. Show that x = 0 is an ordinary point of $(x^2-1)y'' + xy' - y = 0$, but x = 1 is a regular singular point .
14. Show that x = 0 and x = -1 are singular points of $x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0$ where the first is irregular and the other is regular.
15. Solve by power series method $y' - y = 0$.

Unit - III

16. Prove that $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ *and* $H_{2n+1}(0) = 0.$

17. Find Hermit Polynomials for n=0, 1, 2, 3, 4 .

18. Prove that $H_n'' = 4n(n-1)H_{n-2}$

19. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$

20. Prove that $H_n(-x) = (-1)^n H_n(x)$.

21. Prove that, if $m < n,\ \frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^m n!}{(n-m)!}H_{n-m}(x).$

22. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x).$

Unit - IV

23. Prove that $P_n(-x) = (-1)^n P_n(x)$ *and hence deduce that* $P_n(-1) = (-1)^n$
24. Prove that $P_n' = \frac{n(n+1)}{2}$
25. Prove that $(2n+1)P_n = P_{n+1}' - P_{n-1}'.$
26. Prove that $xP_n' - P_{n-1}' = nP_n.$
27. Prove that $(n+1)P_n = P_{n+1}' - xP_n'.$
28. Prove that $(1-x^2)P_n' = n(P_{n-1} - xP_n).$
29. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x).$

Unit – V

30. Prove that, when n is a positive integer $J_{-n}(x) = (-1)^n J_n(x)$.

31. Show that $J_n(-x) = (-1)^n J_n(x)$ for positive or negative integers.

32. Prove that $J_n(x) = \frac{x}{2n}[J_{n-1}(x) + J_{n+1}(x)]$

33. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$

34. Prove that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\, sin\theta)\, d\theta$

35. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$.

36. Show that $\int_0^{\infty} e^{-ax} J_0(bx)dx = \frac{1}{\sqrt{(a^2+b^2)}}$ $, a > 0$

Essay Questions

Unit –I

1. Show that $\Gamma\left(n+\frac{1}{2}\right) = \frac{1.3.5....(2n-1)\sqrt{\pi}}{2^n}$, when n is a positive integer.

2. When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-1)^n 2^n\sqrt{\pi}}{1.3.5.....(2n-1)}$

3. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

4. Prove that $\int_0^{\pi/2} sin^{2l-1}\theta . cos^{2m-1}\theta\, d\theta = \frac{\Gamma(l)\Gamma(m)}{2\Gamma(l+m)}$.

5. Evaluate $i)\int_0^a \frac{dx}{(a^n-x^n)^{\frac{1}{n}}}$ $ii)\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx$

6. Show that $\int_0^{\infty}(\tan x)^n\, dx = \frac{\pi}{2} sec\frac{n\pi}{2}$ $, 0 < n < 1$

Unit – II

1. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty}|a_n|^{\frac{1}{n}} = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for |x| < R and divergent for |x| > R .

2. Find the radius of convergence the exact interval of convergence of the power series $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$

3. Determine the interval of convergence of the power series $\sum\{\frac{1}{n}(-1)^{n+1}(x-1)^n\}$

4. Determine whether x = 0 is an ordinary point or regular singular point for the differential equation $2x^2 y'' - xy' + (x-5)y = 0$.

6. Show that x = 0 is an ordinary point and x = 1 is an irregular singular point of $x(x-1)^3 y'' + 2(x-1)^3 y' + 3y = 0$,

5. Find the power series solution of the equation $(x^2+1)y''+xy'-xy=0$ in powers of x.

6. Find the solution in series of $\left(\frac{d^2y}{dx^2}\right)+x\left(\frac{dy}{dx}\right)+x^2y=0$ about x = 0 .

7. Find the general solution of $y''+(x-3)y'+y=0$ near x = 2 .

Unit – III

1. State and Prove generating function of the Hermit's polynomial.
2. State and Prove Rodrigues formula for $H_n(x)$.
3. State and Prove Orthogonal Properties of Hermite Polynomials.
4. Prove that $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$.
5. Prove that $H_n'(x)=2nH_{n-1}(x) \quad n\geq 1$ and $H_0'(x)=0$.
6. Prove that $H_n(x)=(-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$

Unit – IV

1. Show that $P_n(x)$ is the coefficient of h^n in the expansion of $(1-2xh+h^2)^{-1/2}$ in

   Ascending powers of h for | x | ≤ 1 and | h | < 1 .

2. Prove that $P_n(x)=\frac{1}{n!2^n}.\frac{d^n}{dx^n}(x^2-1)^n$.
3. Prove that $\int_{-1}^{1}[P_n(x)]^2dx=\frac{2}{2n+1}$.
4. Prove that $\int_{-1}^{1}P_m(x).P_n(x)dx=0 \;\; if\; m\neq n.$ and 2/(2n+1) if m = n .
5. Prove that $(2n+1)xP_n=(n+1)P_{n+1}+nP_{n-1}$
6. Express $P(x)=x^4+2x^3+2x^2-x-3$ in terms of Legendre's Polynomials.
7. Prove that $\int_{-1}^{1}(x^2-1)P_{n+1}P_n'\,dx=\frac{2n(n+1)}{(2n+1)(2n+3)}$.
8. Prove that $\int_{-1}^{1}x^n\,P_n(x)dx=\frac{2^{n+1}(n!)^2}{(2n+1)!}$

Unit V

1. Prove that when n is a positive integer, $J_n(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x(z-\frac{1}{z})}{2}}$ in ascending and descending powers of z.

2. Prove that $xJ_n'(x)=nJ_n(x)-xJ_{n+1}(x)$.

3. Prove that $xJ_n'(x)=-nJ_n(x)+xJ_{n-1}(x)$.

4. Prove that $x^2J''_n(x)=(n^2-n-x^2)J_n(x)+xJ_{n+1}(x)$

5. Prove that $i)\frac{d}{dx}[x^nJ_n(x)]=x^nJ_{n-1}(x)\;\;(ii)\frac{d}{dx}[x^{-n}J_n(x)]=-x^{-n}J_{n+1}(x)$.

6. Prove that $\sqrt{\frac{\pi x}{2}}J_{3/2}(x)=\frac{1}{x}\sin x-\cos x$.

7. Show that $\cos x = J_0 - 2J_2 + 2J_4 - \cdots ....$ and

$\sin x = 2J_1 - 2J_3 + 2J_5 - \cdots ...$

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	P.R.Government College (Autonomous) **KAKINADA**	**Program&Semester** IIIB.Sc. (VSem)			
CourseCode MAT-601B	**TITLE OF THE COURSE** **6B- Multiple Integrals and Applications of Vector Calculus**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Integration and vectors	5	1	-	5

Course Objectives:

This course will cover the geometry of space, multivariate and vector-valued functions from a graphical, numerical, and symbolic perspective, differentiation and integration of vector-valued functions, partial differentiation, and multiple integration of multivariate functions.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Learn multiple integrals as a natural extension of definite integral to a function of two variables in the case of double integral / three variables in the case of triple integral.
CO2	Determine the gradient, divergence and curl of a vector and vector identities.
CO3	Evaluate line, surface and volume integrals.
CO4	Understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral (Green's theorem), relation between line and surface integral (Stokes theorem) .

