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|---|---|---|---|---|---|---|
| | **P.R.Government College (Autonomous)**<br>**KAKINADA** | **Program&Semester**<br>III B.Sc. (VSem) | | | | |
| CourseCode<br>**MAT-701A / 5281** | **TITLEOFTHECOURSE**<br>**7A -Mathematical Special Functions** | | | | | |
| Teaching | HoursAllocated:60(**Theory**) | L | T | P | C | |
| Pre-requisites: | BasicMathematicsKnowledge on Integration | 6 | 1 | - | 5 | |

## Course Objectives:

This course will cover the particular mathematical functions that have more or less established names and notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications.

## Course Outcomes:

| On Completion of the course, the students will be able to- | |
|---|---|
| CO1 | Understand the Beta and Gamma functions, their properties and relation between these two functions, understand the orthogonal properties of Chebyshev polynomials and recurrence relations. |
| CO2 | Find power series solutions of ordinary differential equations. |
| CO3 | Solve Legendre equation and write the Legendre equation of first kind, also find the generating function for Legendre Polynomials, understand the orthogonal properties of Legendre Polynomials. |
| CO4 | Solve Bessel equation and write the Bessel equation of first kind of order n, also find the generating function for Bessel function understand the orthogonal properties of Bessel unction. |

**Course with focus on employability/entrepreneurship /Skill Development modules**

| | | | | | | |
|---|---|---|---|---|---|---|
| Skill Development | | | Employability | | | Entrepreneurship |

**II. Syllabus: (Hours: Teaching: 75 (incl. unit tests etc. 05), Training: 15)**

**Unit – 1: Beta and Gamma functions. (15h)**

1. Euler's Integrals-Bet and Gamma Functions, Elementary properties of Gamma Functions, Transformation of Gamma Functions.
2. Another form of Beta Function, Relation between Beta and Gamma Functions.

**Unit– 2: Power series and Power series solutions of ordinary differential equations (15h)**

1. Introduction, summary of useful results, power series, radius of convergence, theorems on Power series
2. Introduction of power series solutions of ordinary differential equation
3. Ordinary and singular points, regular and irregular singular points, power series solution about the ordinary point $x = x_0$.

**Unit – 3: Hermite polynomials (15h)**

1. Hermite Differential Equations, Solution of Hermite Equation, Hermite polynomials, generating function for Hermite polynomials.
2. Other forms for Hermite Polynomials, Rodrigues formula for Hermite Polynomials, to find first few Hermite Polynomials.
3. Orthogonal properties of Hermite Polynomials, Recurrence formulae for Hermite Polynomials.

**Unit – 4: Legendre polynomials (15h)**

1. Definition, Solution of Legendre's equation, Legendre polynomial of degree n, generating function of Legendre polynomials. ,
2. Definition of $P_n(x)$ and $Q_n(x)$, General solution of Legendre's Equation (derivations not required )to show that $P_n(x)$ is the coefficient of $h^n$ , in the expansion of $(1 - 2xh + h^2)^{-1/2}$
3. Orthogonal properties of Legendre's polynomials, Recurrence formulas for Legendre's Polynomials.

**Unit – 5: Bessel's equation (15h)**

1. Definition, Solution of Bessel's equation, Bessel's function of the first kind of order n, Bessel's function of the second kind of order n.
2. Integration of Bessel's equation in series form=0, Definitionof $J_n(x)$, recurrence formulae for $J_n(x)$. 3. Generating function for $J_n(x)$, orthogonally of Bessel functions.

Additional Inputs :

Chebyshev Polynomiaals

**II. Reference Books:**

1. Dr.M.D.Raisinghania, Ordinary and Partial Differential Equations, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
2. J.N.Sharma and Dr.R.K.Gupta, Differential equations with special functions, Krishna Prakashan Mandir. 3. Shanti Narayan and Dr.P.K.Mittal, Integral Calculus, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.
4. George F.Simmons, Differential Equations with Applications and Historical Notes, Tata McGRAW-Hill Edition, 1994.
5. Shepley L.Ross, Differential equations, Second Edition, John Willy & sons, New York, 1974.
6. Web resources suggested by the teacher and college librarian including reading material.