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

II. Syllabus: (Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Multiple integrals-I (15h)

1. Introduction, Double integrals, Evaluation of double integrals, Properties of double integrals.
2. Region of integration, double integration in Polar Co-ordinates,
3. Change of variables in double integrals, change of order of integration.

Unit – 2: Multiple integrals-II (15h)

1. Triple integral, region of integration, change of variables.
2. Plane areas by double integrals, surface area by double integral.
3. Volume as a double integral, volume as a triple integral.

Unit – 3: Vector differentiation (15h)

1. Vector differentiation, ordinaryderivatives of vectors.
2. Differentiability, Gradient, Divergence, Curl operators,
3. Formulae involving the separators.

Unit – 4: Vector integration (15h)

1. Line Integrals with examples.
2. Surface Integral with examples.
3. Volume integral with examples.

Unit – 5: Vector integration applications (15h)

1. Gauss theorem and applications of Gauss theorem.
2. Green's theorem in plane and application of Green's thseorem.
3. Stokes's theorem and applications of Stokes theorem.

III. Reference Books:

1. Dr.M Anitha, Linear Algebra and Vector Calculus for Engineer, Spectrum University Press, SR Nagar, Hyderabad-500038, INDIA.
2.Dr.M.Babu Prasad, Dr.K.Krishna Rao, D.Srinivasulu, Y.AdiNarayana, Engineering Mathematics-II, Spectrum University Press, SR Nagar, Hyderabad-500038,INDIA.

3. V.Venkateswararao, N. Krishnamurthy, B.V.S.S.Sarma and S.Anjaneya Sastry, A text Book of B.Sc., Mathematics Volume-III, S. Chand & Company, Pvt. Ltd., Ram Nagar,NewDelhi-110055.
4. R.Gupta,Vector Calculus, Laxmi Publications.
5. P.C.Matthews, Vector Calculus, Springer Verlag publications.
6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory:

1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).

1. The methods of evaluating double integrals and triple integrals in the class room and train to evaluate These integrals of different functions over different regions.
2. Applications of line integral, surface integral and volume integral.
3. Applications of Gauss divergence theorem, Green's theorem and Stokes's theorem.

2. **For Student: Fieldwork/Project work** Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the following aspects.

1. Going through the web sources like Open Educational Resources to find the values of double and triple integrals of specific functions in a given region and make conclusions. (or)
2. Going through the web sources like Open Educational Resources to evaluate line integral, surface integral and volume integral and apply Gauss divergence theorem, Green's theorem and Stokes theorem and make conclusions.

3**. Max. Marks for Fieldwork/Project work Report: 05.**

4. **Suggested Format for Fieldwork/Project work Report:** Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VI - B

Paper – VI –B : Multiple integrals and applications of Vector calculus

UNIT	TOPIC	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	**Multiple integrals-I**	02	02	26
II	**Multiple integrals-II**	02	02	26
III	**Vector differentiation**	02	02	26
IV	**Vector integration**	01	02	21
V	**Vector integration applications**	01	02	21
Total		08	10	120

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA
III B.SC MATHEMATICS – Semester V (w.e.f. 2020-2021)
Skill Enhancement Course (Elective) :VI – B- Multiple integrals and applications of Vector calculus
Paper – VI – B : MODEL PAPER

Time: 2 hrs 30 Min **Max. Marks : 60 M**

SECTION – A

Answer any **FOUR** of the following. Each question carries 3marks. 4 x 4 = 20 M

11. Change the order of integration, evaluate $\int_0^1 \int_1^{e^x} dy\,dx.$

12. Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} x^2\, dy\, dx$ by changing the polar coordinates.

13. Find the length of the cardioid given by r= a (1-cosθ).

14. Determine the area bounded by the curves xy=2 and x^2=4y and y=4.

15. Find the directional derivative of the function f = x^2- y^2+2 z^2 at the point P(1,2,3) in

the direction of the line PQ where Q(5,0,4).

16. If f = x^2yz , g = $xy-3z^2$ find div(grad f x grad g).

17. Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F} = 3x^2\,\bar{i} + (2xz-y)\,\bar{j} + z\,\bar{k}$ along the straight line C from (0,0,0) to (2,1,3).

18. Show that $\int_S (ax\,\bar{i} + by\,\bar{j} + cz\,\bar{k}).\bar{N}\,ds = \frac{4\pi}{3}(a+b+c)$ where S is the surface of the sphere $x^2+y^2+z^2=1$.

Section – B

Answer **all** questions. Each question carries 8 marks. 5 X 8 = 40 M.

1. Evaluate $\iint_R x^2\,dxdy$ where R is the region bounded by the rectangle hyperbola xy=16 and the lines y=x, y=0 and x=8.

(OR)

2. Find the value of $\int (x+y^2)dx + (x^2-y)dy$, takenintheclockwisesense along the closed curve C formed by $y^2=x$ and y=x between (0,0) and (1,1).

3. Change the order of integration, evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$.

(OR)

4. Find the area of the surface $x^2+y^2+z^2=a^2$ that lies inside the surface $(x^2+y^2)^2=a^2(x^2-y^2)$.

5. Prove that the necessary and sufficient condition for f(t) to have constant direction is $f \times \frac{df}{dt} = 0$.

(OR)

6. Prove that Curl ($\bar{A}$x $\bar{B}$) =$\bar{A}$ div $\bar{B}$ – $\bar{B}$ div $\bar{A}$ + $(B.\nabla)A-(A.\nabla)B$

7. If $\bar{F} = 4xz\bar{\imath} - y^2\bar{\jmath} + yz\bar{k}$, evaluate $\int \bar{F}.\bar{N}dS$ where S is the surface of the cube bounded by $x=0, x=a, y=0, y=a, z=0, z=a$.

(OR)

8. If $\bar{F} = 2xz\,\bar{i} - x\,\bar{j} + y^2\,\bar{k}$, evaluate $\int_V \bar{F}.dV$ where V is the region bounded by the surfaces x=0,x=2,y=0,y=6,Z=x^2,Z=4.

9. State and Prove Green’s theorem in a plane.

Or

10. Verify Stoke's theorem for $\bar{F} = -y^3\bar{\imath} + x^3\bar{\jmath}$ where S is the circular disc

$x^2 + y^2 \leq 1\,, z = 0.$

	P.R.Government College (Autonomous): KAKINADA	**Program&Semester** IIIB.Sc. (VSem)			
CourseCode **MAT-701B**	**TITLEOFTHECOURSE** **7B- Integral Transforms with Applications**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Integration	5	1	-	5

Course Objectives:

To describe the ideas of Fourier and Laplace Transforms and indicate their applications in the fields such as application of PDE, Digital Signal Processing, Image Processing, Theory of wave equations, Differential Equations and many others.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Evaluate Laplace transforms of certain functions, find Laplace transforms of derivatives and of integrals.

CO2	Determine properties of Laplace transform which may be solved by application of special functions namely Dirac delta function, error function, Bessel function and periodic function.
CO3	Understand properties of inverse Laplace transforms, find inverse Laplace transforms of derivatives and of integrals.
CO4	Solve ordinary differential equations with constant/ variable coefficients by using Laplace transform method.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

II. Syllabus :(Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Laplace transforms- I **(15h)**

1. Definition of Laplace transform, linearity property-piecewise continuous function.

2. Existence of Laplace transform, functions of exponential order and of class A.

3. First shifting theorem, second shifting theorem and change of scale property.

Unit – 2: Laplace transforms- II **(15h)**

1. Laplace Transform of the derivatives, initial value theorem and final value theorem. Laplace transforms of integrals.

2. Laplace transform of tn . f (t), division by t, evolution of integrals by Laplace transforms.

3. Laplace transform of some special functions-namely Dirac delta function, error function, Bessel function and Laplace transform of periodic function.

Unit – 3: Inverse Laplace transforms **(15h)**

1. Definition of Inverse Laplace transform, linear property, first shifting theorem, second shifting theorem, change of scale property, use of partial fractions.

2. Inverse Laplace transforms of derivatives, inverse, Laplace transforms of integrals, multiplication by powers of ‘p’, division by ‘p’.

3. Convolution, convolution theorem proof and applications.

Unit – 4: Applications of Laplace transforms **(15h)**

1. Solutions of differential equations with constants coefficients, solutions of differential equations with variable coefficients.

2. Applications of Laplace transforms to integral equations- Abel’s integral equation.

3. Converting the differential equations into integral equations, converting the integral equations into differential equations.

Unit – 5: Fourier transforms **(15h)**

1. Integral transforms, Fourier integral theorem (without proof), Fourier sine and cosine integrals.

2. Properties of Fourier transforms, change of scale property, shifting property, modulation theorem. Convolution.

3. Convolution theorem for Fourier transform, Parseval's Identify, finite Fourier transforms.

III. Reference Books:

1. Dr. S.Sreenadh, S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr. V.Ramesh Babu, Fourier series and Integral Transforms, S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.

2. A.R. Vasistha, Dr. R.K. Gupta, Laplace Transforms, Krishna Prakashan Media Pvt. Ltd. Meerut.

3. M.D.Raisinghania, H.C. Saxsena , H.K. Dass, Integral Transforms, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

4. Dr. J.K. Goyal, K.P. Gupta, Laplace and Fourier Transforms, Pragathi Prakashan, Meerut.

5. Shanthi Narayana , P.K. Mittal, A Course of Mathematical Analysis, S. Chand & Company Pvt.Ltd. Ram Nagar, New Delhi-110055.

6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory:

1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).

1. Demonstrate on sufficient conditions for the existence of the Laplace transform of a function.

2. Evaluation of Laplace transforms and methods of finding Laplace transforms.

3. Evaluations of Inverse Laplace transforms and methods of finding Inverse Laplace transforms.

4. Fourier transforms and solutions of integral equations.

2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources on Applications of Laplace transforms and Inverse Laplace transforms to find solutions of ordinary differential equations with constant /variable coefficients and make conclusions. (or)

2. Going through the web sources like Open Educational Resources on Applications of convolution theorem to solve integral equations and make conclusions. (or)

3. Going through the web source like Open Educational Resources on Applications of Fourier transforms to solve integral equations and make conclusions.

4. Max. Marks for Fieldwork/Project work Report: 05.

4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates

2. Visits to research organizations, Statistical Cells, Universities, ISI etc.

3. Invited lectures and presentations on related topics by experts in the specified area.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VII - B

Paper – VII –B : Integral transforms with Applications

UNIT	TOPIC	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted

I	**Laplace transforms- I**	02	02	26
II	**Laplace transforms- II**	02	02	26
III	**Inverse Laplace transforms**	02	02	26
IV	**Applications of Laplace transforms**	01	02	21
V	**Fourier transforms**	01	02	21
Total		08	10	120

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA
III B.SC MATHEMATICS – Semester V (w.e.f. 2020-2021)
Skill Enhancement Course (Elective) :VII – B- Integral transforms with Applications
Paper – VII – B : MODEL PAPER

Time: 2 hrs 30 Min **Max. Marks : 60 M**

SECTION – A

Answer any **FOUR** of the following. Each question carries 3 marks. 4 x 5 = 20 M

1. Find L$\{cos^3 3t\}$.

2. State and prove second shifting theorem.

3. Evaluate L$\{t^2 e^{-2t}\}$.

4. Find L$\left\{\frac{sin3t\ cost}{t}\right\}$.

5. Find $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$ by using partial fractions.

6. Find the inverse Laplace transform of $log\left(\frac{p+1}{p-1}\right)$.

7. Solve $\frac{d^2y}{dx^2} + y = 0$ under the conditions that y = 1,$\frac{dy}{dx} = 0\ when\ t$ = 0.

8. Find the finite cosine transform of $(1-\frac{x}{\pi})^2$.

Section – B

Answer all questions. Each question carries 7 marks. 5 X 8 = 40 M.

1. Using the expansion $sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - - -$. Show that $L\{sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}e^{\frac{1}{4p}}$

(OR)

2. State and prove the First Shifting Theorem on Laplace Transforms.
3. State and prove initial value theorem.

(OR)

4. i) Find L$\left\{\frac{1-cost}{t^2}\right\}$ ii) Show that $\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}dt = log2$.
5. Find the Inverse Laplace Transform of $\left[\frac{s^2}{s^4+4a^4}\right]$.