**IV. Co-Curricular Activities:**

**A) Mandatory: 1. For Teacher:** Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Beta and Gamma functions.
2. Power series, power series solutions of ordinary differential equations,

3. Procedures of finding series solutions of Hermite equation, Legendre equation and Bessel equation.
4. Procedures of finding generating functions for Hermite polynomials, Legendre Polynomials and Bessel's function.

**2. For Student:** Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work, make observations and conclusions and submit a report not exceeding 10 pages in the given format on the work-done in the areas like the following, by choosing any one of the aspects.

1. Going through the web sources like Open Educational Resources on the properties of Beta and Gamma functions, Chebyshev polynomials, power series solutions of ordinary differential equations. (or)
2. Going through the web sources like Open Educational Resources on the properties of series solutions of Hermite equation, Legendre equation and Bessel equation.

**3. Max. Marks for Fieldwork/Project work Report: 05.**

**4. Suggested Format for Fieldwork/Project work Report:** Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

**5. Unit tests (IE).**

**b) Suggested Co-Curricular Activities:**

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area.

CO-PO Mapping:

**(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)**

| | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PSO1 | PSO2 | PSO3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO1 | 3 | 3 | 2 | 3 | 3 | 3 | 1 | 2 | 2 | 3 | 2 | 3 | 2 |
| CO2 | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 1 | 3 | 3 | 3 | 2 | 1 |
| CO3 | 2 | 3 | 2 | 3 | 2 | 3 | 2 | 2 | 2 | 3 | 2 | 2 | 3 |
| CO4 | 3 | 2 | 3 | 2 | 2 | 1 | 3 | 3 | 1 | 1 | 3 | 1 | 2 |

BLUE PRINT FOR QUESTION PAPER PATTERN,

Skill Enhancement Course (Elective): VII - A

Paper – VII –A :Mathematical Special Functions

| UNIT | TOPIC | S.A.Q (including choice) 5 M | E.Q (including choice) 8 M | Marks Allotted |
|---|---|---|---|---|
| I | Beta and Gamma functions, | 02 | 02 | 30 |
| II | Power series and Power series solutions of ordinary differential equations | 02 | 01 | 20 |
| III | Hermite polynomials | 01 | 01 | 15 |
| IV | Legendre polynomials | 01 | 01 | 15 |
| V | Bessel's equation | 01 | 01 | 15 |
| Total | | 07 | 06 | 95 |

S.A.Q. = Short answer questions (5 marks)
E.Q . = Essay questions ( 10 marks)

Short answer questions : 4 x 5 M = 20
Essay questions : 3 x 10 M = 30

Total Marks : = 50

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**P.R. Government College (Autonomous), Kakinada**

**III Year B.Sc., Degree Examinations - V Semester**

**Mathematics Course : SPECIAL FUNCTIONS**

**Paper -VIIA (Model Paper w.e.f. 2023-24)**

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**Time: 2Hrs** **Max. Marks: 50**

**SECTION-A**

**Answer Any Three Questions, Selecting At Least One Question From Each Part**

**Part – A** **3 X 10 = 30**

1. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
2. Show that $\int_0^\infty (\tan x)^n \, dx = \frac{\pi}{2} sec \frac{n\pi}{2}$ , $0 < n < 1$
3. Find the radius of convergence the exact interval of convergence of the power series $\sum \frac{(n+1)}{(n+2)(n+3)} x^n$

Part – B

4. State and Prove Rodrigues formula for $H_n(x)$.
5. Prove that $\int_{-1}^{1} P_m(x).P_n(x)dx = 0 \;\; if\ m \neq n$. and 2/(2n+1) if m = n
6. Prove that $\sqrt{\frac{\pi x}{2}} J_{3/2}(x) = \frac{1}{x}\sin x - \cos x$.

**SECTION-B**

**Answer any four questions** **4 X 5 M = 20 M**

7. Prove that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-y^{1/n}} dy$ *and hence show that* $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

8. Show that $\Gamma\left(\frac{1}{2}+x\right)\Gamma\left(\frac{1}{2}-x\right) = \frac{\pi}{\cos \pi x}$

9. Find the radius of convergence of the series $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$

10. Show that x = 0 is an ordinary point of $(x^2-1)y'' + xy' - y = 0$ , but x = 1 is a regular singular point .

11. Prove that $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$

12. Prove that $P_3(x) = \frac{1}{2}(5x^3 - 3x)$.

14. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

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