(OR)

6. State and prove Convolution Theorem?
7. Solve Dx +Dy = t,^2x −y = e^{-t} if $x(0) = 3, x^1(0) = -2, y(0) = 0$.

(OR)

8. Solve the integral equation $\int_0^1 \frac{F(u)du}{(t-u)^{\frac{1}{3}}}$= t(1+t).
9. Find the finite cosine transform of (x) if $f(x)$ = -$\frac{\cos k(\pi-x)}{ksin\ k\pi}$

. (OR)

10. State and Prove Parsvel's identity for Fourier Transforms.

	P.R.Government College (Autonomous) KAKINADA	Program&Semester IIIB.Sc. (VSem)
CourseCode **MAT-601C**	**TITLEOFTHECOURSE** **6C- Partial Differential Equations & Fourier Series**	

		L	T	P	C
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on Ordinary differential equations	5	1	-	5

Course Objectives:

Partial differential equations are used to mathematically formulate, and thus aid the solution of, physical and other problems involving functions of several variables, such as the propagation of heat or sound, fluid flow, elasticity, electrostatics, electrodynamics, etc.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Classify partial differential equations, formation of partial differential equations and solve Cauchy’s problem for first order equations.
CO2	Solve Lagrange's equations by various methods, find integral Surface passing through a given curve and Surfaces orthogonal to a given system of Surfaces.

CO3	Find solutions of nonlinear partial differential equations of order one by using Charpit's method and Jacobi's method.
CO4	Understand Fourier series expansion of a function f(x) and Parseval's theorem.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development			Employability			Entrepreneurship	

II. Syllabus: (Hours: Teaching: 75 (incl. unit tests etc.05), Training: 15)

Unit – 1: Introduction of partial differential equations (15h)

1. Partial Differential Equations, classification of first order partial differential equations, Rule I, derivation of a partial differential equations by the elimination of arbitrary constants 2. Rule II, derivation of a partial differential equation by the elimination of arbitrary function φ from the equations ∅(u, v) = 0 where u and v are functions of x, y and z.

3. Cauchy's problem for first order equations

Unit – 2: Linear partial differential equations of order one (15h)

1. Lagrange's equations, Lagrange's method of solving Pp+Qq=R, where P, Q and R are functions of x, y and z, type 1 based on Rule I for solving dx / p = dy / Q = dz / R ,type 2 based on Rule II for solving dx / p = dy / Q = dz / R .

2. Type 3 based on Rule III for solving dx / p = dy / Q = dz / R , type 4 based on Rule IV for solving dx / p = dy / Q = dz / R .

3. Integral Surface passing through a given curve, the Cauchy problem, Surfaces orthogonal to a given system of Surfaces.

Unit – 3: Non-linear partial differential equations of order one-I (15h)

1. Complete integral, particular integral, singular integral and general integral, geometrical interpretation of integrals of f (x, y, z, p, q) = 0, method of getting singular integral from the PDE of first order, compatible system of first order equations.

2. Char pit's method, Standard form I, only p and q present.

3. Standard form II, Clairaut equations.

Unit – 4: Non-linear partial differential equations of order one-II (15h)

1. Standard Form III, only p, q and z present.

2. Standard Form IV, equation of the formf1 (x, p) = f2(y, q).

3. Jacobi's method, Jacobi's method for solving partial differential equations with three or more independent variables, Jacobi's method for solving a non-linear first order partial differential equations in two independent variables.

Unit – 5: Fourier series (15h)

1. Introduction, Euler's formulae for Fourier series expansion of a function f(x), Dirichlet's conditions for Fourier series, convergence of Fourier series.

2. Functions having arbitrary periods. Change of interval, Half range series.

3. Parseval's theorem, illustrative examples based on Parseval's theorem, some particular series.

III. Reference Books:

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

2. Dr. S.Sreenadh, S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr. V.RameshBabu, Fourier Series and Integral Transforms, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

3. Prof T.Amaranath, An Elementary Course in Partial Differential Equations Second Edition, Narosa Publishing House, New Delhi.

4. Fritz John, Partial Differential Equations, Narosa Publishing House, New Delhi, 1979.

5. I.N.Sneddon, Elements of Partial Differential Equations by McGraw Hill, International Edition, Mathematics series.

6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory:

1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).

1. On classification of first order partial differential equations, formation of partial differential equations. 2. Various methods of finding solutions of partial differential equations.

3. Integral Surface passing through a given curve and Surfaces orthogonal to a give system of Surfaces.

2) For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the Following, by choosing any one of the aspects.

1. Going through the web source like Open Educational Resources to find solutions of partial differential equations by using Lagrange's method, Charpit’s method and Jacobi’s method and make conclusions. (or) 2. Going through the web source like Open Educational Resources to find Integral Surface passing through a given curve and Surfaces orthogonal to a given system of Surfaces and make conclusions. (or) 3. Going through the web source like Open Educational Resources to find Fourier series expansions of some functions and applications of Parseval’s theorem and make conclusions.

3. Max. Marks for Fieldwork/Project work Report: 05.

4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

B) Suggested Co-Curricular Activities

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates

2. Visits to research organizations, Statistical Cells, Universities, ISI etc.

3. Invited lectures and presentations on related topics by experts in the specified area.

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VI - C

Paper – VI –C : Partial differential equations & Fourier series

UNIT	TOPIC	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	**Introduction of partial differential equations**	02	02	26
II	**Linear partial differential equations of order one**	02	02	26
III	**Non-linear partial differential equations of order one-I**	02	02	26
IV	**Non-linear partial differential equations of order one-II**	01	02	21

V	**Fourier series**	01	02	21
Total		08	10	120

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P. R. GOVERNMENT COLLEGE (AUTOMOMOUS), KAKINADA

III B.SC MATHEMATICS – Semester V (w.e.f. 2020-2021)

Skill Enhancement Course (Elective) :VI – C- Partial differential equations & Fourier series

Paper – VI – C : MODEL PAPER

Time: 2 hrs 30 Min **Max. Marks : 60 M**

SECTION – A

Answer any **FOUR** of the following. Each question carries 5 marks. 4 x 5 = 20 M

1. Form a partial differential equation from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

2. Form a partial differential equation from the equation $xyz = \emptyset(x + y + z)$.

3. Solve $y^2p - xyq = x(z - 2y)$.

4. Solve xp-yq=xy.

5. Find a complete integral of $p^2 + q^2 = 1$.

6. Find a complete integral of $zpq = p + q$.

7. Find a complete integral of $p^2 + q^2 = x + y$.

8. Express $f(x) = \frac{x}{2}$ as a Fourier series in the interval $-\pi < x < \pi$.

Section – B

Answer all questions. Each question carries 8 marks. 5 X 8 = 40 M.

1. Find a partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

(OR)

2. Eliminate the arbitrary function Øfrom $\emptyset(x^2+y^2+z^2, z^2-2xy)=0$.

3. Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$.

(OR)

4. Find the surface which intersects the surfaces of the system z(x+y)=c(3z+1) orthogonally and which passes through the circles $x^2+y^2=1$, z=1.

5. Find a complete and singular integral of $(p^2+q^2)y=qz$by Charpit's method.

(OR)

6. Show that the equations $xp-yq=x \; and \; x^2p+q=xz$ are compatible and solve them.

7. Solve $z^2(p^2x^2+q^2)=1$.

(OR)

8. Find a complete integral of $p_1p_2p_3=z^3x_1x_2x_3$ by Jacobi's method.

9. Obtain the Fourier series for $f(x)=e^{-x}$ in the interval $0<x<2\pi$.

(OR)

10. Obtain the Fourier series for the function $f(x)=\begin{cases} \pi x, & 0\le x\le 1 \\ \pi(2-x), & 1\le x\le 2 \end{cases}$

Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots+\infty=\frac{\pi^2}{8}$.

	P.R.Government College (Autonomous) **KAKINADA**	**Program&Semester** IIIB.Sc. (VSem)
CourseCode **MAT-701C**	**TITLEOFTHECOURSE** **7C- Number Theory**	

		L	T	P	C
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	BasicMathematicsKnowledge on number system	5	1	-	5

Course Objectives:

The main goal of number theory is to discover interesting and unexpected relationships between different sorts of numbers and to prove that these relationships are true.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Find quotients and remainders from integer division, study divisibility properties of integers and the distribution of primes.
CO2	Understand Dirichlet multiplication which helps to clarify interrelationship between various arithmetical functions.
CO3	Understand the concepts of congruencies, residue classes and complete residues systems.

CO4	Comprehend the concept of quadratic residues mod p and quadratic non residues mod p.

Course with focus on employability/entrepreneurship /Skill Development modules

SkillDevelopment			Employability			Entrepreneurship	

I. **Syllabus:** (Hours: Teaching:75 (incl. unit tests etc.05), Training:15)

Unit – 1: Divisibility **(15h)**

1. Introduction, Divisibility, Greatest Common Divisor.
2. Prime numbers, The fundamental theorem of arithmetic, The series of reciprocals of the primes.
3. The Euclidean algorithm, The greatest common divisor of more than two numbers.

Unit – 2: Arithmetical Functions and Dirichlet Multiplication **(15h)**

1. Introduction, The Mobius function$\mu(n)$, The Euler totient function$\varphi(n)$, A relation connecting φ and μ , A product formula for $\varphi(n)$.
2. The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, The Mangoldt function$\Lambda(n)$.
3. Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function $\lambda(n)$, The divisor functions $\sigma\alpha(n)$.

Unit – 3: Averages of Arithmetical Functions **(15h)**

1. Introduction, The big oh notation. Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas.
2. The average order of d(n), The average order of the divisor functions$\sigma\alpha(n)$, The average order of $\varphi(n)$.
3. The average order of $\mu(n)$ and $\Lambda(n)$, The partial sum of a Dirichlet product, Applications of $\mu(n)$ and $\Lambda(n)$.

Unit – 4: Congruence's **(15h)**

1. Definition and basic properties of congruences, Residue classes and complete residue systems.
2. Linear congruences, reduced residue systems and the Euler-Fermat theorem. Polynomial congruences modulo p. Lagrange's theorem.
3. Applications of Lagrange's theorem, Simultaneous linear congruences. The Chinese remainder theorem. Applications of the Chinese remainder theorem.

Unit – 5: Quadratic Residues and the Quadratic Reciprocity Law **(15h)**

1. Quadratic Residues, Legendre's symbol and its properties, Evaluation of (-1/p) and (2/p), Gauss lemma,

2. The Quadratic reciprocity law, Applications of the reciprocity law, The Jacobi Symbol.

3. Gauss sums and the quadratic reciprocity law, the reciprocity law for quadratic Gauss sums. Another proof of the quadratic reciprocity law.

III. Reference Books:

1. Tom M.Apostol, Introduction to Analytic Number theory, Springer International Student Edition.
2. David, M. Burton, Elementary Number Theory, 2nd Edition UBS Publishers.
3. Hardy & Wright, Number Theory, Oxford Univ, Press.
4. Dence, J. B & Dence T.P, Elements of the Theory of Numbers, Academic Press.

5. Niven , Zuckerman &Montgomery, Introduction to the Theory of Numbers.
6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities:

A) Mandatory:

1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking Relevant outside data (Field/Web).

1. Finding quotient and numbers from integer division and the method of solving congruences. Further problems related to the theory of quadratic residues.
2. Applications of Lagrange's theorem.
3. Applications of the Chinese remainder theorem.
4. Applications of the reciprocity law.

2.For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources and list out Applications of Lagrange's theorem, and make conclusions.(or)
2. Going through the web sources like Open Educational Resources and list out Applications of the Chinese remainder theorem and make conclusions.(or)
3. Going through the web sources like Open Educational Resource and list out Applications of the reciprocity law and make conclusions.

3. Max. Marks for Fieldwork/Project work Report: 05.

4. **Suggested Format for Fieldwork/Project work Report**: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area.

CO-PO Mapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	3	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	3	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	2	3	3	1	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VII - C

Paper – VII –C : Number theory

UNIT	TOPIC	S.A.Q (including choice) 5 M	E.Q (including choice) 8 M	Marks Allotted
I	**Divisibility**	02	02	26
II	**Arithmetical Functions and Dirichlet Multiplication**	02	02	26
III	**Averages of Arithmetical Functions**	02	02	26
IV	**Congruence's**	01	02	21
V	**Quadratic Residues and the Quadratic Reciprocity Law**	01	02	21
Total		08	10	120

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions (8 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 5 x 8 M = 40

Total Marks : = 60

P. R. GOVERNMENT COLLEGE (AUTOMONOUS), KAKINADA

III B.SC MATHEMATICS – Semester V (w.e.f. 2020-2021)

Skill Enhancement Course (Elective) :VII - C Number Theory

Paper – VII – C : MODEL PAPER

Time: 2 hrs 30 Min **Max. Marks : 60 M**

SECTION - A

Answer any **FOUR** questions. Each question carries **5** Marks. **4 x 5 = 20 M**

1. If (a, b) = 1 then prove that (a+b, a-b) is either 1 or 2.

2. If c/ab and (b, c) =1. Then prove that c/a.

3. If n≥1, prove that $\varphi(n) = n\prod_{p/n}\left(1-\frac{1}{p}\right)$, where p is prime.

4. If n ≥ 1, Prove that $\sum_{d/n} \varnothing(d) = n$.

5. For x≥1, Prove that (i) $\sum_{n\le x}\mu(n)\,[x/n] = 1$.

6. State and prove Chinese remainder theorem.

7. Solve the congruence 25x ≡ 15 (mod 120).

8. Define the Jacobi symbol.

Section – B

Answer all questions. Each question carries 8 marks. 5 X 8 = 40 M.

1. State and prove Fundamental theorem of arithmetic.

(OR)

2. Let d = (826, 1890). Use the Euclidean algorithm to compute d, then express d as a linear combination of 826 and 1890.

3. a) If n≥1, prove that $\sum_{d/n}\mu(d) = [1/n] = \begin{cases} 1, if n = 1 \\ 0, if n > 1 \end{cases}$.

b) State and Prove Mobius Inversion Formula.

(OR)

4. If f is multiplicative then prove that $\sum_{d/n}\mu(d)\,f(d) = \prod_{p/n}(1-f(p))$.

5. State and prove Euler's summation formula.

(OR)

6. For x ≥ 1, prove that $\sum_{n\le x}\varnothing(n) = \frac{3}{\pi^2}x^2 + O(x\log x)$.

7. State and Prove Lagrange's Theorem.

(OR)

8. State and prove Fermat’s little theorem.

9. Determine whether 219 is a quadratic residue or non residue mod 383.

(OR)

10. Let p be an odd prime. Then prove that for all n, $(n/p) \equiv (p-1)/2 \pmod{p}$.

P.R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS

Massive Open Online Course (MOOCS) CERTIFICATE COURSE

Additional Credits: Achieved Credits

Guidelines of this course:

After completion of the course the student is able to get 2 additional credits through the examination cell under the following conditions.

- Completed the course through the online platforms Swayam, UGC, CEC, NPTEL, AICTE, NCERT, etc.
- Course related to any Mathematical subject or interdisciplinary with mathematics one of the subject.
- Course contains at least a minimum of 4 weeks.
- Course completion certificate must be submitted to the Examination cell through the department.

For more details about online courses go through the following links:

- http://www.apcce.gov.in/SwC
- https://swayam.gov.in/
- http://free.aicte-india.org/
- https://ugcmoocs.inflibnet.ac.in/
- https://swayam.gov.in/nc_details/CEC
- https://swayam.gov.in/nc_details/NCERT

P.R.GOVERNMENT COLLEGE (A) : KAKINADA

DEPARTMENT OF MATHEMATICS

SYLLABUS FOR CERTIFICATE COURSE

PAPER – MAT CM - COMPETATIVE MATHEMATICS

Total Hours-45 Hours / Additional Credits: 2

Objectives of the Course:

1. Use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real life contexts.

2. Use the binomial theorem to calculate the probability of success or failure in a Bernoulli trial.

UNIT-I : **Algebra** **(10 Hrs)**

Sets, Relations and functions:

Set, Roster form and set builder form, Types of sets, Venn Diagrams, Basic operations on Sets, Relations, Functions, Types of functions.

Matrix Algebra:

Introduction - Definition - Types of matrices - Scalar multiplication of matrix - Equality of matrices - Matrix operations - Addition and Subtraction Multiplication of Matrices - Properties - Transpose of Matrix - Determinants of a square matrix - Determinants of order two.

Binomial Theorem:

Binomial theorem, number of terms, General term, Middle term, Independent term, Binomial coefficients, Numerically greatest term.

Progressions:

Arithmetic Progressions, nth term of an Arithmetic Progressions, Sum of n terms in A.P, Geometric Progressions, nth term of a G.P, Harmonic Progression.

UNIT-II - Trigonometry: **(05 Hrs)**

Trigonometry; Trigonometry ratios; Trigonometry Ratios of Standard Angles; Trigonometric Identities; Heights and Distances.

UNIT-III : Geometrical Ability **(08 Hrs)**

Coordinate Geometry:

Distance between two points, Section formula, Centroid of a Triangle, Collinearity, Slop of the straight line, slop of a line joining two points.

UNIT-IV : Arithmetic Ability **(12 Hrs)**

Numbers and Divisibility, Rational Numbers, LCM and GCD, Surds, Laws of Indices, Ratio and Proportion, Percentage, Profit and Loss, Partnership, Time, Distance and Work.

UNIT-V : Statistical Ability **(10 Hrs)**

Simple problems on Probability, Mean, Median and Mode, Standard Deviation, Correlation.

References:

1. R.S.Agarwal, Quantitative Aptitude for competitive examinations , S.chand publications.
2. R.V.Praveen, Quantitative Aptitude and Reasoning. PHI publishers
3. Pratogitaprakasan , Kic X, Quantitative Aptitude: Numerical Ability (fully solved) Objective questions, Kiran Prakasan Publishers.
4. Abhijitguha , Quantitative Aptitude for competitive examination ,TMG Hill Publications.
5. Business Mathematics by D.C.Sancheti, V.K.Kapoor, S.Chand& Sons Publications,2000 Edition.
6. Old question papers of the Exams conducted by (Wipro, TCS, Infosys etc.) at their Recruitment Process, source- internet.

BLUE PRINT FOR CERTIFICATE COURSE

PAPER – MAT CM - COMPETATIVE MATHEMATICS

Unit	**TOPIC**	**M.C.Q**	**Marks allotted to the Unit**
I	**Algebra**	20	20
II	**Trigonometry**	05	05
III	**Geometrical Ability**	05	05
IV	**Arithmetic Ability**	10	10
V	**Statistical Ability**	10	10
Total		50	50

Question Paper pattern

M.C.Q. = Multiple choice questions (1 marks)

Multiple choice questions : 50 x 1 = 50M

……………….

Total Marks = 50M

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P.R. GOVERNMENT COLLEGE (A), KAKINADA

DEPARTMENT OF MATHEMATICS

CERTIFICATE COURSE ON "BASIC MATHEMATICS"

Paper –Mat BM - BASIC MATHEMATICS (FOR NON-MATHS STUDENTS)

Total Hours – 45 hours /Additional Credits: 2

Unit-1: Matrix Algebra (10 Hrs)

Introduction - Definition - Types of matrices - Scalar multiplication of matrix - Equality of matrices - Matrix operations - Addition and Subtraction

Unit - 2 : Matrix Multiplication (05 Hrs)

Multiplication of Matrices - Properties - Transpose of Matrix - Determinants of a square matrix - Determinants of order two

Unit-3 : Quadratic Equations (10 Hrs)

Solutions of quadratic equations – Preliminaries - Solving pure quadratic equations - Solving affected quadratic equations - Nature of Roots - Symmetric expression of a roots of a quadratic equation

Unit-4 : Arithmetic Progression (10 Hrs)

Finding general terms of Arithmetic Progression - Sum of finite numbers of terms in Arthematic Progression - Arithmetic means - To insert 'n' terms between two given quantities in Arithmetic Progression.

UNIT-V : Arithmetic Ability (10 Hrs)

Numbers and Divisibility, Rational Numbers, LCM and GCD, Surds, Laws of Indices, Ratio and Proportion, Percentage, Profit and Loss, Partnership, Time, Distance and Work.

Prescribed Text Book :

1. Business Mathematics by D.C.Sancheti, V.K.Kapoor, S.Chand& Sons Publications,2000 Edition.

2. R.S.Agarwal, Quantitative Aptitude for competitive examinations , S.chand publications.

BLUE PRINT FOR CERTIFICATE COURSE

Paper –Mat BM - BASIC MATHEMATICS (FOR NON-MATHS STUDENTS)

Unit	**TOPIC**	**M.C.Q**	**Marks allotted to the Unit**
I	**Matrix Algebra**	20	20
II	Matrix **Multiplication**	05	05
III	**Quadratic Equations**	05	05
IV	**Arithmetic Progression**	10	10
V	**Arithmetic Ability**	10	10
Total		50	50

Question Paper pattern

M.C.Q. **=** Multiple choice questions (1 marks)

Multiple choice questions : 50 x 1 = 50M

……………….

Total Marks = 50M

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P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

DEPARTMENT OF MATHEMATICS

WORK LOAD FOR THE YEAR 2022-2023 (ODD SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **324** (actual)

No. of Permanent posts sanctioned : 05

No. of Permanent staff working : Nil

No. of Contract faculty : 05

No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC EM1	4	2	2	4	8	
2	I MPC EM2	4	2	2	4	8	
3	I MPE	4	2	2	4	8	
4	I MPCs	4	2	2	4	8	
5	I MECs	4	2	2	4	8	
6	I MCAC	4	2	2	4	8	
7	I MCPC	4	2	2	4	8	
8	I MSCs	4	2	2	4	8	
9	I MCCs	4	2	2	4	8	
10	I MSAS	4	2	2	4	8	
11	I MEIOT	4	2	2	4	8	
12	I BVOC	4	2	2	4	8	
13	II MPC EM1	6	-	-	-	6	
14	II MPC EM2	6	-	-	-	6	
15	II MPE	6	-	-	-	6	
16	II MPCs	6	-	-	-	6	
17	II MECs	6	-	-	-	6	
18	II MCAC	6	-	-	-	6	
19 -	II MCPC	6	-	-	-	- 6	
20	II MSCs	6	-	-	-	6	
21	II MCCs	6	-	-	-	6	
22	II MSAS	6	-	-	-	6	
23	II MEIOT	6	-	-	-	6	
24	II BVOC	6	-	-	-	6	
25	Analytical Skills	60	-	-	-	60	
26	III MPC TM	12	-	-	-	12	
27	III MPC EM	12	-	-	-	12	
28	III MPE	12	-	-	-	12	
29	III MPCs	12	-	-	-	12	
30	III MECs	12	-	-	-	12	
31	III MCAC	12	-	-	-	12	
32	III MCPC	12	-	-	-	12	

33	III MSCs	12	-	-	-	12	
34	III MCCs	12	-	-	-	12	
35	III MSAS	12	-	-	-	12	
36	III MEIOT	12	-	-	-	12	
Total Work load for the subject Mathematics						360	

GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2022-2023 (ODD SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **200** (adjusted)

No. of Permanent posts sanctioned : 05

No. of Permanent staff working : NIL

No. of Contract faculty : 05

No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC EM1	4	2	2	4	8	
2	I MPC EM2	4	2	2	4	8	
3	I MPE	4	2	2	4	8	
4	I MPCs, MECs	4	2	2	4	8	
5	I MCAC,MCPC	4	2	2	4	8	
6	I MSCs,MCCs	4	2	2	4	8	
7	I MSAS,MEIOT	4	2	2	4	8	
8	I BVOC	4	2	2	4	8	
9	II MPC EM1	6	-	-	-	6	
10	II MPC EM2	6	-	-	-	6	
11	II MPE	6	-	-	-	6	
12	II MPCs, MECs	6	-	-	-	6	
13	II MCAC,MCPC	6	-	-	-	6	
14	II MSCs,MCCs	6	-	-	-	6	
15	II MSAS,MEIOT	6	-	-	-	6	
16	II BVOC	6	-	-	-	6	
17	Analytical Skills	32	-	-	-	32	
18	III MPC TM	12	-	-	-	12	
19	III MPC EM ,MPE	12	-	-	-	12	
20	III MPCs,MECs	12	-	-	-	12	
21	III MCAC,MEIOT	12	-	-	-	12	
22	III MCPC,MSAS	12	-	-	-	12	
23	III MSCs,MCCs	12	-	-	-	12	
Total Work load for the subject Mathematics						216	

In addition to these hours there are activity hours @ 2 hours for each class for 1st and 2nd years and 1 hour for 3rd year.

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2022-2023 (EVEN SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **252** (actual)
No. of Permanent Posts sanctioned : 05
No. of Permanent staff working : NIL
No. of Contract faculty : 05
No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC EM1	4	2	2	4	8	
2	I MPC EM2	4	2	2	4	8	
3	I MPE	4	2	2	4	8	
4	I MPCs	4	2	2	4	8	
5	I MECs	4	2	2	4	8	
6	I MCAC	4	2	2	4	8	
7	I MCPC	4	2	2	4	8	
8	I MSCs	4	2	2	4	8	
9	I MCCs	4	2	2	4	8	
10	I MSAS	4	2	2	4	8	
11	I MEIOT	4	2	2	4	8	
12	I BVOC	4	2	2	4	8	
13	II MPC EM1	12	-	-	-	12	
14	II MPC EM2	12	-	-	-	12	
15	II MPE	12	-	-	-	12	
16	II MPCs	12	-	-	-	12	
17	II MECs	12	-	-	-	12	
18	II MCAC	12	-	-	-	12	
19 -	II MCPC	12	-	-	-	- 12	
20	II MSCs	12	-	-	-	12	
21	II MCCs	12	-	-	-	12	
22	II MSAS	12	-	-	-	12	
23	II MEIOT	12	-	-	-	12	
24	II BVOC	12	-	-	-	12	
25	III MPC TM	-	-	-	-	-	
26	III MPC EM	-	-	-	-	-	
27	III MPE	-	-	-	-	-	
28	III MPCs	-	-	-	-	-	
29	III MECs	-	-	-	-	-	
30	III MCAC	-	-	-	-	-	
31	III MCPC	-	-	-	-	-	
32	III MSCs	-	-	-	-	-	
33	III MCCs	-	-	-	-	-	
34	III MSAS	-	-	-	-	-	
35	III MEIOT	-	-	-	-	-	
Total Work load for the subject Mathematics						252	

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS
WORK LOAD FOR THE YEAR 2022-2023 (EVEN SEMESTERS)

Name of the Subject : Mathematics
Total No. of Hours : **160** (adjusted)
No. of Permanent Posts sanctioned : 05
No. of Permanent staff working : NIL
No. of Contract faculty : 05
No. of Part – Time Faculty : 03

S. No	Name of the class	No. of Theory hours	No. of Practical Hours	No.of Batches	Total Practical Hours	Total hrs.(Theory + Practical)	Names of the Faculty allotted to the class
1	I MPC EM1	4	2	2	4	8	
2	I MPC EM2	4	2	2	4	8	
3	I MPE	4	2	2	4	8	
4	I MPCs, MECs	4	2	2	4	8	
5	I MCAC,MCPC	4	2	2	4	8	
6	I MSCs,MCCs	4	2	2	4	8	
7	I MSAS,MEIOT	4	2	2	4	8	
8	I BVOC	4	2	2	4	8	
9	II MPC EM1	12	-	-	-	12	
10	II MPC EM2	12	-	-	-	12	
11	II MPE	12	-	-	-	12	
12	II MPCs, MECs	12	-	-	-	12	
13	II MCAC,MCPC	12	-	-	-	12	
14	II MSCs,MCCs	12	-	-	-	12	
15	II MSAS,MEIOT	12	-	-	-	- 12	
16	II BVOC	12	-	-	-	12	
18	III MPC TM	-	-	-	-	-	
19	III MPC EM ,MPE	-	-	-	-	-	
20	III MPCs,MECs	-	-	-	-	-	
21	III MCAC,MEIOT	-	-	-	-	-	
22	III MCPC,MSAS	-	-	-	-	-	
23	III MSCs,MCCs	-	-	-	-	-	
Total Work load for the subject Mathematics						160	

In addition to these hours there are activity hours @ 2 hours for each class for 1st and 2nd years and 1 hour for 3rd year.

P. R. GOVERNMENT COLLEGE (A), KAKINADA

Department of Mathematics

Budget Proposal for the Academic Year 2022-23

S.No.	PURPOSE	EXPENDITURE ESTIMATED	REMARKS
1.	Guest Faculty	14400 x 5 = 10 M = 720000	
2.	National /International Seminars/Workshops/Conference	50000	
3.	Guest Lecture	15000	
4.	Field Trip / Industrial Tour	5000	
5. `	AMC for all Labs	4000	
6.	Certificate / Add on Course	40000	
7.	Seed money	10000	
8.	Furniture (Wooden / Steel)	10000	
9.	Stationary	20000	
10	Computer & peripherals	15000	
	TOTAL	Rs: 9,10,000	

Budget estimated in Rupees : Nine Lakhs ten thousand only

BOS MEMBERS RECOMMENDED SUGGESTIONS

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
KAKINADA 533 001-ANDHRA PRADESH
An AUTONOMOUS and NAAC Accredited Institution(A Grade- 3.17 CGPA)
(Affiliated to ADI KAVI NANNAYA UNIVERSITY, Rajamahendravarm.)

ACADEMIC CELL

(Certificate to be issued by the UniversityNomine/Subject Expert/Member of BOS)

Department Name :MATHEMATICS

Name of the BOS Member :

(UniversityNomine/Subject Expert/Industrilist/ Member)

I certify that the syllabus submitted by the Mathematics Department is verified by me and I recommend the following suggestions:

1. Cyclic groups may be put in Additional Inputs.
2. Bolzano-Weierstrass theorem statement must be included.
3. The Practicals for First year students may be implemented as per
4. the university nominee.
5.

The syllabus is approved with the above suggested modification

V. Ananta [illegible] 2/11/22
Signature with Date

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
KAKINADA 533 001-ANDHRA PRADESH
An AUTONOMOUS and NAAC Accredited Institution(A Grade- 3.17 CGPA)
(Affiliated to ADI KAVI NANNAYA UNIVERSITY, Rajamahendravarm.)

ACADEMIC CELL

(Certificate to be issued by the UniversityNomine/Subject Expert/Member of BOS)

Department Name :MATHEMATICS

Name of the BOS Member : Dr. P. SUBHASHINI, Principal, GDC, Pithapuram.
(UniversityNomine/Subject Expert/Industrilist/ Member)

I certify that the syllabus submitted by the Mathematics Department is verified by me and I recommend the following suggestions:

1. Better to include Absolute convergence and conditional
2. convergence in the IV semester
3. Better to include Practical exams pattern as per
4. university decision
5.

The syllabus is approved with the above suggested modification

Signature with Date

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
KAKINADA 533 001-ANDHRA PRADESH
An AUTONOMOUS and NAAC Accredited Institution(A Grade- 3.17 CGPA)
(Affiliated to ADI KAVI NANNAYA UNIVERSITY, Rajamahendravarm.)

ACADEMIC CELL

(Certificate to be issued by the UniversityNomine/Subject Expert/Member of BOS)

Department Name :MATHEMATICS

Name of the BOS Member :

(UniversityNomine/Subject Expert/Industrilist/ Member)

I certify that the syllabus submitted by the Mathematics Department is verified by me and I recommend the following suggestions:

1. → Kept Bolzano-weierstrass theorem (without proof) in paper IV of Real Analysis
2. → Keep Absolute convergence and condition convergence
3. in paper IV of Real Analysis
4. → Remove chebyshev polynomial unit-1 paper VII-A of special functions
5. → cyclic groups for advanced learners as additional input.

→ follow sop of CIA given by CCE, AP, VJY

The syllabus is approved with the above suggested modification

Signature with Date 02/11/22.

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
KAKINADA 533 001-ANDHRA PRADESH
An AUTONOMOUS and NAAC Accredited Institution(A Grade- 3.17 CGPA)
(Affiliated to ADI KAVI NANNAYA UNIVERSITY, Rajamahendravarm.)

ACADEMIC CELL

(Certificate to be issued by the UniversityNomine/Subject Expert/Member of BOS)

Department Name :MATHEMATICS

Name of the BOS Member : PSR SUBRAHMANYAM

(UniversityNomine/Subject Expert/Industrilist/ Member)

I certify that the syllabus submitted by the Mathematics Department is verified by me and I recommend the following suggestions:

1. Bolzano - Weierstrass theorem (without proof) may be included
2. Absolute convergence & condition conv may be included.
3. cyclic groups may be taught for advanced learners.
4. Chebyshev polynomial may be deleted in Unit I
5.

The syllabus is approved with the above suggested modification

Subrahmanyam
Signature with Date
(PSRSUBRAHMANYAM)


Thank
you